OPTIMIZATION OF THE 2P FIFTH DEGREE CONVOLUTION KERNEL IN THE SPECTRAL DOMAIN

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ABSTRACT

The first part of the paper describes a two-parameter (2P) fifth-order interpolation kernel, *r*. After that, from the 2P kernel, the kernel components were created. By applying the Fourier transformation to each kernel component, the spectral components of the 2P kernel were obtained. The spectral characteristic of the 2P kernel, *H*, was created from the spectral components. After that, the algorithm, that optimizes the parameters of the 2P kernel so as to eliminate the ripple of the spectral characteristics, is described. The optimization was performed in such a way that the spectral characteristic developed in the Taylor series, H_T. With the condition for the elimination of the members of the Tylor series, which greatly affect the ripple of the spectral characteristic, the optimal kernel parameters (α_{opt}, β_{opt}) were determined. The second part of the paper describes an Experiment, in which the interpolation accuracy of the 2P kernel was tested. Convolution interpolation, with the 2P kernel, was performed over the signals from the Test base. The Test base is created with musical signals. By analyzing the interpolation error, which is represented by the Mean Square Error, MSE, the precision of the interpolation was determined. The results (α_{opt}, β_{opt}, MSE_{min}) are presented on tables and graphs. Detailed comparative analysis showed higher interpolation precision with the proposed 2P interpolation kernel, compared to the interpolation precision with, 1P interpolation kernel. Finally, the numerical values of the optimal kernel parameters, which are determined by the optimization algorithm proposed in this paper, were experimentally verified.

Keywords: Convolution, Interpolation, Polynomial kernel, Taylor series.

INTRODUCTION

Interpolation is the estimation of data between regular samples. It is the construction of new data based on a known set of discrete data. In digital signal processing (DSP), there is often a need to apply interpolation (audio and speech signal processing, image processing, numerical differentiation, integration, ...). A characteristic example is the spatial transformation of the image (rotation, translation,...). Most often, interpolation should be realized in real-time. The application of numerical interpolation formulas (Lagrangian, Newtonian, Gaussian, Stirling, Bessel, Chebyshev,...) requires knowledge of a large amount of data, sometimes the complete signal. For this reason, interpolation formulas are often of an impractically large order, that is, of great numerical complexity. The consequence of the high numerical complexity is an impractically long interpolation time.

One of the interpolations, which largely fulfills the criteria for working in real-time, is convolutional interpolation. Convolutional interpolation is realized by convolution between a discrete signal and a convolutional interpolation kernel. Theoretical analysis showed that the ideal convolutional interpolation kernel with time-spatial form is $r = sin(x)/x$ (Keys, 1981). The usual notation for this kernel in DSP is *sinc*.

The definition range of the *sinc* kernel is ($-\infty \leq x \leq +\infty$). This fact indicated that the *sinc* kernel is not possible to practically realize (Meijering & Unser, 2003). The solution to the problem of the infinite length of the interpolation kernel is the truncation of the kernel length. However, truncation of the *sinc* kernel causes deformations of the spectral characteristic of the *sinc* kernel. Namely, the spectral characteristic of the *sinc* kernel is a *box* characteristic (ideally flat in the pass-band and stop-band, and with an ideal slope in the transition area). The truncation length of the *sinc* kernel spectral characteristic causes a ripple in the pass-band and stop-band, and a final slope in the transition region (Dodgson, 1997). For the previously described reason, simple truncation *sinc* kernel does not give satisfactory results. For practical reasons, in DSP, the truncated *sinc* kernel is less often used. The current scientific task is to find other interpolation convolution kernels, which would realize interpolation with high precision and with high speed. Current solutions, which are proposed for use in DSP, are interpolation kernels of small lengths, which are defined on the segment [-*L*, *L*], where $L \leq 8$. In addition, in order to increase the speed of interpolation, it is desirable that the kernel is presented as a mathematical function of low complexity (Milivojević et al., 2022).

Interpolation polynomial kernels are extremely relevant. The chronology of the development of the polynomial kernels is detailed in (Milivojević et al., 2022). Numerically, the least complex is the nearest-neighbor, i.e. zero-degree polynomial

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kernel (Dodgson, 1997). The interpolation speed is extremely high. Unfortunately, the precision of the interpolation is extremely low (Rukundo & Maharaj, 2014). These features have led to a minor application of this kernel. First-degree interpolation kernel, allows linear interpolation between samples (Rifman, 1973). A quadratic, second-degree interpolation kernel is described in (Dodgson, 1997) and (Deng, 2010). A cubic, third-degree interpolation kernel, is described in (Keys, 1981). The analysis of the interpolation error, in the case of image interpolation, which is based on Taylor expansion in the spatial domain, was realized. The analysis showed that the precision of interpolation with the polynomial third-degree interpolation kernel is higher than the precision with the zero-, first- and second-degree polynomial kernels.

The paper (Keys, 1981) describes a third order polynomial kernel with an inserted parameter α. In this way, the parameterization of the kernel was performed, that is, a one-parameter (1P) kernel was formed. It is shown that by choosing the value of the parameter α, the interpolation kernel can be better adapted to the specific signal. Subsequent scientific activities went in the direction of parametrizing other kernels, with the aim of achieving higher interpolation precision. Later, in honor of the author Roberts B. Keys, the 1P interpolation kernel, described in (Keys, 1981), was named the 1P Keys kernel. The problems of reducing the ripple of the spectral characteristic were analyzed in (Meijering et al., 1999). In (Park & Schowengerdt, 1982) it was shown that the ripple of the spectral characteristic of the 1P Keys kernel is greatly reduced for the optimal parameter $\alpha = -0.5$.

The paper (Hanssen & Bamler, 1999) describes a two-parameter (α, β) third-order interpolation kernel. The construction of the 2P interpolation kernel is based on the extension of the 1P Keys kernel. For this reason, this 2P kernel is called 2P Keys kernel. The possibility of optimization of the two parameters provided greater possibilities of adaptation of the 2P kernel to specific signals. Examples of estimating the fundamental frequency, F_0, of a speech signal, by interpolation in the spectral domain, are presented in (Milivojević & Brodić, 2013) and (Milivojević et al., 2017). As a measure of the error estimate of the fundamental frequency MSE was used. The tendency to further increase the precision of the convolutional interpolation led to the construction of the 1P fifth-order interpolation kernel (Meijering et al., 1999). A further increase in the precision of the interpolation led to the construction of the 2P fifth-order interpolation kernel (Savić et al., 2021). The 2P kernel was created by expanding the 1P kernel. Examples of convolutional interpolation of audio signals with a 2P kernel are presented in (Savić et al., 2022). It is shown that the precision of interpolation is higher with interpolation with 2P compared to interpolation with 1P kernel. The interpolation

error with application of Septic-convolution Kernel is analyzed in (Savić & Milivojević, 2022).

In the paper (Milivojević et al., 2022), the spectral characteristic of the 2P fifth order kernel was determined. The spectral characteristic of the 2P kernel is done as follows. It is first done by decomposing the 2P kernel into components. Then, by applying the Fourier transform to each kernel component, the spectral characteristics of each component were determined. In this way, the spectral components of the kernel are determined. Finally, taking into account all spectral components as well as the kernel parameters α and β, the spectral characteristic of the 2P kernel were determined.

In this work, the parameters (α, β) of the interpolation 2P kernel, which is described in (Savić et al., 2021), were optimized. In the first part of the paper, the analytical form of the fifth order 2P interpolation kernel is described and the components of the kernel are shown. After that, the spectral components, which are determined by applying the Fourier transformation over the kernel components, are shown (Milivojević et al., 2022). The spectral characteristic of the 2P kernel is formed by spectral components. Then the optimization of the kernel was performed. The optimization, with the criterion of eliminating the ripples of the spectral characteristic, was performed. First, the spectral characteristic was developed in the Taylor series. After that, members of the Tylor series, which dominantly affect the ripples of the spectral characteristic, were eliminated. Optimal parameters $(\alpha_{opt}, \beta_{opt})$ were determined from the conditions of Tylor member elimination. The Experiment is described in the second part of the paper. In the Experiment, convolutional interpolation of the Test signal from the Base was performed. The Base is created from musical signals. Musical signals (tones A0, A1, ..., A7) were obtained by recording the tones interpreted on a Steinway B piano, the world-renowned piano manufacturer Steinway & Sons. The goal of the Experiment was to determine the precision of convolutional interpolation using 1P and 2P kernels, whose parameters were obtained theoretically, that is, by optimization, and whose spectral characteristics are ripple-free. The second goal of the experiment was the experimental determination of the optimal parameters of kernels in the interpolation of musical signals. Experimental determination involves minimizing the interpolation error MSE (Mean Square Error), and thus determining the optimal parameters. The results of the Experiment are presented in tabular and graphical form. At the end, a comparative analysis of the results related to the precision of interpolation was performed, and, based on it, conclusions were drawn about the efficiency of the 2P kernel.

Further organization of this paper is as follows. In Section II, the spectral characteristic of the 2P kernel is described. Section III describes the kernel parameter

optimization. In Section IV the Experiment is described. Section V is the Conclusion.

FIFTH DEGREE POLYNOMIAL CONVOLUTION KERNEL

In the paper (Meijering et al., 1999) the 1P fifth degree polynomial convolution kernel is described. The shape of the 1P kernel is, in accordance with the recommendations on the values in the nodes in the specified interval, defined in the time domain. Optimization of the kernel parameter α in the spectral domain was performed. The optimized parameter, α_{opt} = 3/64, leads to the minimization of the ripple spectral characteristic. In this way, it was achieved that the 1P kernel better approximates the spectral characteristic of the ideal $sinc$ kernel. In Fig. 1 shows the time form of the shortened ideal kernel sinc, r_{sinc}. In addition, in Fig. 1 shows the time shape of the optimal 1P kernel r_α.

Two parameter fifth-order kernel

The paper (Savić et al., 2021) describes a new, two-parameter, polynomial interpolation convolution kernel of the fifth order (quintic-convolution kernel). The form of the 2P kernel is:

$$r(x) = \begin{cases} \left(10\alpha - 10\beta - \frac{21}{16}\right)|x|^5 + \\ \left(-18\alpha + 18\beta + \frac{45}{16}\right)|x|^4 + \\ \left(8\alpha - 8\beta - \frac{5}{2}\right)|x|^2 + 1, & |x| \le 1 \\ \left(11\alpha - 11\beta - \frac{5}{16}\right)|x|^5 + \\ \left(-88\alpha + 88\beta + \frac{45}{16}\right)|x|^4 + \\ (270\alpha - 270\beta - 10)|x|^3 + \\ \left(-392\alpha + 392\beta + \frac{35}{2}\right)|x|^2 + & 1 < |x| \le 2 \\ (265\alpha - 265\beta - 15)|x| + \\ (-66\alpha + 66\beta + 5), \\ \alpha|x|^5 + (-14\alpha + 3\beta)|x|^4 + \\ (78\alpha - 30\beta)|x|^3 + \\ (-216\alpha + 112\beta)|x|^2 + & 2 < |x| \le 3 \\ (297\alpha - 185\beta)|x| + \\ (-162\alpha + 114\beta), \\ \beta|x|^5 - 19\beta|x|^4 + 144\beta|x|^3 - \\ 544\beta|x|^2 + 1024\beta|x| - 768\beta, & 3 < |x| \le 4 \\ 0, & |x| > 4 \end{cases} \quad (1)$$

where α and β are kernel parameters. The 2P kernel can be presented in the form (Milivojević et al., 2022):

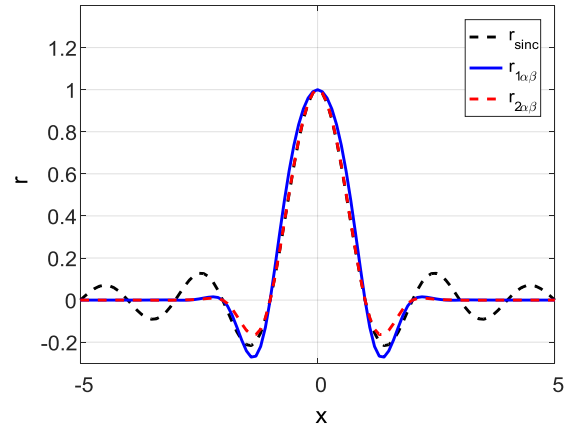$$r(x) = r_0(x) + \alpha r_1(x) + \beta r_2(x), \quad (2)$$

where r_0, r_1 and r_2 are kernel components:

$$r_0(x) = \begin{cases} \frac{-21}{16}|x|^5 + \frac{45}{16}|x|^4 - \\ \quad -\frac{5}{2}|x|^2 + 1 & |x| \le 1 \\ \frac{-5}{16}|x|^5 + \frac{45}{16}|x|^4 - 10|x|^3 + \\ \quad \frac{35}{2}|x|^2 - 15|x| + 5 & 1 < |x| \le 2 \\ 0 & |x| > 2 \end{cases} \quad (3)$$
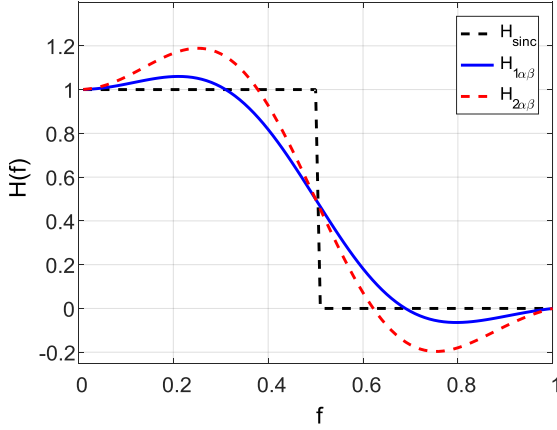
$$r_1(x) = \begin{cases} 10|x|^5 - 18|x|^4 + 8|x|^2, & 0 < |x| \le 1 \\ 11|x|^5 - 88|x|^4 + 270|x|^3 - 392|x|^2 + \\ +265|x| - 66, & 1 < |x| \le 2 \\ |x|^5 - 14|x|^4 + 78|x|^3 - 216|x|^2 + \\ 297|x| - 162, & 2 < |x| \le 3 \\ 0; & |x| > 3 \end{cases} \quad (4)$$

and

$$r_2(x) = \begin{cases} -10|x|^5 + 18|x|^4 - 8|x|^2 & 0 \le |x| \le 1 \\ -11|x|^5 + 88|x|^4 - 270|x|^3 + & 1 < |x| \le 2 \\ +392|x|^2 - 265|x| + 66 \\ 3|x|^4 - 30|x|^3 + 112|x|^2 - & 2 < |x| \le 3 \\ -185|x| + 114 \\ |x|^5 - 19|x|^4 + 144|x|^3 - & 3 < |x| \le 4 \\ -544|x|^2 + 1024|x| - 768 \\ 0 & |x| > 4 \end{cases} \quad (5)$$

Figure 1. Interpolation 2P kernel: time domain, r_{sinc} - ideal $sinc$ kernel, $r_{1\alpha\beta}$ - (α = 0.1, β = -0.01), $r_{2\alpha\beta}$ - (α = 0.2, β = 0.01).

Figure 2. Interpolation 2P kernel: spectral domain , H_{sinc} - *box*, $H_{1\alpha\beta}$ - (α = 0.1, β = -0.01), $H_{2\alpha\beta}$ - (α = 0.2, β = 0.01).

The effect of the kernel parameters α and β on time shape kernels is shown in Fig. 1: a) $r_{1\alpha\beta}$ (α = 0.1, β = -0.01) and b) $r_{2\alpha\beta}$ (α = 0.2, β = 0.01). r_{sync} is a time shape of the truncate *sinc* kernel. The spectral characteristics are shown in Fig. 2.: a) $H_{1\alpha\beta}$ (α = 0.1, β = -0.01) and b) $H_{2\alpha\beta}$ (α = 0.2, β = 0.01). H_{sinc} is the spectral *box* characteristic of the ideal kernel.

Spectral characteristic of the 2P kernel

The spectral characteristic of the 2P polynomial convolutional interpolation fifth order kernel, r, is presented in (Milivojević et al., 2022). The 2P kernel is obtained by applying a Fourier transform over the time shape of the kernel Eq. (1). More precisely, the Fourier transform is applied over each kernel component r_0, r_1 and r_2 Eq. (3 - 5). In this way, the spectral components of the 2P kernel H_0, H_1 and H_2, were calculated. The spectral components of the 2P kernel are:

$$H_0 = \frac{15\sin[f\pi]}{32f^6\pi^6}\left(\begin{array}{c}-2f\pi(17\cos[f\pi]+\cos[3f\pi])+\\21\sin[f\pi]+5\sin[3f\pi]\end{array}\right), \quad (6)$$

$$H_1 = \frac{3\sin[2f\pi]}{2f^6\pi^6}\left(\begin{array}{c}66f\pi+50\sin[2f\pi]-5\sin[4f\pi]+\\2f\pi(26\cos[2f\pi]+\cos[4f\pi])\end{array}\right), \quad (7)$$

and

$$H_2 = \frac{\sin[2f\pi]}{2f^6\pi^6}\left(\begin{array}{c}-2f\pi(87+4f^2\pi^2+72\cos[2f\pi])\\-150\sin[2f\pi]+15\sin[4f\pi]-15\sin[6f\pi]\\+2f\pi((21-8f^2\pi^2)\cos[4f\pi]+3\cos[6f\pi])\end{array}\right). \quad (8)$$

Finally, the spectral characteristic is:

$$H(f) = H_0(f) + \alpha H_1(f) + \beta H_2(f). \quad (9)$$

OPTIMIZATION OF THE 2P KERNEL

By changing the value of the parameters of the 2P kernel (α, β), its time-spatial shape (Eq. (1)) as well as the shape of

the spectral characteristic (Eq. (9)) are affected. In this way, it is achieved that the spectral characteristic is more similar to the spectral characteristic of the ideal, *sinc* kernel (box function). Therefore, the precision of convolutional interpolation with 2P kernel increases.

In the further part of this paper, applying the criterion of reducing the ripple of the spectral characteristic, the optimization of kernel parameters is described (Fig. 1). The optimization process was carried out in the following steps. Over the spectral components H_0, H_1 and H_2, (Eq. (6 - 8)) Taylor expansion was applied.

In this way, the Taylor spectral components H_{T0}, H_{T1} and H_{T2}, were obtained:

$$H_{T0}(f) = 1 - \frac{3f^2\pi^2}{14} - \frac{f^4\pi^4}{105} + \frac{89f^6\pi^6}{10395}$$
$$- \frac{458f^8\pi^8}{315315} + O\left((f\pi)^{10}\right), \quad (10)$$

$$H_{T1}(f) = \frac{32f^2\pi^2}{7} - \frac{352f^4\pi^4}{105} +$$
$$\frac{2144f^6\pi^6}{3465} + \frac{75904f^8\pi^8}{945945} + O\left((f\pi)^{10}\right), \quad (11)$$

and

$$H_{T2}(f) = -\frac{176f^2\pi^2}{35} + \frac{1936f^4\pi^4}{315} - \frac{14512f^6\pi^6}{2475} +$$
$$\frac{22309568f^8\pi^8}{4729725} + O\left((f\pi)^{10}\right), \quad (12)$$

respectively.

The spectral characteristic of the 2P kernel, composed of Taylor components, is

$$H_T(f) = H_{T0}(f) + \alpha H_{T1}(f) + \beta H_{T2}(f) =$$
$$1 - \frac{1}{70}(15 - 320\alpha + 352\beta)(f\pi)^2 -$$
$$\frac{1}{315}(3 + 1056\alpha - 1936\beta)(f\pi)^4 +$$
$$\frac{1}{51975}(445 + 32160\alpha - 304752\beta)(f\pi)^6 +$$
$$\frac{1}{4729725}(-6870 + 379520\alpha + 22309568\beta)(f\pi)^8 +$$
$$O\left((f\pi)^{10}\right) \quad (13)$$

The second and third elements in the Taylor series (Eq. (13)) have a dominant influence on the ripple of the spectral characteristics (Fig. 2.). For this reason, the second and third terms should be eliminated in the optimization process, so that the corresponding coefficients should be equal to zero:

$$\begin{cases} 15 - 320\alpha + 352\beta = 0 \\ 3 + 1056\alpha - 1936\beta = 0 \end{cases}. \quad (14)$$

By solving the system of equations (Eq. (14)), the values of the optimal parameters of the 2P kernel were calculated

$$\alpha_{opt} = \frac{171}{1408} \approx 0.121, \quad \beta_{opt} = \frac{525}{7744} \approx 0.068 \, .$$

In Fig. 3, in addition to the time shape of the shortened ideal kernel $sinc$, (r_{sinc}), and the time shape of the optimal 1P kernel (r_α), the time shape of the optimal 2P kernel $r_{\alpha\beta}$ is shown. It can be seen that the time shape of the 2P kernel is more similar to the $sinc$ kernel, compared to the shape of the 1P kernel.

In Fig. 4 spectral characteristics: a) ideal $sinc$ kernel (H_{sinc} - box characteristic), b) optimal 1P kernel (H_α, α_{opt} = 3/64) and c) optimal 2P kernel ($H_{\alpha\beta}$, α_{opt} = 171 / 1408, β_{opt} = 525/7744), are shown. It can be seen that the spectral characteristics of 1P and 2P kernels are smooth, that is, ripples are eliminated, which was the optimization criterion. In addition, the spectral characteristic of the 2P kernel better approximates the box characteristic of the ideal kernel, compared to the spectral characteristic of the 1P kernel.
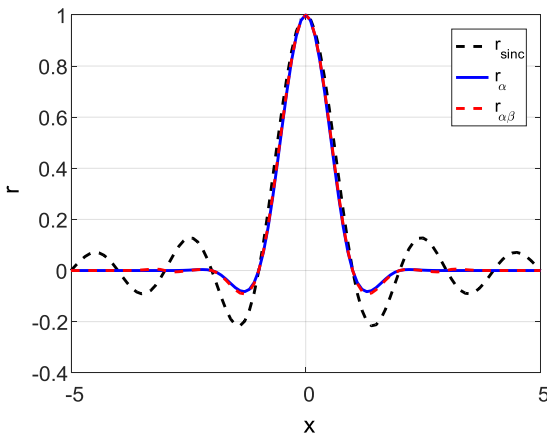

Figure 3. Interpolation kernel (time domain): a) r_{sinc} - ideal kernel b) r_α - optimal 1P kernel (α_{opt} = 3 / 64) and c) $r_{\alpha\beta}$ - optimal 2P (α_{opt} = 171 / 1408, β_{opt} = 525 / 7744).
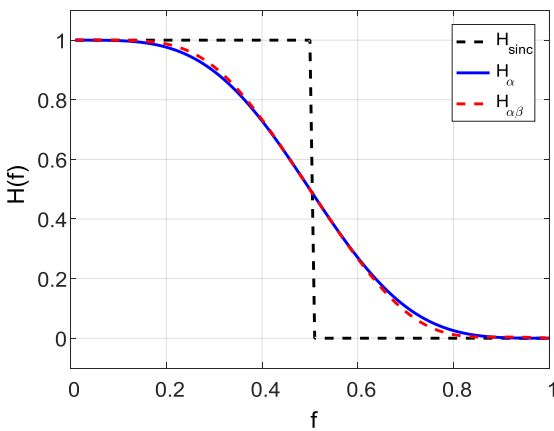
Figure 4. Spectral characteristic of interpolation kernel: a) H_{sinc} - ideal kernel b) H_α - optimal 1P kernel (α_{opt} = 3 / 64) and c) $H_{\alpha\beta}$ - optimal 2P (α_{opt} = 171 / 1408, β_{opt} = 525 / 7744).

In Fig. 4 spectral characteristics: a) ideal $sinc$ kernel (H_{sinc} - box characteristic), b) optimal 1P kernel (H_α, α_{opt} = 3 / 64) and c) optimal 2P kernel ($H_{\alpha\beta}$, α_{opt} = 171 / 1408, β_{opt} = 525 / 7744) , are shown. It can be seen that the spectral characteristics of 1P and 2P kernels are smooth, that is, ripples are eliminated, which was the optimization criterion. In addition, the spectral characteristic of the 2P kernel better approximates the box characteristic of the ideal kernel, compared to the spectral characteristic of the 1P kernel.

**EXPERIMENTAL RESULTS AND ANALYSIS**

Experiment

The optimal parameter of the fifth order 1P kernel was calculated in (Meijering et al., 1999) (α_{opt} = 3 / 64 = 0.046). In this paper the optimal parameters of the 2P kernel were determined (α_{opt} = 171 / 1408 = 0.121 and β_{opt} = 525 / 7744 = 0.068). Using the Experiment, a comparative analysis of the accuracy of convolutional interpolation with 1P kernel and 2P kernel was analyzed. The interpolation is performed on the Test signals from the Base. The Base is created from music signals. As a measure of interpolation precision, the Root Mean Square error (MSE) was applied. First, convolutional interpolation with optimal 1P and 2P interpolation kernels was performed. By comparative analysis of interpolation precision, the efficiency of the kernels, which were optimized in relation to minimizing the ripples spectral characteristic, was determined. After that, in the second part of the Experiment, the interpolation of musical signals in a wide range for the kernel parameters (α = -1, ..., 0.5, β = -1, ..., 1), was performed. By minimizing the MSE interpolation error, the optimal parameters of the kernels are determined (1P kernel $\Rightarrow$ $\alpha_{opt} = \arg\min_{\alpha} (\text{MSE}(\alpha))$ Fig. 7, and 2P kernel $\Rightarrow$ (α_{opt}, β_{opt}) = $\arg\min_{\alpha,\beta} (\text{MSE}(\alpha,\beta))$ Fig. 8. The results are presented using tables. At the end, a comparative analysis of the results was performed and the higher precision of the 2P compared to the 1P kernel was determined.

Base

For the purposes of the Experiment, the Base, which is composed of musical signals. Musical tones A0 - A7 are interpreted on a Steinway B piano, the world-renowned piano manufacturer Steinway & Sons. The recording was made at the University of Iowa and is part of the RWC Music Database (Goto et al., 2003). Recording was done with Fs = 44100 Hz and 16 bps. Musical Test signals are shown in: a) Fig. 5 (time domain) and b) Fig. 6 (spectral domain).
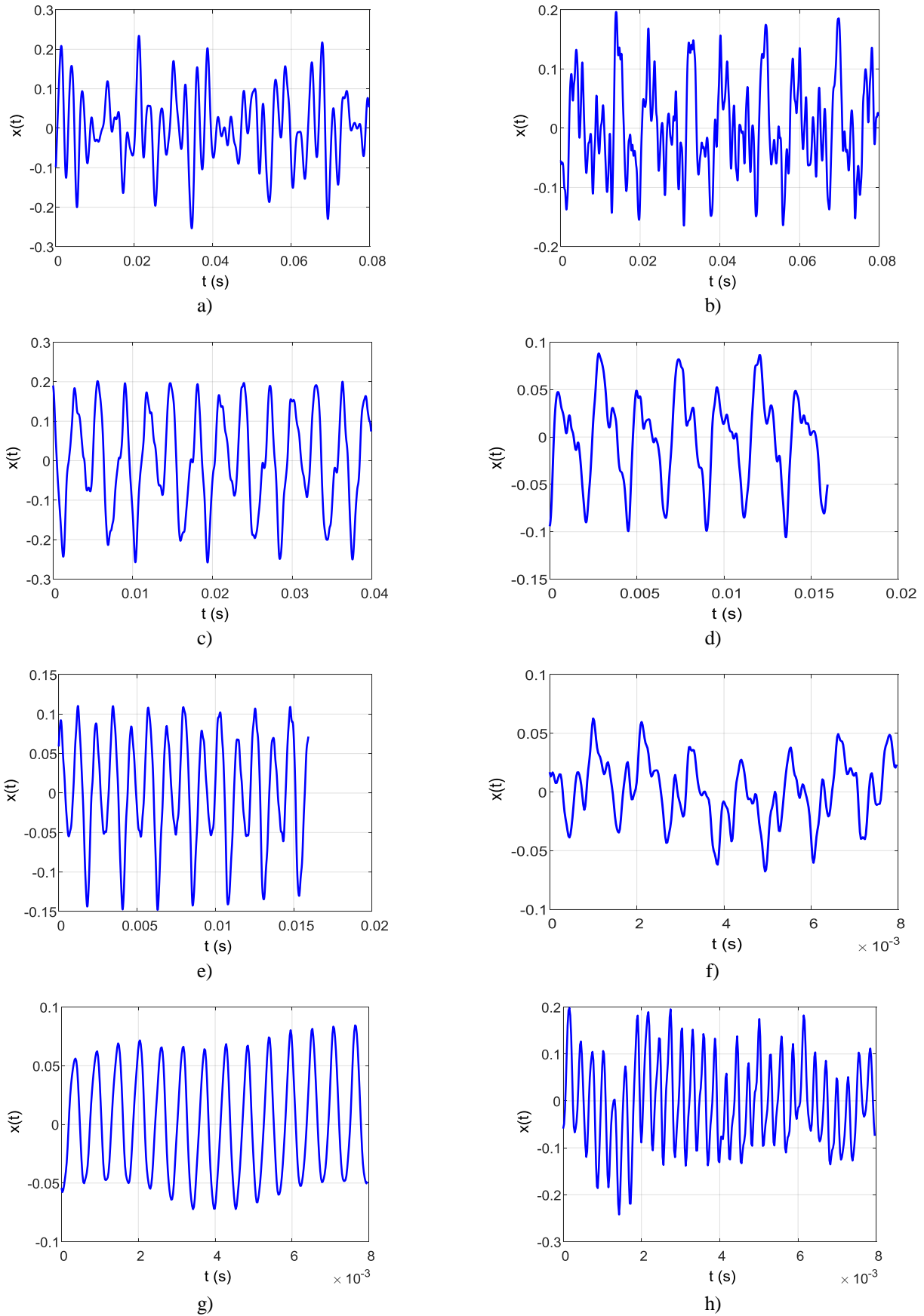
Figure 5. Musical Test signals in the time domain: a) A0, b) A1, c) A2, d) A3, e) A4, f) A5, g) A6 and h) A7.
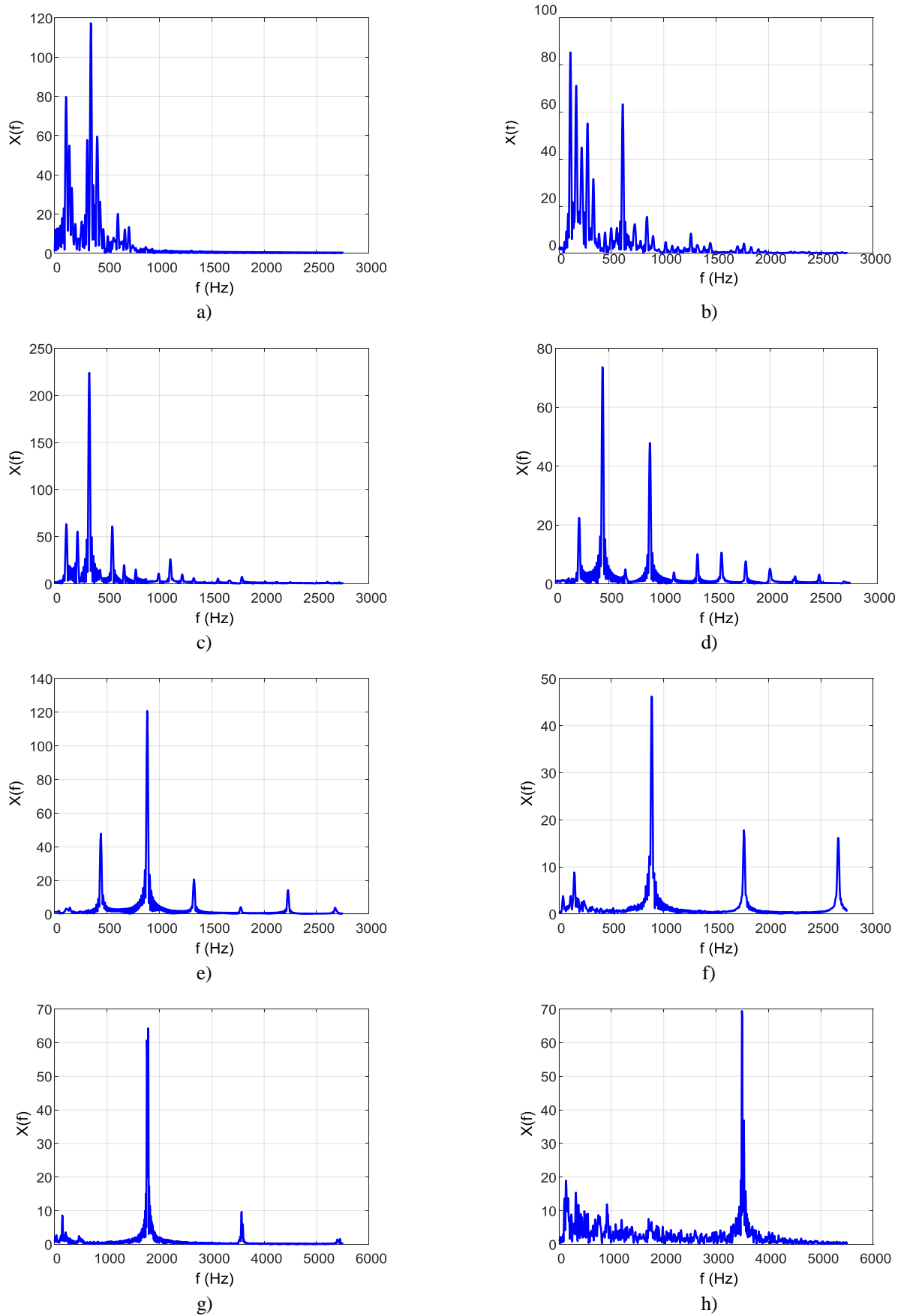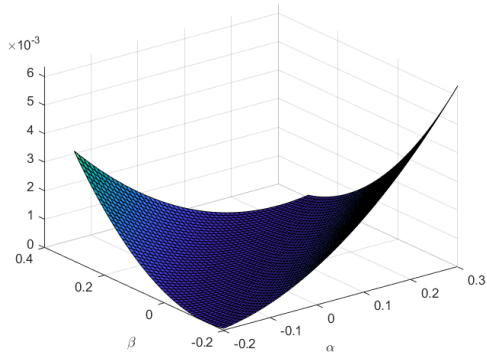
Figure 6. Musical test signals in the spectral domain: a) A0, b) A1, c) A2, d) A3, e) A4, f) A5, g) A6 and h) A7.
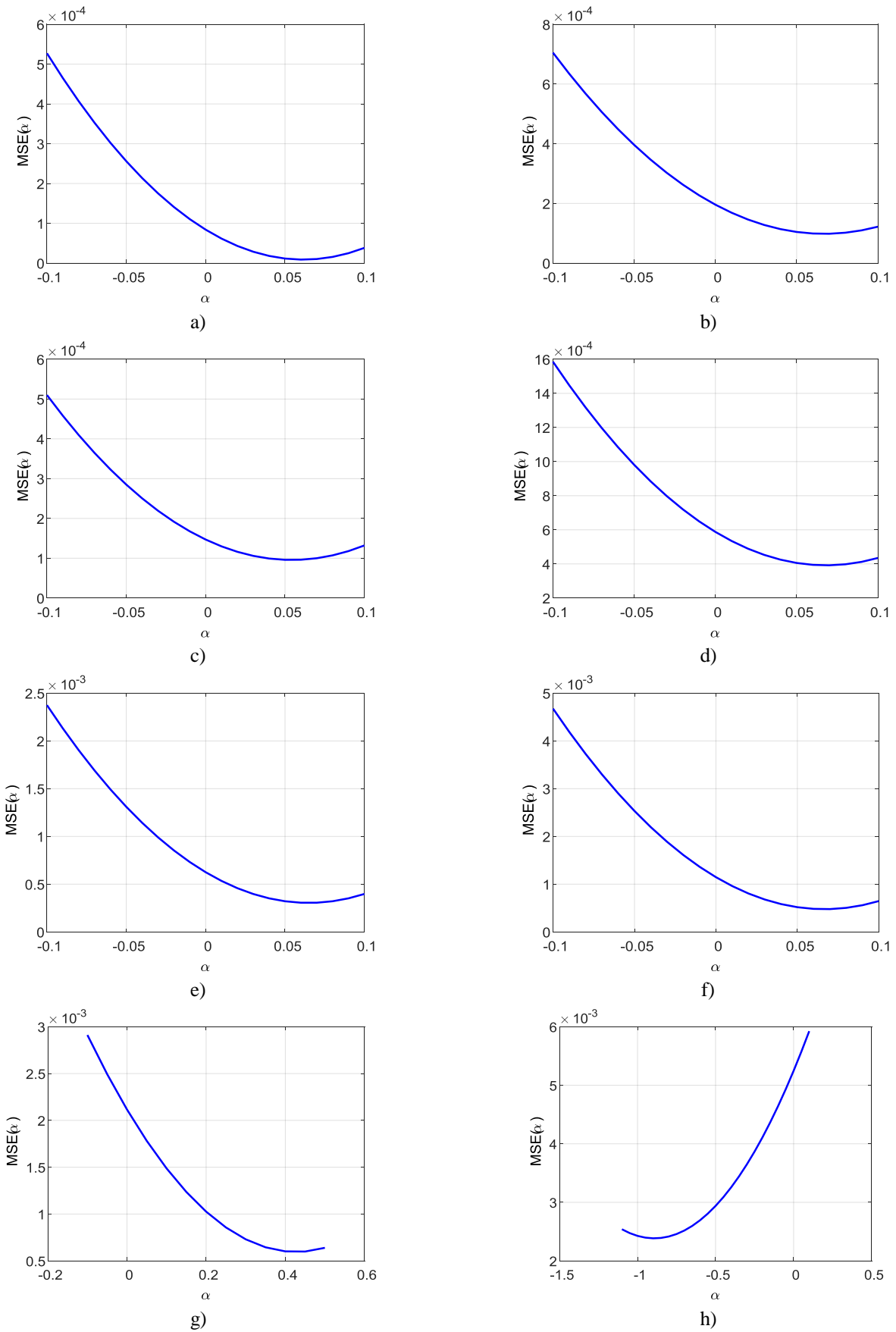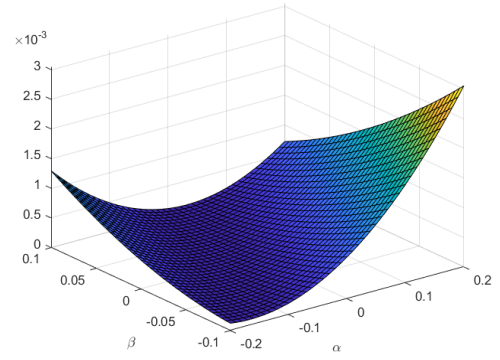
Figure 7. MSE(α) for 1P kernel for musical tones: a) A0, b) A1, c) A2, d) A3, e) A4, f) A5, g) A6 and h) A7.
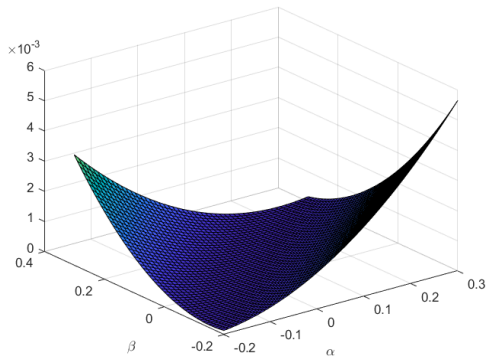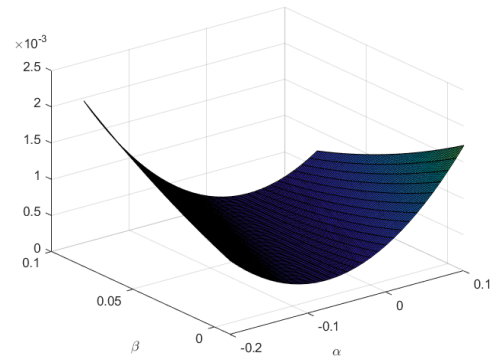
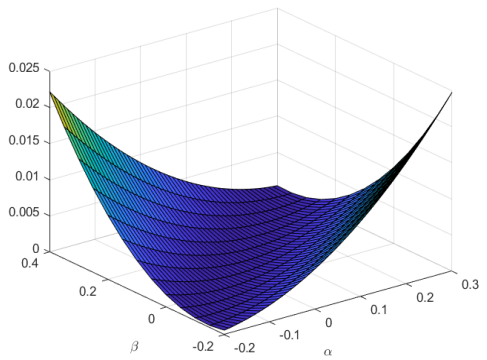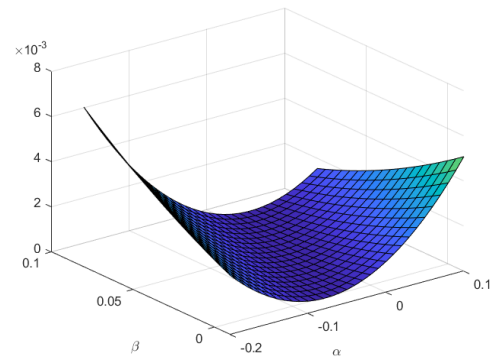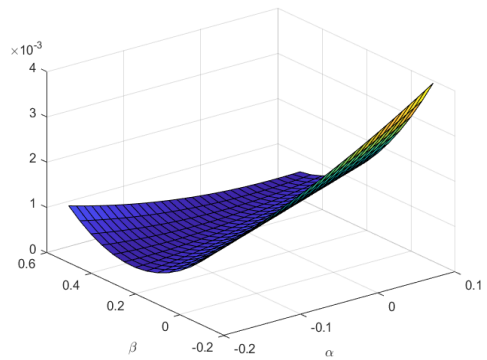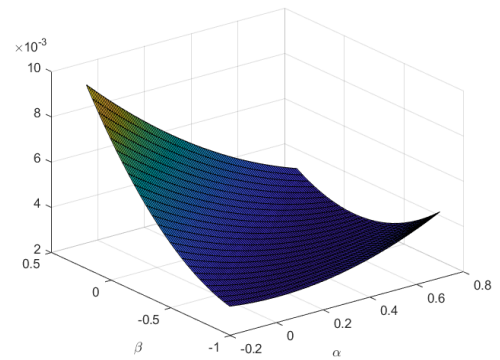**Figure 8.** MSE(α, β) for 2P kernel for musical tones: a) A0, b) A1, c) A2, d) A3, e) A4, f) A5, g) A6 and h) A7.

Results

The interpolation errors MSE of musical Test signals, in convolutional interpolation with optimal 1P and 2P fifth-order kernels, are shown in Table 1. The minimum MSE and optimal parameters α_{opt}, for the 1P kernel are shown in Table 2. The minimum MSE and the optimal parameters α_{opt} and β_{opt} for the 2P kernel are shown in Table 3. The interpolation errors are shown by graphs for: a) 1P kernel (Fig. 7, MSE(α)) and 2P kernel (Fig. 8, MSE(α, β)).

Table 1. Interpolation errors MSE in with optimal 1P and 2P interpolation kernels.

Tone	MSE	
	1P	2P
A0	$1.3052 \cdot 10^{-5}$	$1.0012 \cdot 10^{-5}$
A1	$1.0686 \cdot 10^{-4}$	$1.0283 \cdot 10^{-4}$
A2	$9.6588 \cdot 10^{-5}$	$9.5695 \cdot 10^{-5}$
A3	$4.1070 \cdot 10^{-4}$	$4.0181 \cdot 10^{-4}$
A4	$3.2884 \cdot 10^{-4}$	$3.1281 \cdot 10^{-4}$
A5	$5.3470 \cdot 10^{-4}$	$4.9720 \cdot 10^{-4}$
A6	$1.8008 \cdot 10^{-3}$	$1.7859 \cdot 10^{-3}$
A7	$5.5589 \cdot 10^{-3}$	$5.5555 \cdot 10^{-3}$
	$\overline{MSE_\alpha}$	$\overline{MSE_{\alpha\beta}}$
	0.0011	0.0010

Table 2. Minimum MSE and optimal parameters α_{opt}, for the 1P kernel.

tone	α_{opt}	MSE_α
A0	0.0600	$8.8281 \cdot 10^{-6}$
A1	0.0700	$9.8313 \cdot 10^{-5}$
A2	0.0500	$9.5954 \cdot 10^{-5}$
A3	0.0700	$3.9186 \cdot 10^{-4}$
A4	0.0600	$3.0580 \cdot 10^{-4}$
A5	0.0700	$4.7789 \cdot 10^{-4}$
A6	0.4500	$5.9965 \cdot 10^{-4}$
A7	-0.9000	0.0024
	$\overline{\alpha_{opt}}$	$\overline{MSE_\alpha}$
	-0.0087	$5.4729 \cdot 10^{-4}$

Table 3. Minimum MSE and the optimal parameters α_{opt} and β_{opt} for the 2P kernel.

Tone	α_{opt}	β_{opt}	$MSE_{\alpha\beta}$
A0	0.2800	0.2200	$7.9855 \cdot 10^{-6}$
A1	-0.0550	-0.1200	$9.7994 \cdot 10^{-5}$
A2	0.2000	0.1400	$9.5462 \cdot 10^{-5}$
A3	-0.0020	-0.0700	$3.9151 \cdot 10^{-4}$
A4	0.3500	0.2800	$2.9661 \cdot 10^{-4}$
A5	0.0750	0.0100	$4.7624 \cdot 10^{-4}$
A6	0.2500	-0.1700	$5.8914 \cdot 10^{-4}$
A7	-0.1000	0.7900	0.0023
	$\overline{\alpha_{opt}}$	$\overline{\beta_{opt}}$	$\overline{MSE_{\alpha\beta}}$
	0.1247	0.135	$5.3187 \cdot 10^{-4}$

Analysis of results

In accordance with the results shown in Table 1 (1P and 2P kernels with optimal parameters), it is concluded that MSE, when applying 2P kernel compared to 1P Keys kernel $\overline{MSE_\alpha}$ / $\overline{MSE_{\alpha\beta}}$ = 0.0011/0.0010 = 1.1 times smaller.

Based on the experimental results shown in Table 2 and Table 3, by comparative analysis of MSE, it is concluded that: a) MSE, when applying 2P kernel compared to 1P kernel $\overline{MSE_\alpha}$ / $\overline{MSE_{\alpha\beta}}$ = $5.4729 \cdot 10^{-4}$ / $5.3187 \cdot 10^{-4}$ = 1.029 times smaller. b) the range of optimal values of the parameter of the 1P kernel (Table 2) $\alpha_{opt} \in [-0.9 \div 0.45]$ and that the mean value $\overline{\alpha_{opt}}$ = - 0.0087. c) range of optimal parameter values of the 2P kernel (Table 3) $\alpha_{opt} \in [-0.1 \div 0.35]$ and $\beta_{opt} \in [-0.17 \div 0.79]$ and mean value $\overline{\alpha_{opt}}$ = 0.1247 and $\overline{\beta_{opt}}$ = 0.135. d) the estimation error of the parameters α is $\Delta\alpha = |\alpha_{opt} - \overline{\alpha_{opt}}|$ = $|0.1214 - 0.1247|$ = 0.0033, where the theoretical α_{opt} = 171 / 1408 = 0.1214. e) the estimation error of the parameters β is $\Delta\beta = |\beta_{opt} - \overline{\beta_{opt}}|$ = $|0.0678 - 0.135|$ = 0.0672, where the theoretical β_{opt} = 0.0678. The total parameter estimation error for musical tones is $E_T = \sqrt{\Delta_\alpha^2 + \Delta_\beta^2}$ = 0.0045. In accordance with the performed comparative analysis of MSE for 1P and 2P interpolation kernels of the fifth order, it is concluded that convolutional interpolation with 2P kernel is more accurate compared to interpolation with 1P kernel.

CONCLUSION

The paper presents an algorithm for optimizing the parameters of the 2P fifth order interpolation kernel. Parameter optimization was performed in the spectral domain by minimizing the ripple of the spectral characteristic. First, the spectral characteristic was developed in the Taylor series, and, after that, the members of the Taylor series that had a great effect on increasing the ripple of the spectral characteristic, were eliminated. From the conditions of elimination of the dominant members of the Taylor series, the optimal values of the parameters of the 2P kernel (α_{opt} = 0.121, β_{opt} = 0.068), were determined. Verification of the accuracy of the 2P kernel when interpolating musical signals (tones A0, A1, ..., A7) was performed experimentally. The interpolation accuracy is expressed through the MSE interpolation error. Detailed comparative analysis showed that the 2P kernel, with theoretically determined optimal parameters, has a higher interpolation accuracy compared to the 1P kernel 1.1 times. In addition, comparative analysis showed that the 2P kernel, with experimentally determined optimal parameters, has a higher interpolation accuracy compared to the 1P kernel 1,029 times. Based on the presented results, it is concluded that the 2P fifth-order kernel is superior to the 1P kernel, and that the

interpolation error is smaller. The 2P fifth-order kernel with optimal parameters α_{opt} and β_{opt}, compared to the ideal *sinc* kernel, has less numerical complexity, and therefore, it is suitable for implementation in convolutional interpolations for operation in real-time systems.

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