

SPECTRAL CHARACTERISTICS OF TWO PARAMETER FIFTH DEGREE POLYNOMIAL CONVOLUTION KERNEL

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ABSTRACT

In this paper, the spectral characteristic of a polynomial two parameter convolutional fifth - order interpolation kernel is determined. The spectral characteristic is determined as follows. First, the kernel is decomposed into components. After that, the spectral characteristics of each kernel component were calculated using the Fourier transform. Finally, the spectral characteristic of the interpolation two parameter kernel using a combination of the spectral components of the kernels and the kernel parameters, α and β , is formed. Through a numerical example and a graphical representation of the spectral characteristics of the one parameter and two parameter kernels, greater similarity of the spectral characteristics of the 2 P kernel, relative to the ideal box characteristic, is shown.

Keywords: Convolution, Interpolation, PCC interpolation, Polynomial kernel.

INTRODUCTION

Theoretical analyzes of convolutional interpolation have shown that, for interpolation of band limited signals, the interpolation kernel $\text{sinc}(x) / x$ (in the notation *sinc*) should be applied (Keys, 1981). The *sinc* interpolation kernel is called the ideal interpolation kernel. The boundaries of the *sinc* interpolation kernel are $-\infty \leq x \leq +\infty$ (Meijering & Unser, 2003). The spectral characteristic of the *sinc* interpolation kernel is a rectangular or, in some notations, box function. From a practical point of view, it is not possible to implement a kernel with boundaries $-\infty \leq x \leq +\infty$. The solution to this problem was found in the truncated sinc kernel, so that the kernel becomes a finite length L . The process of truncated the kernel to finite length is called windowizing (Dodgson, 1997). Truncated distinguishes the spectral characteristic of the interpolation kernel from the ideal spectral characteristic, ie from the box characteristic. The spectral characteristics of the truncated kernel, *sincw*, have: a) ripple in the passband and stopband, and b) finite slope in the transition band.

In order to reduce the numerical complexity of the interpolation kernel, and thus reduce the interpolation time, approximation of the *sinc* kernel with low-degree polynomial functions is performed. Reducing of the interpolation time is especially important for convolutional interpolation in real-time systems. A zero-degree polynomial kernel performs nearest-neighbor interpolation (Dodgson, 1997). The nearest-neighbor interpolation has a very high computational speed. However, a large interpolation error is generated (Rukundo & Maharaj, 2014).

A linear, first-degree interpolation kernel is described in (Rifman, 1973). A quadratic, second-degree interpolation kernel is described in (Dodgson, 1997) and (Deng, 2010). A cubic, third-degree interpolation kernel is described in (Keys, 1981). Detailed numerical analysis of the interpolation error, which is presented in great detail in (Keys, 1981), showed that the interpolation error, when the cubic kernel was applied, is smaller than the interpolation errors when the nearest-neighbor and linear interpolation kernels were applied. Thus, precision, as one of the parameters for estimating the quality of interpolation, is increased.

A cubic kernel with interpolation parameter α is shown in (Keys, 1981). Later, in the scientific literature, the one parametric interpolation kernel from (Keys, 1981), in honor of the author Roberts B. Keys, the 1P Keys interpolation kernel, was named. By adjusting of the parameter α it is possible to minimize of the interpolation error in various applications (image interpolation, audio signal interpolation, etc.) The process of changing the kernel parameter for customization is called parameter optimization. Changing the kernel parameter α affects, among other things, to the ripple of the spectral characteristic. Reduction of ripples was achieved by eliminating members of the Taylor series that predominantly influence on the ripple (Park & Schowengerdt, 1982). The analysis presented in (Meijering et al., 1999) indicates that $\alpha = -0.5$ is the optimal value of the kernel parameter for reducing the ripple of the spectral characteristic in the passband and stopband. Convolution interpolation realized by the parameterized cubic kernel is called PCC (Parametric Cubic Convolution) interpolation.

Increasing of the interpolation precision of the cubic interpolation kernel by constructing a two parameter (α, β) interpolation kernel (2P), was achieved (Hanssen & Bamler, 1999). The two parameter interpolation kernel is based on the

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additional parameterization of the 1P Keys kernel. In the scientific literature, this kernel is called the 2P Keys kernel. Optimization of the 2P Keys kernel parameters in the estimation of the fundamental frequency, F_0 , of the speech signal was determined in (Milivojević & Brodić, 2013) and (Milivojević et al., 2017). As a measure of the error estimate of the fundamental frequency MSE was used.

One parameter fifth-degree interpolation kernel, length $L=8$, described in (Meijering et al., 1999). In order to increase of the precision of interpolation, the construction of a two parameter fifth-degree interpolation kernel was performed (Savić et al., 2021). The 2P fifth-degree interpolation kernel is constructed by the extended parameterization of the 1P fifth-degree interpolation kernel. The optimal parameters of the 2P fifth-degree interpolation kernel, when interpolating the audio signal, ($\alpha_{opt} = 0.1$, $\beta_{opt} = 0.0571$), were determined using the experiment (Savić et al., 2022). The precision of the interpolation at 2P (MSE = $1.3253 \cdot 10^{-6}$) is higher than the precision of the interpolation at 1P kernel (MSE = $1.8096 \cdot 10^{-6}$).

In the previously cited papers, the analytical form of the spectral characteristic of the 2P fifth degree polynomial kernel, has not been determined. In this paper, the spectral characteristic of the 2P kernel, whose parameterization was done in (Savić et al., 2021), was calculated. The spectral characteristic of the 2P kernel is done as follows. It is first done by decomposing the 2P kernel into components. Then, by applying the Fourier transform to each kernel component, the spectral characteristics of each component were determined. In this way, the spectral components of the kernel are determined. Finally, taking into account all spectral components as well as the kernel parameters α and β , the spectral characteristic of the 2P kernel was determined. With the knowledge of the analytical form of the spectral characteristic, it is possible to change its shape by changing the kernel parameters, in order to bring its shape closer to the ideal box characteristic. In addition, the precision of interpolation in Digital Image, Audio and Speech processing can be affected by changing the kernel parameters, and thus minimize the interpolation error. This optimizes the kernel parameters. In this way, the scope of application of parametric convolutional kernels is increased.

Further organization of this paper is as follows. Section II describes: ideal kernel, 1P fifth order kernel, and 2P fifth order interpolation kernel. Section III describes the process of determining the spectral characteristic of the kernel. Section IV is the Conclusion

INTERPOLATION KERNELS

Ideal interpolation kernel

The ideal interpolation kernel for interpolating band limited signals is the form $sinc = \sin(x) / x$ (Keys, 1981). The definition of the $sinc$ kernel is in the interval $(-\infty, +\infty)$.

Therefore, due to its infinite length, the $sinc$ kernel cannot be realized. For this reason, it was necessary to truncated the $sinc$ interpolation kernel to a finite length L (interval $[-L/2, L/2]$). The truncated $sinc$ kernel, $sincw$, has a spectral characteristic that differs from the box characteristic. The difference is reflected in: a) the appearance of ripples of the spectral characteristics in the passband and stopband and b) finite slope of the spectral characteristics in the transition band (Savić et al., 2021). The ideal interpolation kernel $sinc$ in the range $(-20, 20)$ is shown in Fig. 1a. The spectral characteristics of the truncated kernel $sincw$ for some lengths L (3, 5, 10, 20, $+\infty$) are shown in Figs. 1.b. The spectral characteristics of the ideal kernel $sinc$ ($L \rightarrow \infty$) and the spectral characteristics in which the ripple increases with decreasing length L are indicated.

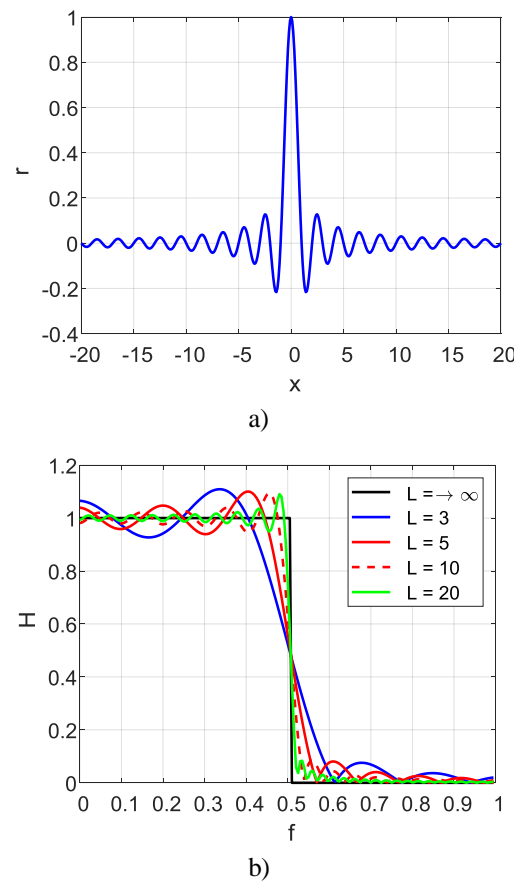


Figure 1. Interpolation kernel: a) time domain and b) spectral domain.

In order to reduce the numerical complexity, the $sinc$ function is approximated by simpler mathematical functions. In the field of the Digital Signal Processing, DSP, and especially in the field of the Digital Image Processing, the approximation of the $sinc$ function with polynomial functions is intensively used. In the field of Digital Image Processing, the interpolation kernels, formed from the third-order polynomials, are very popular. The most significant are the parameterized 1P, 2P and 3P Keys kernels (Hanssen & Bamler, 1999). The need to increase precision of the

interpolation has led to the construction of fifth-order polynomial interpolation kernels.

In the further part of this paper, the definition of one parameter and two parameter fifth-order kernels and their parameterization is described. After that, the spectral characteristic of the two parameter kernel was determined by applying the Fourier transform.

One parameter fifth-order kernel

The general form of the convolutional fifth-degree interpolation kernel is (Meijering et al., 1999):

$$r(x) = \begin{cases} a_5 |x|^5 + a_4 |x|^4 + a_3 |x|^3 + a_2 |x|^2 + a_1 |x| + a_0, & |x| \leq 1 \\ b_5 |x|^5 + b_4 |x|^4 + b_3 |x|^3 + b_2 |x|^2 + b_1 |x| + b_0, & 1 < |x| \leq 2 \\ c_5 |x|^5 + c_4 |x|^4 + c_3 |x|^3 + c_2 |x|^2 + c_1 |x| + c_0, & 2 < |x| \leq 3 \\ d_5 |x|^5 + d_4 |x|^4 + d_3 |x|^3 + d_2 |x|^2 + d_1 |x| + d_0, & 3 < |x| \leq 4 \\ 0, & |x| > 4 \end{cases} \quad (1)$$

The coefficients of the kernel are determined subject to the following conditions: a) $r(0) = 1$, b) $r(x) = 0$ for $|x| = 1, \dots, 4$; and c) $r^{(l)}(x)$ are continuous for $|x| = 0, \dots, 4$. By parameterizing of the kernel, taking into account the previous conditions, form of the one parameter fifth-degree interpolation kernel is determined. The form of the 1P fifth-degree interpolation kernel is:

$$r(x) = \begin{cases} \left(10\alpha - \frac{21}{16}\right)|x|^5 + \left(-18\alpha + \frac{45}{16}\right)|x|^4 + \left(8\alpha - \frac{5}{2}\right)|x|^2 + 1, & |x| \leq 1 \\ \left(11\alpha - \frac{5}{16}\right)|x|^5 - \left(88\alpha - \frac{45}{16}\right)|x|^4 + (270\alpha - 10)|x|^3 - \left(392\alpha - \frac{35}{2}\right)|x|^2 + (265\alpha - 15)|x| - (66\alpha - 5), & 1 < |x| \leq 2 \\ \alpha|x|^5 - 14\alpha|x|^4 + 78\alpha|x|^3 - 216\alpha|x|^2 + 297\alpha|x| - 162\alpha, & 2 < |x| \leq 3 \\ 0, & |x| > 3 \end{cases} \quad (2)$$

where α is the kernel parameter.

Two parameter fifth-order kernel

In order to increase of the interpolation precision, a two parameter fifth-degree convolutional kernel is created. The process of kernel parameterization is presented in great detail

in (Savić et al., 2021). The final form of the parametric 2P kernel is:

$$r(x) = \begin{cases} \left(10\alpha - 10\beta - \frac{21}{16}\right)|x|^5 + \left(-18\alpha + 18\beta + \frac{45}{16}\right)|x|^4 + \left(8\alpha - 8\beta - \frac{5}{2}\right)|x|^2 + 1, & |x| \leq 1 \\ \left(11\alpha - 11\beta - \frac{5}{16}\right)|x|^5 + \left(-88\alpha + 88\beta + \frac{45}{16}\right)|x|^4 + (270\alpha - 270\beta - 10)|x|^3 + \left(-392\alpha + 392\beta + \frac{35}{2}\right)|x|^2 + (265\alpha - 265\beta - 15)|x| + (-66\alpha + 66\beta + 5), & 1 < |x| \leq 2 \\ \alpha|x|^5 + (-14\alpha + 3\beta)|x|^4 + (78\alpha - 30\beta)|x|^3 + (-216\alpha + 112\beta)|x|^2 + (297\alpha - 185\beta)|x| + (-162\alpha + 114\beta), & 2 < |x| \leq 3 \\ \beta|x|^5 - 19\beta|x|^4 + 144\beta|x|^3 - 544\beta|x|^2 + 1024\beta|x| - 768\beta, & 3 < |x| \leq 4 \\ 0, & |x| > 4 \end{cases} \quad (3)$$

where α and β are kernel parameters.

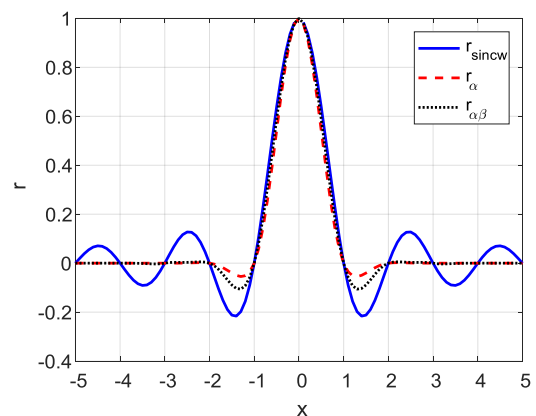


Figure 2. Truncated *sinc* ideally kernel, r_{sincw} , and fifth-order polynomial kernels: a) one parameter, r_α , and b) two parameter, $r_{\alpha\beta}$.

In fig. 2 are shown: a) truncated, i.e. windowizing ideally kernel, r_{sincw} , and b) 1P kernel, r_α , (Eq. (2)) and 2P kernel $r_{\alpha\beta}$ (Eq. (3)). Kernel parameters are $\alpha = 0.025$, $\beta = -0.04$. It can be

observed that the form of the two parameter, $r_{\alpha\beta}$, has a smaller deviation compared to the kernel r_{sinc} .

SPECTRAL CHARACTERISTICS OF THE 2P FIFTH-ORDER INTERPOLATION KERNEL

The algorithm for determining the spectral characteristic of the fifth-order kernel (Eq. (3)) was implemented in the following steps: a) decomposing the kernel into components, b) determining the spectral characteristics of each kernel component, and c) determining the spectral characteristics of the kernel depending on the parameters α and β .

2P kernel components

The 2P fifth-degree kernel (Eq. (3)) can be written in the form:

$$r(x) = r_0(x) + \alpha r_1(x) + \beta r_2(x), \quad (4)$$

where r_0 , r_1 and r_2 are kernel components:

$$r_0(x) = \begin{cases} \frac{-21}{16}|x|^5 + \frac{45}{16}|x|^4 - \frac{5}{2}|x|^2 + 1, & |x| \leq 1 \\ \frac{5}{16}|x|^5 + \frac{45}{16}|x|^4 - 10|x|^3 + \frac{35}{2}|x|^2 - 15|x| + 5, & 1 < |x| \leq 2, \\ 0, & |x| > 2 \end{cases} \quad (5)$$

$$r_1(x) = \begin{cases} 10|x|^5 - 18|x|^4 + 8|x|^2, & 0 < |x| \leq 1 \\ 11|x|^5 - 88|x|^4 + 270|x|^3 - 392|x|^2 + 265|x| - 66, & 1 < |x| \leq 2 \\ |x|^5 - 14|x|^4 + 78|x|^3 - 216|x|^2 + 297|x| - 162, & 2 < |x| \leq 3 \\ 0; & |x| > 3 \end{cases} \quad (6)$$

and

$$r_2(x) = \begin{cases} -10|x|^5 + 18|x|^4 - 8|x|^2 & 0 \leq |x| \leq 1 \\ -11|x|^5 + 88|x|^4 - 270|x|^3 + 392|x|^2 - 265|x| + 66 & 1 < |x| \leq 2 \\ 3|x|^4 - 30|x|^3 + 112|x|^2 - 185|x| + 114 & 2 < |x| \leq 3 \\ |x|^5 - 19|x|^4 + 144|x|^3 - 544|x|^2 + 1024|x| - 768 & 3 < |x| \leq 4 \\ 0 & |x| > 4 \end{cases} \quad (7)$$

As an example, the kernel components r_0 , r_1 and r_2 are shown in Fig. 3.a. The ideal interpolation kernel, r_{sinc} , and the

2P fifth degree kernel, $r_{\alpha\beta}$, for $\alpha = 0.025$ and $\beta = -0.04$ (Eq. (4)), on the interval $[-5, 5]$, are shown in Fig. 3.b.

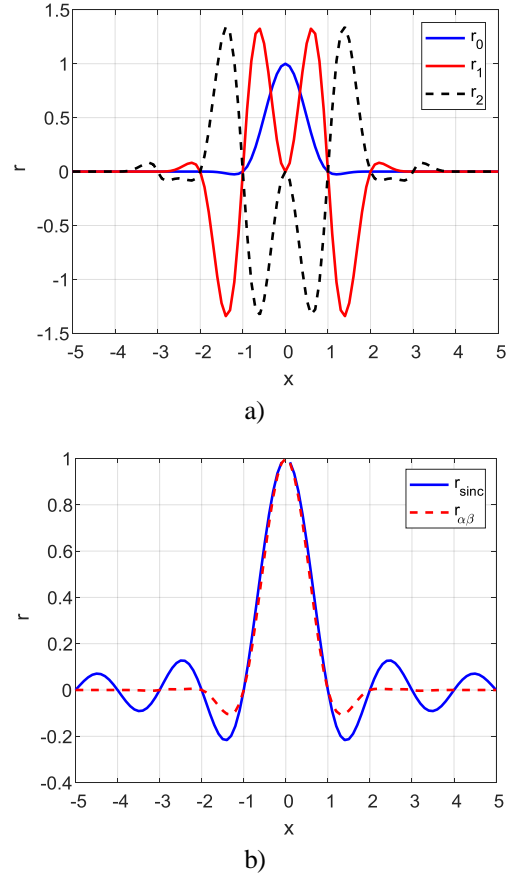


Figure 3. Form of: a) kernel components r_0 , r_1 and r_2 , and b) ideal interpolation kernel, r_{sinc} , and the 2P fifth degree kernel, $r_{\alpha\beta}$, for $\alpha = 0.025$ and $\beta = -0.04$.

Spectral characteristic of the kernel component

By applying of the Fourier transform over the 2P kernel r (Eq. 4), the spectral characteristic of the kernel is obtained:

$$H(f) = FT(r_0(x) + \alpha r_1(x) + \beta r_2(x)) = H_0(f) + \alpha H_1(f) + \beta H_2(f) \quad (8)$$

where H_0 , H_1 and H_2 are the spectral components of the 2P kernel, respectively:

$$H_0(f) = \int_{-\infty}^{\infty} r_0(x) e^{-2\pi x f i} dx, \quad (9)$$

$$H_1(f) = \int_{-\infty}^{\infty} r_1(x) e^{-2\pi x f i} dx, \quad (10)$$

and

$$H_2(f) = \int_{-\infty}^{\infty} r_2(x) e^{-2\pi x f i} dx. \quad (11)$$

By replacing Eq. (5), (6) and (7) in (9), (10) and (11), respectively, the spectral components become:

$$H_0(f) = \int_{-2}^{-1} \left(\frac{5}{16}x^5 + \frac{45}{16}x^4 + 10x^3 + \frac{35}{2}x^2 + 15x + 5 \right) e^{-2\pi xfi} dx + \int_{-1}^0 \left(\frac{21}{16}x^5 + \frac{45}{16}x^4 - \frac{5}{2}x^2 + 1 \right) e^{-2\pi xfi} dx + \int_0^1 \left(\frac{-21}{16}x^5 + \frac{45}{16}x^4 - \frac{5}{2}x^2 + 1 \right) e^{-2\pi xfi} dx + \int_1^2 \left(\frac{-5}{16}x^5 + \frac{45}{16}x^4 - 10x^3 + \frac{35}{2}x^2 - 15x + 5 \right) e^{-2\pi xfi} dx, \quad (12)$$

$$H_1(f) = \int_{-3}^{-2} \left(-x^5 - 14x^4 - 78x^3 - 216x^2 - 297x - 162 \right) e^{-2\pi xfi} dx + \int_{-2}^{-1} \left(-11x^5 - 88x^4 - 270x^3 - 392x^2 - 265x - 66 \right) e^{-2\pi xfi} dx + \int_{-1}^0 \left(-10x^5 - 18x^4 + 8x^2 \right) e^{-2\pi xfi} dx + \int_0^1 \left(10x^5 - 18x^4 + 8x^2 \right) e^{-2\pi xfi} dx + \int_1^2 \left(11x^5 - 88x^4 + 270x^3 - 392x^2 + 265x - 66 \right) e^{-2\pi xfi} dx + \int_2^3 \left(x^5 - 14x^4 + 78x^3 - 216x^2 + 297x - 162 \right) e^{-2\pi xfi} dx, \quad (13)$$

and

$$H_2(f) = \int_{-4}^{-3} \left(-x^5 - 19x^4 - 144x^3 - 544x^2 - 1024x - 768 \right) e^{-2\pi xfi} dx + \int_{-3}^{-2} \left(3x^4 + 30x^3 + 112x^2 + 185x + 114 \right) e^{-2\pi xfi} dx + \int_{-2}^{-1} \left(11x^5 + 88x^4 + 270x^3 + 392x^2 + 265x + 66 \right) e^{-2\pi xfi} dx + \int_{-1}^0 \left(10x^5 + 18x^4 - 8x^2 \right) e^{-2\pi xfi} dx + \int_0^1 \left(-10x^5 + 18x^4 - 8x^2 \right) e^{-2\pi xfi} dx + \int_1^2 \left(-11x^5 + 88x^4 - 270x^3 + 392x^2 - 265x + 66 \right) e^{-2\pi xfi} dx + \int_2^3 \left(3x^4 - 30x^3 + 112x^2 - 185x + 114 \right) e^{-2\pi xfi} dx + \int_3^4 \left(x^5 - 19x^4 + 144x^3 - 544x^2 + 1024x - 768 \right) e^{-2\pi xfi} dx, \quad (14)$$

Finally, the spectral components of the 2P kernel are

$$H_0 = \frac{15 \sin(f\pi)}{32f^6\pi^6} \left(-2f\pi(17 \cos(f\pi) + \cos(3f\pi)) + 21 \sin(f\pi) + 5 \sin(3f\pi) \right), \quad (15)$$

$$H_1 = \frac{3 \sin(2f\pi)}{2f^6\pi^6} \left(66f\pi + 50 \sin(2f\pi) - 5 \sin(4f\pi) + 2f\pi(26 \cos(2f\pi) + \cos(4f\pi)) \right), \quad (16)$$

$$H_2 = \frac{\sin(2f\pi)}{2f^6\pi^6} \left(-2f\pi(87 + 4f^2\pi^2 + 72 \cos(2f\pi)) - 150 \sin(2f\pi) + 15 \sin(4f\pi) - 15 \sin(6f\pi) + 2f\pi((21 - 8f^2\pi^2) \cos(4f\pi) + 3 \cos(6f\pi)) \right). \quad (17)$$

In fig. 4 shows the spectral components of the 2P kernel, H_0 , H_1 and H_2 .

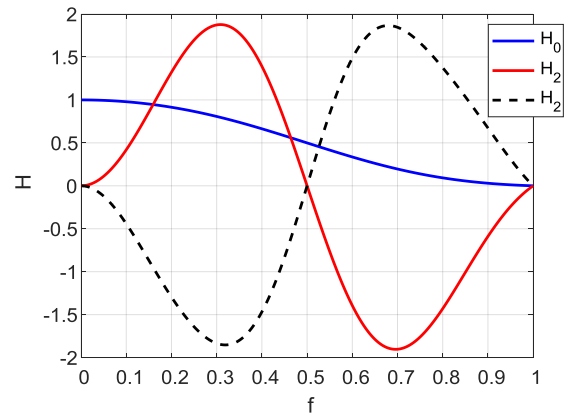


Figure 4. Spectral components of the 2P kernel, H_0 , H_1 and H_2 .

Spectral characteristics of the 2P kernel

The spectral characteristic of the 2P kernel, $H_{\alpha\beta}$, was obtained by substituting Eq. (15), (16) and (17) into (8). As an example, the spectral characteristics of: a) ideal interpolation kernel, H_{sinc} , and b) 1P (H_α) and 2P ($H_{\alpha\beta}$) kernels, for $\alpha = 0.025$ and $\beta = -0.04$, are shown in Fig. 5.

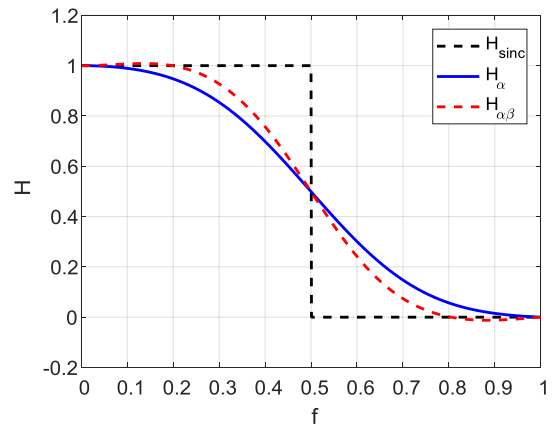


Figure 5. Spectral characteristics: a) ideal kernel, H_{sinc} , and b) 1P kernel, H_α , and 2P kernel, $H_{\alpha\beta}$, for kernel parameters $\alpha = 0.025$ and $\beta = -0.04$.

It can be seen that the spectral characteristic of the 2P kernel, $H_{\alpha\beta}$, in relation to the spectral characteristic of the 1P kernel, H_{α} , has a better similarity with the box characteristic.

Further research will be based on optimizing the spectral characteristics of the 2P kernel in the spectral domain. The optimization will be aimed at minimizing the ripple of the spectral characteristic, and thus, towards greater similarity with the ideal box spectral characteristic.

CONCLUSION

In this paper, the spectral characteristic of two parametric fifth order polynomial interpolation kernels is calculated. By applying the Fourier transform, the spectral characteristic of the interpolation kernel was determined. Due to the finite length of the kernel, the spectral characteristic has ripples in the passband and stopband, as well as finite slopes in the transition band. Therefore, there is a deviation of the spectral characteristic of the 2P kernel from the characteristics of the ideal *sinc* interpolation kernel, that is, the box characteristic. By adjusting the kernel parameters α and β it is possible to minimize the ripple, and thus increase the similarity of the spectral characteristics of the kernel, with the box characteristic. Examples of minimizing the interpolation error, in Digital Image Processing, as well as in Audio and Speech Processing, are described in the scientific literature. In this way, the optimal values of the kernel parameters are determined. MSE is most often used as a measure of the interpolation precision.

Further research will be aimed at minimizing the ripples of spectral characteristic in the spectral domain. Minimization of the ripple will be carried out by analyzing the effect of the Taylor series members of the spectral characteristics that predominantly affect the ripple.

REFERENCES

Deng T. B. 2010. Frequency-domain weighted-least-squares design of signal-dependent quadratic interpolators, *IET Signal Process.*, 4(1), pp. 102-111.

Dodgson N. 1997. Quadratic Interpolation for Image Resampling, *IEEE Transactions On Image Processing*, 6(9), pp. 1322-1326.

Hanssen R. & Bamler R. 1999. Evaluation of Interpolation Kernels for SAR Interferometry, *IEEE Transactions on Geoscience and Remote Sensing*, 37(1), pp. 318-321.

Keys, R. G. 1981. Cubic convolution interpolation for digital image processing' *IEEE Trans. Acoust. Speech, & Signal Processing*, ASSP-29, pp. 1153-1160.

Meijering E. & Unser M. 2003. A Note on Cubic Convolution Interpolation, *IEEE Transactions on Image Processing*, 12(4), pp. 447-479.

Meijering E., Zuiderveld K. & Viergever M. 1999. Image Reconstruction by Convolution with Symmetrical Piecewise n^{th} -Order Polynomial Kernels', *IEEE Transactions on Image Processing*, 8(2), pp. 192-201.

Milivojević Z. & Brodić D. 2013. Estimation Of The Fundamental Frequency Of The Real Speech Signal Compressed By MP3 Algorithm, *Archives of Acoustics*, 38(3), pp. 363-373.

Milivojević Z., Savić N. & Brodić D. 2017. Three-Parametric Cubic Convolution Kernel For Estimating The Fundamental Frequency Of The Speech Signal, *Computing and Informatics*, 36, pp. 449-469.

Park K. S. & Schowengerdt R. A. 1982. Image reconstruction by parametric cubic convolution, *Computer Vision, Graphics & Image Processing*, 23, pp. 258-272.

Rifman S. S. 1973. Digital rectification of ERTS multispectral imagery, in *Proc Symp. Significant Results Obtained From the Earth Resources Technology Satellite-1*, 1(B) pp. 1131-1142.

Rukundo O. & Maharaj B. 2014. Optimization of image interpolation based on nearest neighbor algorithm, 2014 *International Conference on Computer Vision Theory and Applications (VISAPP)*, 1, pp. 641-647.

Savić N. & Milivojević Z. 2022. Optimization of Parameters of the 2P Fifth Degree Interpolation Kernel for Interpolation of Audio Signals, 21st *International Symposium INFOTEH-JAHORINA*, pp. 207-210.

Savić N., Milivojević Z. & Prlinčević B. 2021. Development of the 2P Fifth-degree Interpolation Convolutional Kernel, *International Journal of Innovative Research in Advanced Engineering (IJIRAE)*, 1(8), pp. 306-311.