Original Scientific Paper

doi:10.5937/jaes15-15449 Paper number: 15(2017)4, 474, 474 - 479

THE STUDY OF THE STIFFNESS COEFFICIENT OF THE SEAMDEPENDING ON THE QUANTITY OF SYMMETRICALLY LOCATED SHIFT ONNECTION-SIN AN OVAL TWO-LAYERPLATE

Andrey Viktorovich Turkov[1*] *Ekaterina Karpova*[2]

[1]*Southwest State University, Faculty of Building and Architecture, Kursk, Russia*
[2]*Orel State University, Orel, Russia*

The paper considers a two-layer isotropic plate on pliable connections, for which the theory of cal-culation of composite plates by A.R.Rzhanitsinwas adopted. With the help of bending moments Mx(My), the numerical stiffness coefficient of the seam is calculated. The numerical basic studies were carried out on an oval two-layered plate in the case of a rigid and hinged support of the plate along the contour. It proved that the stiffness coefficient of the seam ξ depends on the main coeffi-cient after the hard oscillation frequency ω. The authors constructed the graph of the change of the natural oscillation frequency (ω) and the graph of the change of the stiffness coefficientof the seam (ξ) on the number of shift connections (nss./ne).

Key words: Composite plate, Layer of the composite plate, Stiffness coefficient of the seam, Basic frequency of oscillation, Bending moment, Shift connections

INTRODUCTION

A large number of works are devoted to the calculation of solid and composite plates [01, 02, 03, 04]. In [05-08], composite plates of square and round shapes were studied depending on the number of symmetrically and unevenly located shift connections, and the stiffness coefficients of the seams were determined depending on the frequency of the natural oscillations of composite plates. The authors also studied the stiffness coefficients of the seam for triangular composite plates [09, 10]. The present paper researched the stiffness coefficients of seams depending on the frequency of natural oscillations of oval composite plates with a different number of evenly and symmetrically located shift connections.

Let us consider a composite plate consisting of a number of layers joined together not only by the shift connections, but also by transverse connections, which prevent the removal or rapprochement of the layers. This approach is based on A.R. Rzhanitsin'stheory of composite plates. For each layer, a hypothesis of direct normals is valid. The number of seams or gaps between plates is n, and the total number of layers is n + 1 (Figure 1).

There is a linear dependence between the differences of longitudinal displacements and tangential stresses in the connections of the *i-th* seam shift:

$$\tau_x^i = \xi_i \Delta u_i; \quad \tau_y^i = \xi_i \Delta v_i \qquad 1)$$

where,

$$\Delta u_i = \frac{dw}{dx} c_i + u_{i+1} - u_i$$
$$\Delta v_i = \frac{dw}{dy} c_i + v_{i+1} - v_i \qquad 2)$$

c_i is the distance between the median planes of the layers lying on both sides of the *i-th* seam; ξ_i is the stiffness coefficient of the connections of the *i-th* seam shift.

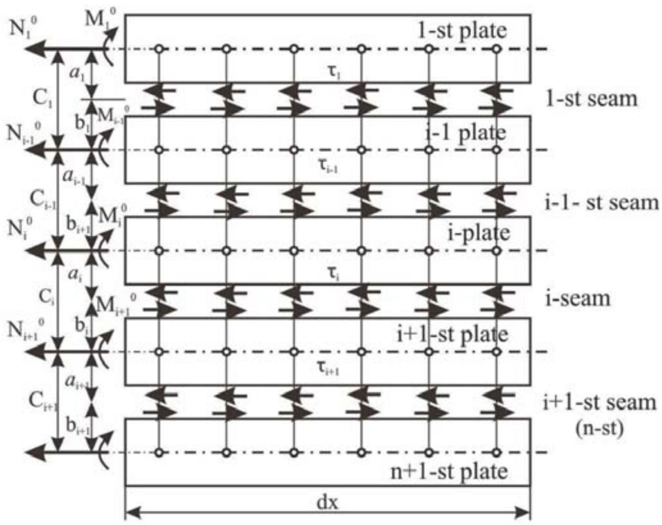


Figure 1: Accepted designations for a composite plate

* *Southwest State University, Street 50 years of October, 94 Kursk, Russia, aturkov@bk.ru*

For bending and twisting moments in the *i-th* seam: M_x^i, M_y^i and M_{xy}^i, one can draw equations:

$$\begin{cases} \dfrac{dM_x^i}{dx} + \dfrac{dM_{xy}^i}{dy} = Q_x^i + m_x^i + m_x^{i-1} \\ \dfrac{dM_y^i}{dy} + \dfrac{dM_{xy}^i}{dx} = Q_y^i + m_y^i + m_y^{i-1} \end{cases} \quad 3)$$

By transforming the right-hand side of the system and expressing lateral forces through the sum of the momentary loads in adjacent layers, we have:

$$\frac{d^2M_x^i}{dx^2} + 2\frac{d^2M_{xy}^i}{dxdy} + \frac{d^2M_y^i}{dy^2} = -q_i + \frac{d(m_x^i + m_x^{i-1})}{dx} + \frac{d(m_y^i + m_y^{i-1})}{dy} \quad 4)$$

As a result of the transformations, we get:

$$\frac{d^2M_x^i}{dx^2} + 2\frac{d^2M_{xy}^i}{dxdy} + \frac{d^2M_y^i}{dy^2} = -q - \sum_{i=1}^{n} c_i\left(\frac{d\tau_x^i}{dx} + \frac{d\tau_y^i}{dy}\right) \quad 5)$$

We express the left side of equation through the biharmonic operator from the deflection:

$$D\nabla^2\nabla^2 w = q + \sum_{i=1}^{n} c_i\left(\frac{d\tau_x^i}{dx} + \frac{d\tau_y^i}{dy}\right) \quad 6)$$

where D_0 is the cylindrical rigidity of a composite plate devoid of shift connections, which is deter-mined by the formula:

$$D_0 = \frac{E}{12(1-\mu^2)}\sum_{i=1}^{n+1} h_i^3 \quad 7)$$

μ isPoisson's ratio; hi is the thickness of the *i-th* layer.

Accepting

$$A_i = \frac{\partial\tau_x^i}{\partial x} + \frac{\partial\tau_{xy}^i}{\partial y} \quad 8)$$

we substitute A_i into equation (8):

$$D\nabla^2\nabla^2 w = q + \sum_{i=1}^{n} c_i A_i \quad 9)$$

and lowering the order, we get:

$$D\nabla^2\nabla^2 W = -M + \sum_{i=1}^{n} c_j T_j \quad 10)$$

Equation (10) is analogous to the equation of the theory of elastic compound rods with absolutely rigid cross connections [02]. Carrying out similar transformations in accordance with [02], we get:

$$\frac{\nabla^2 T_i}{\xi_i} = \sum_{j=1}^{n} \delta_{ij} T + \frac{1-\mu_i^2}{E_i h_i} N_i - \frac{1-\mu_{i+1}^2}{E_{i+1} h_{i+1}} N_{i+1} - \frac{c_i M}{D_0} \quad 11)$$

In the composite plate, T_j is the total shifting force in the *i-th* plate, which is equal to $\int_0^x \tau dx$, and N_i is the longitudinal forces in the *i-th* layer.

The bending moments M_X and M_Y will be considered equal to the total bending moments in the composite plate devoid of shift connections:

$$\sum_{i=1}^{n+1} M_x^i = M_x; \sum_{i=1}^{n+1} M_y^i = M_y; \quad \sum_{i=1}^{n+1} M_{xy}^i = M_{xy}; \sum_{i=1}^{n+1} q_i = q \quad 12)$$

$$M = \frac{M_x + M_y}{1 + \mu_{усл}} \quad 13)$$

where $\mu_{усл}$ is Poisson's ratio of a conventional solid plate.

Let us consider a specific case of a composite plate of two layers. For this, we set n = 1 for equations (12) and (13). We have the system of equations:

$$\begin{cases} \dfrac{\nabla^2 T}{\xi} = \delta T + \dfrac{N_1}{E_1^* h_1} - \dfrac{N_2}{E_2^* h_2} - \dfrac{cM}{D_0} \\ D_0 \nabla^2 W = -M + cT. \end{cases} \quad 14)$$

where D_0 is actual cylindrical stiffness equal to

$$D_0 = \sum_{i=1}^{n} D_i; D_i = \frac{E_i h_i^3}{12(1-\mu^2)} (i = 1,2) \quad \delta = \frac{c^2}{D} + \frac{1}{E_1^* h_1} + \frac{1}{E_2^* h_2} \quad E_i^* = \frac{E_i}{1-\mu_1^2} (i = 1,2) \quad 15)$$

where E_i^* is the modulus of elasticity of the layers in the composition of the composite plate, while the indices of the seams are omitted, since the seam is one.

Eliminating T from the system of equations (14), we get:

$$\begin{cases} T = \frac{D_0}{c}\nabla^2 w + \frac{M}{c}; \\ \frac{D_0}{c\xi}\nabla^2\nabla^2 w + \frac{1}{c\xi}\nabla^2 w = \frac{\delta D_0}{c}\nabla^2 w + \\ + \frac{\delta}{c}M + \frac{N_1}{E_1^* h_1} - \frac{N_2}{E_2^* h_2} - \frac{cM}{D_0} \end{cases} \quad 16)$$

or

$$\nabla^2\nabla^2 W - \xi\delta\nabla^2 W = -\frac{\nabla^2 M}{D_0} + \frac{\xi\delta}{D_0}M + \\ + \frac{c\xi}{D_0}\left(\frac{N_1}{E_1^* h_1} - \frac{N_2}{E_2^* h_2}\right) - \frac{\xi c^2 M}{D_0^2} \quad 17)$$

In the absence of axial loads N_1 and N_2, equation (17) takes the form:

$$\nabla^2\nabla^2 W - \xi\delta\nabla^2 W = -\frac{\nabla^2 M}{D_0} + \xi M\delta\frac{\delta D_0 - c^2}{\delta D_0^2} \quad 18)$$

Knowing that

$$\nabla^2 M = -q, \quad \nabla^2\nabla^2 W = -\frac{\nabla^2 M}{D_{усл}} = \frac{q}{D_{усл}} \\ \nabla^2 w = -\frac{M}{D_{усл}} \quad 19)$$

where Dусл iscylindrical rigidity of a conventional solid plate.

$$\frac{q}{D_{усл}} + \xi\cdot\delta\frac{M}{D_{усл}} = \frac{q}{D_0} + \xi\cdot M\cdot\delta\frac{\delta\cdot D_0 - c^2}{\delta D_0^2} \quad 20)$$

For the plate:

$$D_M = \frac{\delta\cdot D_0^2}{(\delta\cdot D_0 - c^2)} \quad 21)$$

where Dм is cylindrical rigidity of a monolithic plate with a longitudinal modulus of elasticity in the seam zone and it is equal to zero. Then:

$$\frac{q}{D_{усл}} + \xi\cdot\delta\frac{M}{D_{усл}} = \frac{q}{D_0} + \xi\cdot\delta\frac{M}{D_M} \quad 22)$$

We express the stiffness coefficient of the seam from this equation:

$$\xi = \frac{q\left(\frac{1}{D_0} - \frac{1}{D_{усл}}\right)}{\delta\cdot M\left(\frac{1}{D_{усл}} - \frac{1}{D_M}\right)} \quad 23)$$

THE STUDY OF THE SEAM STIFFNESS COEFFICIENT

In the framework of the present study, the research task of studying the stiffness coefficient of the seam ξ depending on the the boundary conditions of the layers and the ratio of the number of elements with shift connections (n_{ss}) to the number of finite elements of one layer (n_e = 288) symmetrically located along the area of the plate was being solved. In turn, we introduce symmetrically shift connections into the net of finite elements, according to schemes a-e in Figure 3, while the rigidity of the shift connections in all cases is constant and equal to EA_{SS} = 10 kN.

The calculation was performed in the SCAD software package by the finite element method [11]. As a result of the calculation, the fundamental frequency of the transverse oscillations and the value of the distributed moments were determined.

The following schemes of composite plates with symmetrically located shift connections were considered (Figure 2).

The studies of oval composite two-layer plates were carried out using the method of finite elements; for this purpose, both layers were conventionally divided into sectors (Figure 2), and this division of the dependence allows approximating the original plate rather precisely. Two conditions were considered for the plate support: hinged and rigid pinching. The supports along the contour of the plate were located at the nodes of the finite elements of the layers, while their boundary conditions were the same.

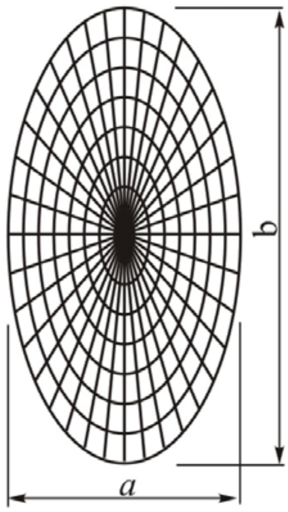
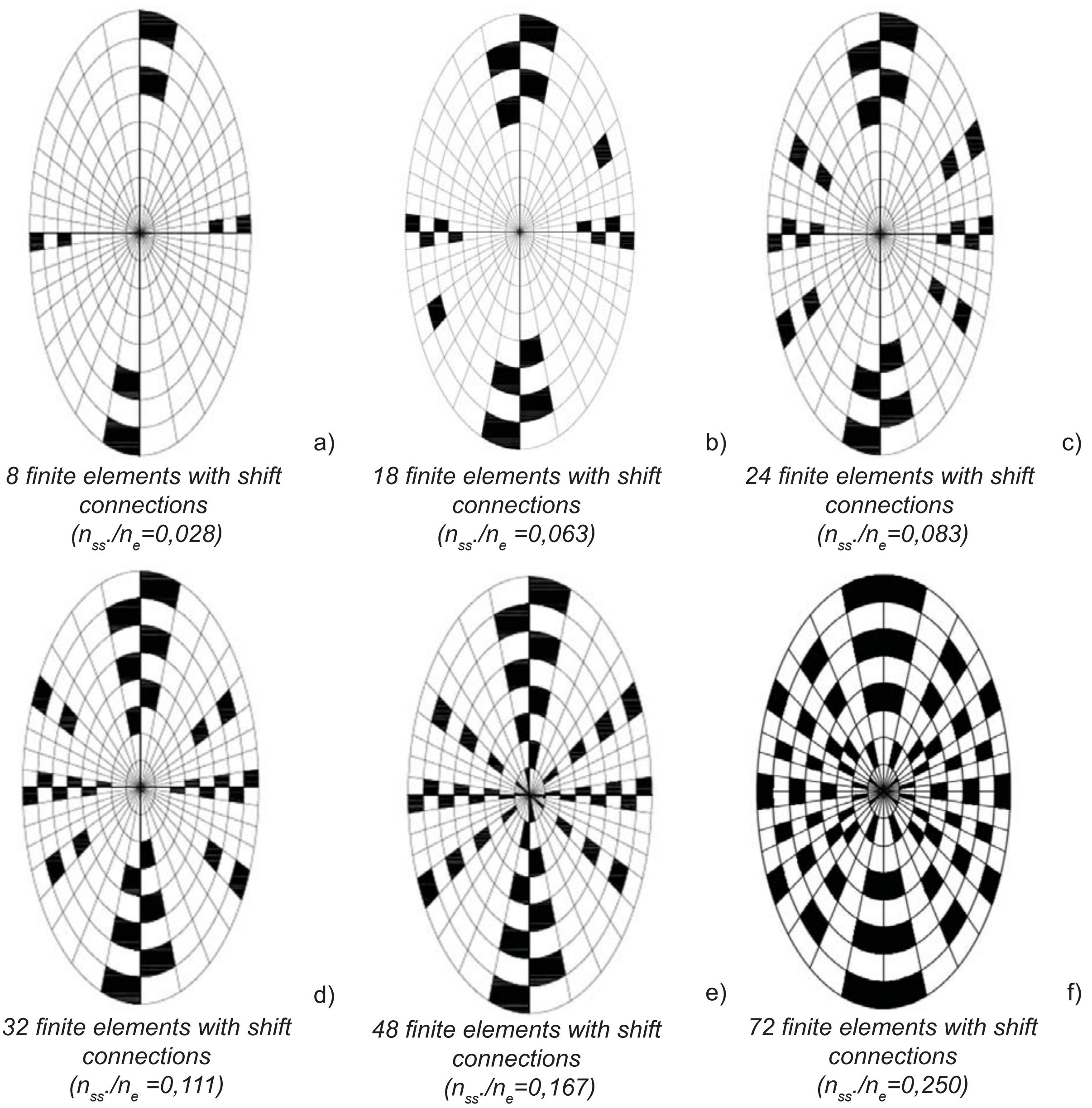


Figure 2: The split of the composite oval plate into the final elements



a) *8 finite elements with shift connections ($n_{ss}./n_e$=0,028)*

b) *18 finite elements with shift connections ($n_{ss}./n_e$ =0,063)*

c) *24 finite elements with shift connections ($n_{ss}./n_e$=0,083)*

d) *32 finite elements with shift connections ($n_{ss}./n_e$ =0,111)*

e) *48 finite elements with shift connections ($n_{ss}./n_e$=0,167)*

f) *72 finite elements with shift connections ($n_{ss}./n_e$=0,250)*

Figure 3: Calculated composite plates with different number of shift connections (n_{ss} is the number of elements with shift connections, n_e = 288 is the number of finite elements in one layer of the composite plate)

The composite material on a wood base (a chipboard plate) with a thickness δ = 10 mm is used as layers. All the characteristics were taken from the product passport: the thickness is δ = 10 mm, the average density is ρ = 740 kg / m^3, the modulus of elasticity at bending is E = 260,000 MPa. To determine the dynamic calculation of the mass in the nodes, the results were collected in ac-cordance with the construction with a volumetric weight and the loading node area. In the case of a static calculation, a uniformly distributed load with intensity of 1kN/ m^2 was applied to the top layer. The distance between the layers was assumed to be equal to the distance between the gravity centers of layers. The calculation was performed in the SCAD software package by the finite ele-ment method [11].

The results of numerical studies of composite plates, which are pivotally supported and rigidly clamped along the contour, are shown in Table 1. By the results of the calculation, the bending moments in the layers of the composite plate and the natural frequencies of the structure oscillations were determined, depending on the ratio nss./ne.

According to the data in Table 1, graphs of the oscillation frequency change depending on the stiffness coefficient of the seam (Figure 4), as well as changes in oscillation frequencies and stiffness coefficient of the seam, depending on the ratio of the number of shift connections to the number of final elements in the layer $n_{ss}./n_e$ (Figures 5 and 6) were constructed.

Table 1: Numerical studies of the composite plate when changing the quantity of symmetrically arranged shift connections in accordance with schemes a-f in Figure 3

№ №	Quantity of finite ele-ments with shift connec-tions, (n_{ss}./n_e)	The circular fre-quency of the basic tone, ω (c^{-1})	Distributed mo-ment, M_x (M_y) (N×m/m)	Maximum mo-ment, M (N×m/m)	Coefficient of stiff-ness of the seam, $\xi\times10^6$(N/m³)
The plate rigidly clamped along the contour					
1	0 (0.000)	161.2229	12.26674	17.7459	97.44331
2	8(0.028)	161.291	12.25693	17.73081	97.52243
3	18(0.063)	162.5401	11.99214	17.32344	99.71077
4	24 (0.083)	163.9788	11.85484	17.11222	100.887
5	32 (0.110)	167.771	11.24681	16.17678	106.4702
6	48(0.167)	172.6747	10.52109	15.06029	114.0551
7	72 (0.250)	184.8238	9.001003	12.7217	134.3309
8	288 (1.000)	224.3665	5.960833	8.044512	211.2151
The plate pivotally supported along the contour					
1	0 (0.000)	101.4194	33.21119	43.97822	51.95415
2	8(0.028)	103.5157	32.39724	42.72599	53.50067
3	18(0.063)	105.3268	31.34793	41.11166	55.6631
4	24 (0.083)	107.5365	30.48494	39.78399	57.60233
5	32 (0.110)	112.9102	28.17058	36.22342	63.68014
6	48(0.167)	119.1424	25.87582	32.69303	71.38537
7	72 (0.250)	132.6553	22.11987	26.91465	89.96421
8	288 (1.000)	169.6148	16.79486	18.72232	148.3558

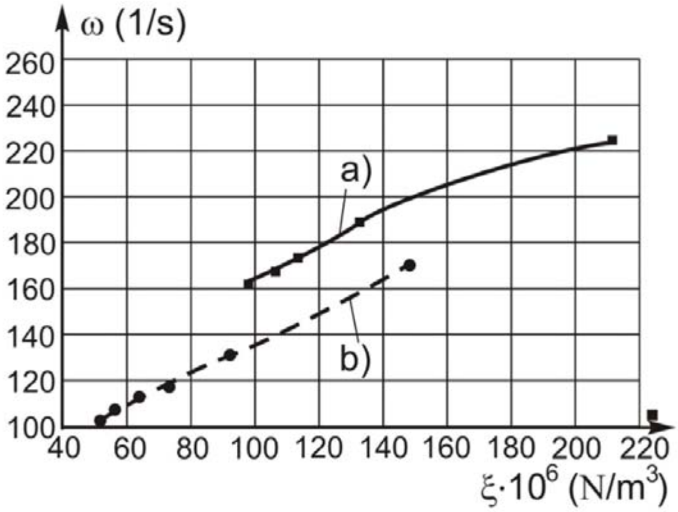


Figure 4: Frequency oscillation graphs depending on the stiffness coefficient of the seam (a –at a rigid pinching of the plate along the contour, b – at a hinged support along the contour)

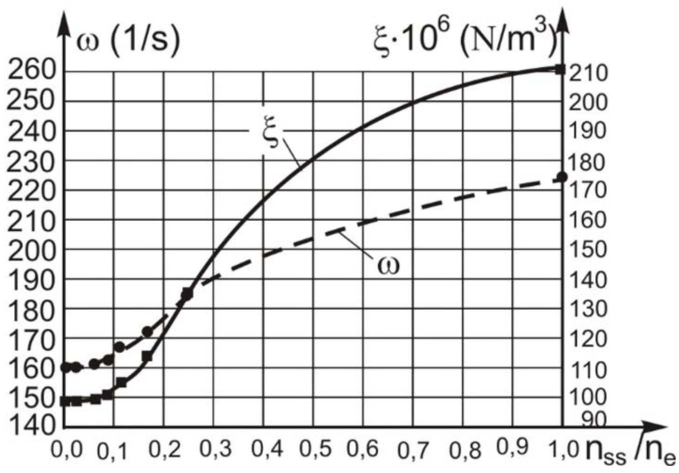


Figure 5:Change in frequencies of natural oscillations (ω) and stiffness of the seam (ξ) from the number of shift connections (nss./ne) at a rigid contour seal

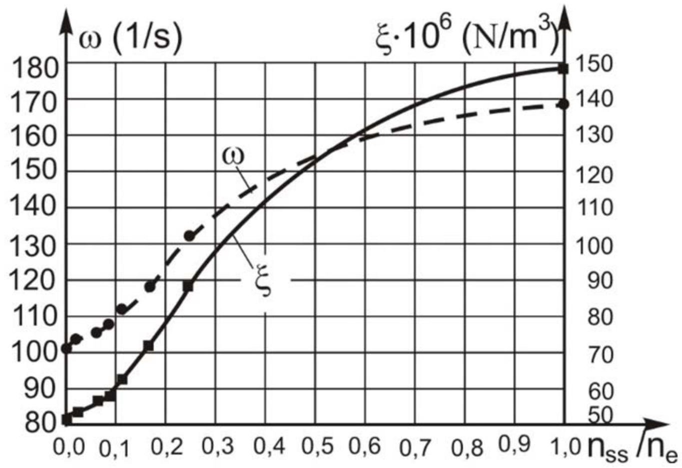


Figure 6: Change in frequencies of natural oscillations (ω) and stiffness of the seam (ξ) from the number of shift connections (nss./ne) at a hinged support along the contour

CONCLUSION

The analysis of the data shows that with the growth of the quantity of symmetrically arranged shift of connections in the two-layer plate increases both the frequency of natural oscillations and the stiffness coefficient of the seam, which does not contradict the physical meaning of the problem. In practical terms, the stiffness of the seam can be estimatedby determining the frequency of the basic tone of the oscillations of the composite plate.

REFERENCES

1. Rzhanitsyn A. (1976) [Calculation of composite plates with absolutely tight cross couplings]. Researches on the theory of constructions, issue XXII, M.: Stroyizdat, pp. 120-133. (in Russian)
2. Rzhanitsyn A. (1986) [Composite rods and plates]. M.: Stroyizdat, 316 p. (in Russian)
3. Kalmanok A. (1950) [Construction mechanics of plates]. M.: Mashstroyizdat, 303 p. (in Russian)
4. Shaposhnikov N. (1968) [Calculation of plates on a bend according to the finite-element method]. Works of MITE, issue 250, pp. 134 – 144. (in Russian)
5. Turkov A.V., Marfin K.V. (2012) [Interrelation of maximum deflections and natural oscillation frequencies of com-posite square isotropic plates depending on the number of symmetrically located shift connections]. Stroitel'stvoirekonstrukcija [Construction and Reconstruction], no 2, pp. 44-48. (in Russian)
6. Turkov A.V., Marfin K.V. (2012) [Determination of the coeffi-cient of the seam stiffness of a square compound isotropic plate based on its fundamental oscil-la-tion frequency]. [Construction mechanics and calculation of structures], no 5, pp. 38-44. (in Russian).
7. Turkov A.V., Marfin K.V. (2013) [Determination of the coefficient of the seam stiffness of a circular composite isotropic plate at its fundamental frequency of oscillations]. [Construction mechanics and calculation of structures], no 4, pp. 58-62. (in Russian).
8. Turkov A.V., Marfin K.V. (2013) Determination of the coefficient of the seam stiffness of a circu-lar composite isotropic plate at its fundamental frequency of oscillations. XVI International scientific and practical conference of students, Master students, postgraduate students and young scientists "Construction – the formation of the environment of life". Moscow, 24-26 April, pp. 187-190.
9. Turkov A., Karpova E. (2015) [The study of a seam stiffness coefficient for a triangular composite isotropic plate depending on its basic frequency of oscillations in case of different rigidness of shift bindings]. Construction mechanics and calculation of constructions, no. 2, pp. 66-69. (in Russian)
10. Karpova E.V. (2015) The study of the coefficient of the seam stiffness of composite triangular isotropic plates, depending on the number of symmetrically located shift connections. Collected Works of the Eighteenth International Interuniversity Scientific and Practical Conference of Stu-dents, Master Students and Young Scientists. Moscow, April 22-24, pp. 238-242.
11. Semenov A., Gabitov A. (2005) [The SCAD project computer system in educational process. Part I. Static calculation]. M.: ASV publishing house, 152 p. (in Russian)

Paper submitted: 24.10.2017.
Paper accepted: 28.11.2017.