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MODIFICATION OF TRANSITION'S FACTOR IN THE COMPACT SURFACE-POTENTIAL- BASED MOSFET MODEL

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ABSTRACT

The modification of an important transition's factor which enables continual behavior of the surface potential in entire useful range of MOSFET operation is presented. The various modifications have been made in order to obtain an accurate and computationally efficient compact MOSFET model. The best results have been achieved by introducing the generalized logistic function (GL) in fitting of considered factor. The smoothness and speed of the transition of the surface potential from the depletion to the strong inversion region can be controlled in this way. The results of the explicit model with this GL functional form for transition's factor have been verified extensively with the numerical data. A great agreement was found for a wide range of substrate doping and oxide thickness. Moreover, the proposed approach can be also applied on the case where quantum mechanical effects play important role in inversion mode.

Key words: MOSFET modeling, generalized logistic functions, surface potential, quantum mechanical effects.

1. INTRODUCTION

The various compact MOSFET models which satisfy basic requirements such as continuity, accuracy, scalability, and simulation performance have been developed over the years. The most accurate among them are the surface-potential-based MOSFET models (henceforth referred to as SPBMs). These inherently single-piece models are essentially short-channel adaptations of the physically based charge sheet model (Eftimie et al., 2007). Although SPBMs continually describe current and its derivatives in all regions of MOSFET operation, they need an iterative solution of a well-known implicit equation (Cunha et al., 1998). The iterative procedure requires expensive times and represents a significant detriment to implement SPB models in popular circuit simulators.

To overcome these difficulties an explicit approximate solution of the mentioned implicit equation has been proposed in (van Langevelde & Klaassen, 2000). This solution, in its present form, introduces a pure empirical fitting factor to control the smoothness of the surface potential behavior. However, the empirical nature of that factor causes the significant deviations of results of approximate SPBM (van Langevelde & Klaassen, 2000) from the implicit ones, especially in regions near and below the threshold (Chen & Gildenblat, 2001).

The purpose of this work is to replace empirical transition's factor with a function that can be precisely determined for given technological characteristics of the MOSFET devices. The first two proposed functions incorporated in the original SPB model (van Langevelde & Klaassen, 2000) have given excellent results for the surface potential of MOS transistors only for certain technological generation. Then, in order to obtain the model that will be applicable to any MOSFET devices, i.e., which will be technologically mapped, we have introduced GL function in fitting of the observed transition's factor. In this way, control of the smoothness as well as the speed of the surface potential transition from the depletion region to the strong inversion region is enabled. Implementation of proposed GL functional form of mentioned factor in original SPB model removes limits in computational efficiency of the model and also increases its accuracy and continuity.

Moreover, the surface potential values obtained from the resulting model have been verified extensively with the numerical data, and a great agreement was found for a wide range of substrate doping and oxide thickness. Finally, the proposed GL approach can be broaden on the compact MOSFET modeling based on surface potential terms, which

takes into account quantum mechanical effects ((Chaudhry et al., 2010), (Pregaldini et al., 2004)).

2. THE BASIC EXPLICIT SPB MODEL

Consider an n-MOS transistor with gate oxide thickness t_{ox}, and the channel homogenously doped with an acceptor concentration of N_A. Under the gradual channel and charge sheet approximation, for the usual range of the n-MOS operation, the surface potential is related to the terminal voltage V_G through the implicit equation (Arora, 1993):

$$V_G - V_{FB} - \psi_s = \gamma \cdot \sqrt{\psi_s + u_T \cdot \exp\left(\frac{\psi_s - 2\phi_F - V_{ch}}{u_T}\right)} \qquad (1)$$

Here, we denoted:

- V_{FB} is the flat band voltage,
- $\gamma = \sqrt{2qN_A\varepsilon_0\varepsilon_{si}/C_{ox}}$ is the body effect coefficient, where $C_{ox} = \varepsilon_{ox} / t_{ox}$ is the oxide capacitance per unit area, ε_{ox} is the oxide permittivity,
- $u_T = kT / q$ is the thermal voltage,
- $\phi_F = u_T \ln(N_A / n_i)$ is Fermi potential,
- V_{ch} is the channel potential defined by the difference between the quasi-Fermi potentials of the carriers forming the channel (ϕ_n) and that of the majority carriers (ϕ_p).

The Eq. (1) can be solved only numerically. The explicit approximate solution of Eq. (1) has been developed in (van Langevelde & Klaassen, 2000) and is expressed as:

$$\psi_s^*(V_G) = f + u_T \ln\left[\frac{1}{\gamma^2 u_T}\left(V_G - V_{FB} - f - \frac{\psi_{s_{wi}} - f}{\sqrt{1+\left(\frac{\psi_{s_{wi}} - f}{4u_T}\right)^2}}\right)^2 - \frac{f}{u_T} + 1\right]. \qquad (2)$$

Here, $\psi_{s_{wi}}$ is the surface potential in the weak inversion regions and is approximately given by (van Langevelde & Klaassen, 2000):

$$\psi_{s_{ei}}(V_G) = \left(-\frac{\gamma}{2} + \sqrt{V_G - V_{FB} + \frac{\gamma^2}{4}}\right)^2. \qquad (3)$$

In Eq. (2), f is the empirical function which changes smoothly from $\psi_{s_{wi}}$ to $2\phi_F + V_{ch}$, and is given by:

$$f(\psi_{s_{ei}}, \varepsilon) = \frac{2\phi_F + V_{ch} + \psi_{s_{ei}}}{2} - \frac{1}{2}\sqrt{(\psi_{s_{ei}} - 2\phi_F - V_{ch})^2 + 4\varepsilon^2}, \qquad (4)$$

where ε is the fitting factor which controls the smoothness of the transition of the function f from weak inversion region to the onset of the strong inversion region and it is fixed at a value of 0.02V (van Langevelde & Klaassen, 2000).

3. MODIFICATION OF THE FITTING PARAMETER

Below the threshold, i.e. for $V_G < V_T$, the expected equality $f = \psi_{s_{wi}}$ can be fulfill only by reducing the value of fitting factor ε. However, the simple annulment the value of ε would make the transition of the function f abrupt at the threshold voltage. This would jeopardize the smoothness of the behavior both of f and ψ_s^*, as functions of the so-called effective voltage $V_E = V_G - V_T$. Instead that, the constant value of parameter ε can be replaced by a function which varies from a value close to zero in the depletion region, to a value close to 0.02V as the threshold voltage V_T is approached. In that purpose, the next form for parameter ε as the function of effective bias V_E is proposed in (Basu & Dutta, 2006):

$$\varepsilon_m(V_E) = 0.01\left(1 + \frac{V_E + 8u_T}{\sqrt{(V_E + 8u_T)^2 + 0.02}}\right) \qquad (5)$$

Values of ψ_s^*, obtained from Eq. (2), with ε_m given by Eq. (5), show better match with results of the implicit Eq. (1) than ones with 0.02V, as proposed in (van Langevelde & Klaassen, 2000).

On the other side, the application of explicit SPB model with a constant ε as well as with ε_m on the scaled MOSFET devices with thinner gate oxides and higher doping concentrations gives results that differ substantially from the results of implicit SPB model. In order to reduce this difference we have proposed following functional form for transition's factor (Kevkić & Petković, 2009):

$$\varepsilon = 0.02 \cdot \left[1 - \exp\left(-\frac{V_G - V_T + 16u_T}{8u_T}\right)\right]. \qquad (6)$$

The results for the surface potential obtained from explicit SPB model with ε given by Eq. (6) are accurate particularly in the case of submicron scaled MOSFET devices. The function (6) increases slower than (5) as we can see from Fig. 1.

However, as CMOS technology scales down aggressively, it approaches a point where quantum mechanical effects become significant. In this case ε must be very quickly approached to the value of 0.02V, so we suggest the next function for transition's smoothing factor (Kevkić & Petković, 2010):

$$\varepsilon = 0.02 \cdot \left[1 - \frac{1}{2} \cdot \exp\left(-\frac{V_G - V_T + 8u_T}{4u_T}\right)\right]. \qquad (7)$$

The transition's factor ε versus gate voltage V_G calculated according to the relations (5), (6) and (7) are shown in Fig.1.

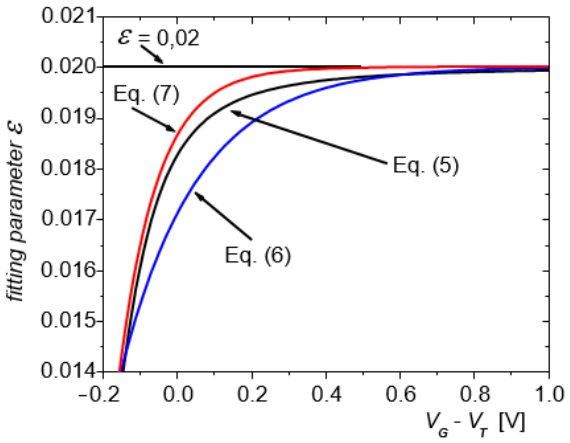

Fig. 1. Fitting parameter ε versus gate voltage V_G, calculated according relations (5), (6) and (7).

Based on the above it is clear that different technological characteristics of MOSFETs require changes of factor ε from 0 to 0.02 with various speeds. In the other words, the manner and speed of continual transition of the function f, and consequently of ψ_s^* between weak and strong inversion determine different

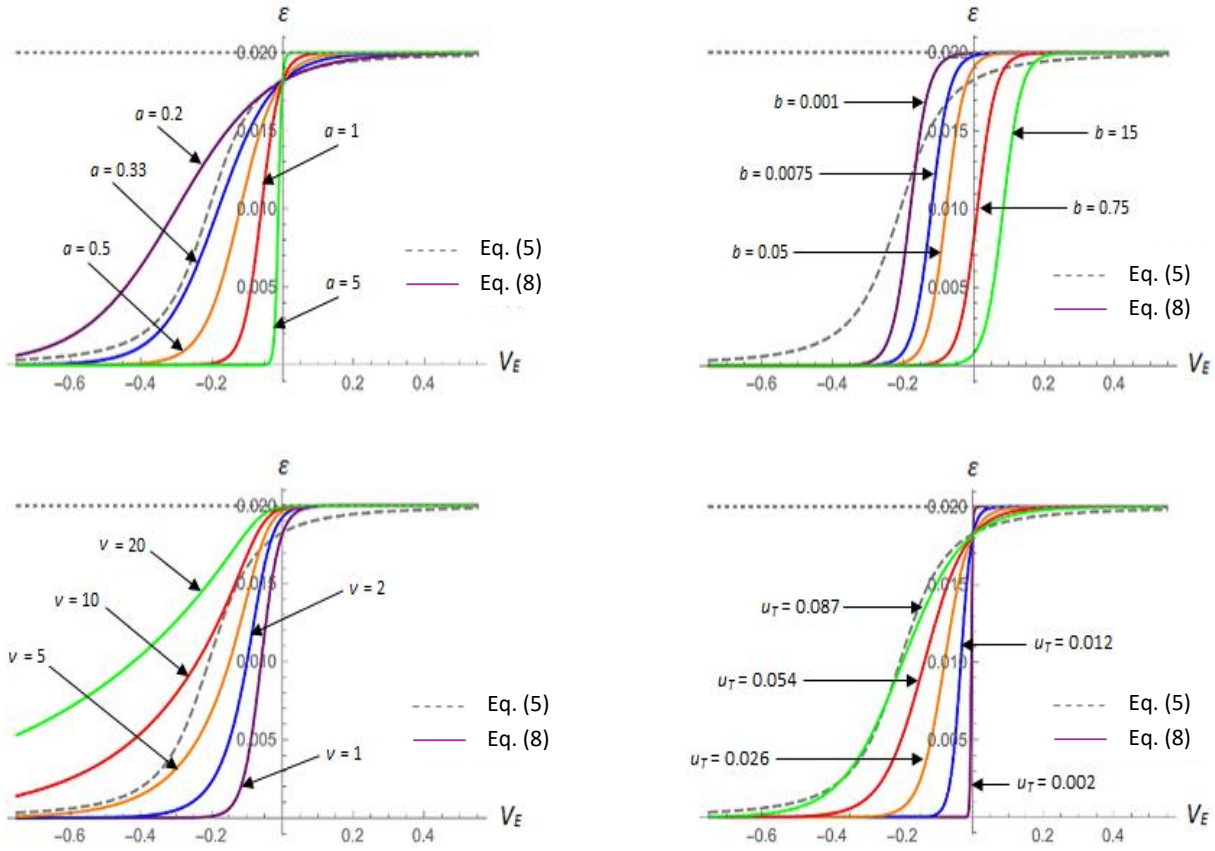

Fig. 2. Graphs of the GL-fitting parameter ε vs. effective voltage $V_E = V_G - V_T$, by using the Eq. (8) (solid lines), compared to the fitting model proposed in Eq. (5) (dashed line). Above left diagram: Varying of the parameter a (b = 0.1, v = 1, u_T = 0.026). Above right diagram: Varying of the parameter b (a = v = 1, u_T = 0.026). Lower left diagram: Varying of the parameter v (a = 1, b = 0.1, u_T = 0.026). Lower right diagram: Varying of the parameter u_T (a = v = 1, b = 0.1).

combinations of gate oxide thickness t_{ox} and dopant concentration N_A of device. Especially, thinner gate oxides and higher dopant concentration of the modern MOSFET devices require faster and smoother transitions. It is clear that technologically mapped SPB model can be obtained if sensitivity of the speed and also the way of the transition of ε on changes in the technological characteristics of MOSFETs are taken into account. Several simulations have shown that it can be achieved by introducing new the *Generalized Logistic (GL)* functional form for ε (Kevkić et al., 2015):

$$\varepsilon_{GL}(V_E) = \frac{0.02}{\left(1 + b\exp\left(-a\frac{V_E}{u_T}\right)\right)^{1/\nu}}. \qquad (8)$$

Here, $a, \nu > 0$ are *the growth parameters*, and $b > 0$ is parameter which determines the shift of the GL curve, related to the value.

$$\varepsilon_{GL}(0) = 0.02(1+b)^{-1/\nu} \lesssim 0.02. \qquad (9)$$

The parameters a, b can be obtained by using a fitting procedure in accordance to MOSFETs technical characteristics (Kevkić et al., 2015). Unlike them, the parameter $\nu > 0$ will be determined in advance, according to the condition of the asymptotic linear growth of the following, *the Logit-function*:

$$L(V_E/u_T) := \ln\frac{\varepsilon_{GL}^{\nu}(V_E)}{0.02^{\nu} - \varepsilon_{GL}^{\nu}(V_E)} = a\frac{V_E}{u_T} - \ln b.$$

The linear form of function $L(V_E/u_T)$ provides easily obtaining the GL-fitted factor ε_{GL}, given by Eq. (8), by using some of the standard fitting techniques (Jukić & Scitovski, 2003). The graphs of $\varepsilon_{GL}(V_E)$, for varying values of a, b, ν, as well as the thermal voltage u_T, are shown in Fig. 2. The graphs of $\varepsilon_m(V_E)$ are also shown in this figure for comparison. As can be readily seen, the diverse and adapted transitions of ε_{GL} from 0 to 0.02 have been realized with simple changes in values of the parameters a, b, ν.

4. VALIDATION OF THE GL-FITTING

The advantages of introducing the GL function in the original explicit SPB model (van Langevelde & Klaassen, 2000) are shown in Table 1 via the mean values of some typical error functions for the transition's factor ε obtained by using Eq. (5) and Eq. (8), and accordingly for the approximate surface potential ψ_s^*. The first two rows of the table show the average values of *the Absolute Error* (*AE*) which is defined as the absolute value of the difference between fitted and reference values of the factor ε, as well as between the corresponding approximate surface potential ψ_s^*, and ψ_s obtained from implicit Eq. (1). The average values of Fractional Error (FE) are show in third and fourth rows of the Table 1. The FE presents the percentage value of the ratio of AE to the reference values of ε and ψ_s. Finally, the last two rows contain average values of the typical statistics error labeled as the Squared Error (SE), which is commonly used in approximation theory. As we can see, all the estimated errors are obviously smaller in the case of GL-fitted factor ε_{GL}, and it is particularly pronounced in the weak inversion region.

Table 1. Estimated errors obtained by various types of the ε-fitting. Device parameters are: t_{ox} = 2.3nm, N_A = 10^{18}cm^{-3}, V_{FB} = -1V, V_{ch} = 0V, u_T = 0.026V.

Errors	Region	ε-fitting		Surface potential approximation (ψ_s^*)	
		Eq.(5)	Eq.(8)	Eq.(5)	Eq.(8)
AE	w.i.	8.62E-04	1.00E-04	5.21E-05	2.71E-06
	s.i.	1.75E-02	1.73E-02	2.23E-03	2.21E-03
FE	w.i.	8.947	0.930	5.74E-03	3.08E-04
	s.i.	46.60	46.01	2.09E-01	2.08E-01
SE	w.i.	1.10E-06	1.40E-08	1.38E-08	2.02E-11
	s.i.	3.17E-04	3.10E-04	5.01E-06	4.92E-06

The absolute errors, defined as $AE = |\psi_s - \psi_s^*|$, are plotted versus V_G in logarithmic scales in Fig. 3. The values of the explicit surface potential ψ_s^* are obtained from Eq. (2) with ε_m, then with ε_{GL} as well as with the constant value of 0.02V, respectively. The both diagrams show that the best matches with reference values are achieved by using the GL-fitted transition's factor.

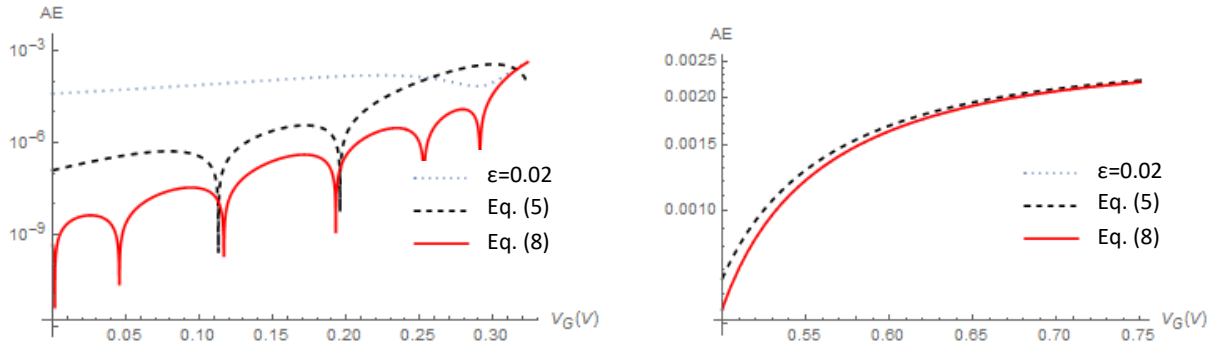

Fig. 3. Absolute errors of the surface potential, $AE = |\psi_s - \psi_s^*|$, vs. V_G in the weak inversion region (left diagrams) and in the strong inversion region (right diagrams). Device parameters are: t_{ox}=2.3nm, N_A = 10^{18}cm^{-3}, V_{FB} = -1V, V_{ch} = 0V, u_T = 0.026V.

5. Modeling the Increase in Surface Potential due to the Quantum Mechanical Effects

The developed GL approach can be applied in the case where severe band bending on the Si side of the Si–SiO_2 interface confines the carriers in a narrow potential well. The energy levels of the carriers are grouped in sub-bands what results in increasing of threshold voltage and surface potential compared to the classical case where the energy band is not split. The decreasing of the inversion charge density and other quantum mechanical effects (QME) are also observed in that case.

The SPB models which include QME require the solution of the Schrödinger's and Poisson's equations which gives the correction of the surface potentials caused by the energy quantization process in the substrate. Under the assumption that only the lowest subband is occupied by the electrons, the variational approach gives a good estimation of the eigen energy of the lowest subband (Stern, 1972):

$$E_0 = (3\hbar^2 b^2)/(8m_{zz}).$$

Here, b is the variational parameter and is obtained by minimizing the ground state energy of the lowest subband with respect to it ((Stern, 1972), (Sho et al., 2016)), m_{zz} is the longitudinal effective mass of electrons, and $\hbar$ is reduced Planck's constant.

The QM correction of the surface potential is equal to the ratio of the energy E_0 and the elementary charge of the electron q:

$$\delta\psi_s = E_0 / q.$$

that the weak inversion approximation gives inaccurate results in the strong inversion region.

Thus, the main task is reduced to finding $\delta\psi_s$ as a function of the applied terminal voltages, and its adequate inclusion in the original SPB model (van Langevelde & Klaassen, 2000). A good way to do that is modification of the function f given by Eq. (4), so that it varies from $\psi_{s_{wi}}$ in weak inversion to $2\varphi_F + V_{ch} + \delta\psi_s$ from the onset of strong inversion. The modified smoothing function has the following form:

$$f_{GL} = \frac{\psi_{S_{wi}}}{2} + \frac{2\phi_F + V_{ch} + \delta\psi_s}{2} - \frac{1}{2}\sqrt{\left(\psi_{S_{wi}} - 2\phi_F - V_{ch} - \delta\psi_s\right)^2 + 4\varepsilon_{GL}^2}, \quad (11)$$

where ε_{GL} is given by Eq. (8). To get an explicit QME incorporated SPB model, we only need to use the function f_{GL} in place of f in Eq. (2).

The results of explicit QME incorporated SPB model as a function of the gate voltage V_G - V_{FB} are shown in Fig. 4. The results of implicit SPB model and weak inversion approximation, given by Eq. (3), are also depicted in this figure. As we can see, the deviation between quantum and classical results is the most obvious in the strong inversion region. This deviation will become much more significant with increasing of dopant concentration and decreasing the thickness of gate oxides, i.e. with greater influence of quantum mechanical effects on the MOSFET's operation. Additionally, from Fig. 4 is quite evident

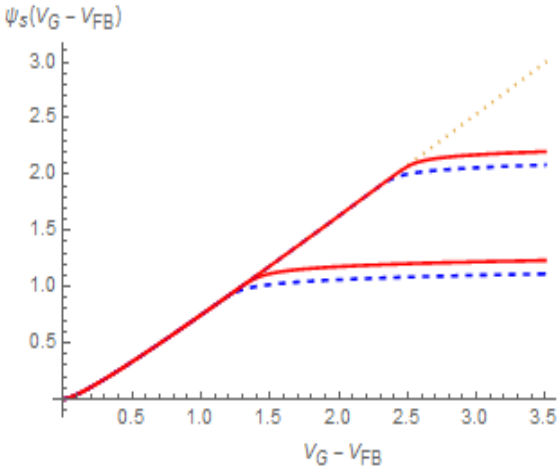

Fig. 4. The electrostatic surface potential ψ_s vs. gate voltage V_G - V_{FB} obtained from the QM incorporated explicit SPB model with ε_{GL} (solid line). The results of the weak inversion approximation (3) (dotted line), and the classical implicit ψ_s model (dashed line) are also shown.

5. Conclusion

The explicit surface- potential -based MOSFET model has been modified in order to increase its accuracy, continuity and simulation performances. The modifications consist in new functional forms for important smoothing factor which controls continuity of the surface potential transition from depletion to strong inversion region. Pure empirical parameter has been avoided and an accurate as well as technology mapped model has been obtained by introducing the generalized logistic function in fitting of mentioned smoothing factor. Very important is that the complexity of calculations increases only marginally over the similar advanced models reported in the literature. The results of the surface potential values obtained from the proposed model have been verified extensively with the numerical results of classical implicit equation on which are based all known SPB models, and a great agreement was found. Moreover, the application of GL-fitted model can be broadened to the case where quantum mechanical effects become important. Finally, the validity of the model was proven by comparisons with full numerical solutions data from different advanced CMOS technologies.

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