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OPTIMIZATION OF AN ECONOMIC PRODUCTION QUANTITY-BASED SYSTEM WITH RANDOM SCRAP AND ADJUSTABLE PRODUCTION RATE

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With the aim of increasing capacity to smooth production planning and coping with existence of random scrap in real fabrication processes, this paper explores an economic production quantity (EPQ)-based inventory system with random scrap and adjustable production rate. Mathematical modeling is used to carefully portray and analyze the problem, and the expected system cost function is derived and proved to be a convex function. Then, differential calculus is employed to help determine the optimal batch size for the proposed system. Numerical example along with sensitivity analysis is provided to demonstrate applicability of the obtained results. Analytical outcomes pointed out that this in-depth exploration to the problem reveals diverse important managerial decision-making required information.

Key words: Optimizer, Randomness, Systems, Production planning, Economic production quantity, Adjustable production rate, Inventory system

INTRODUCTION

This paper explores an economic production quantity (EPQ)-based replenishment system with random scrap and adjustable production rate. Conventional EPQ model [01] assumes perfection in production process and a constant production rate, and it employed mathematical modeling to portray the problem and its relevant parameters, balance setup and inventory holding costs, and determine the most beneficial batch size. In real production environments, owing to diverse unexpected factors, random scrap items produced is inevitable. Mak [02] considered an inventory control problem in which defective items exist and product demand is partial met. Backordering is permitted to satisfy the shortage. The best replenishment theory to the problem was presented and demonstrated by a numerical example. Hariga and Ben-Daya [03] explored an imperfect EPQ problem, in which the production process may shift from in-control status to out-of-control status. For this process transition, they examined effects of both distribution-based and -free bounds on the expected system cost. They also showed the comparison result of optimal solution from their exponential case and the approximate solution from the literature. Jaber [04] studied a lot-size problem considering interruptions for product quality corrections and improvements on reduction of defective rate and consequence quality restoration action. A mathematical model was developed to help analyze the problem, and numerical examples were provided with results discussed. Chakraborty et al. [05] examined an economic manufacturing quantity (EMQ) model with random breakdown, repair, stock threshold level, and variable production rate. They assumed that machine failure is a function of production rate and extra capacity is reserved to cope with possible uncertainties during production process. General failure and repair time distributions were used to build their basic model, and two computational algorithms were proposed to help decide the best production rate and stock threshold level that minimize the expected system cost. Numerical examples were provided to illustrate the key managerial viewpoint insights of their model. Additional studies that addressed diverse aspects of production systems with defective items can also be referred to [06-13].

Moreover, with the aim of increasing machine capacity or reducing production cycle length, expediting production rate can be an effective strategy to achieve these operational goals. de Kok [14] considered a production-inventory control model with a specific (m, M)-policy production rate and a compound Poisson demand process. Approximations to the problem were presented to explore the operating characteristics of the system. As a result, a sequential rule of (M – m) and m was found to be the cost minimization policy subject to a service level constraint. Numerical examples were presented to confirm the accuracy of approximations. Moon et al. [15] examined a single-facility multi-item production system under a rotation production cycle time policy. They assumed that production rates can be reduced from the maximum rate during the production runs. They categorized products by their holding costs and used different production rate to fabricate these different items, with the aim of saving relevant production-inventory cost. Eiamkanchanalai and Banerjee [16] simultaneously determined the optimal runtime and production rate for a single-product fabrication system. They assumed that unit manufacturing cost is a quadratic function of the variable production rate, and explored the effect of variations in production rates on manufacturing cost.

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A numerical example with an iterative solution procedure was provided to show applicability of research result as well as sensitivity analyses. Gharbi et al. [17] studied the problem of production rate control for single-item remanufacturing systems. They considered that in remanufacturing operations for different components repair strategies can be different and executed at different rates, with the objective of defining proper rates to minimize the long-run repair and inventory/shortage costs. A multiple hedging point policy with two thresholds relating to two accelerated repair rates was proposed, to not only determine the parameters of the control policy, but also achieve a close approximation of the optimal repair policy. With the help of design of experiment, simulation modeling, and response surface methodology, they were able to obtain better control policies and demonstrated that the expected cost is lower than what was obtained from using classical hedging point policy. Sicilia et al. [18] investigated the optimal policy for an inventory system with shortage backordering, power demand, and production rate proportional to demand rate. They assumed that during the replenishment period, it is permitted to increase the demand rate, and accordingly, the production rate becomes variable for it is proportional to the demand rate. Stock-out situation is allowed and all shortages are backordered. A solution procedure was proposed to determine the optimal scheduling period, optimal reorder point, and the economic lot size that minimize the total system cost. Numerical examples were provided to demonstrate their model and results. Additional studies related to different aspects of fabrication systems with variable or adjustable production rates can also be found elsewhere [19-27]. In summary, with the aim of addressing aforementioned scrap and capacity increase issues, and providing production managers with a decision support-typed of system to deal with these practical situations, this study develops an exact model to portray the EPQ-based replenishment system with random scrap and adjustable production rate, and explores their effects on the optimal operating decisions. Details on the proposed model are provided as follows.

MATERIALS AND METHODS

The proposed EPQ-based system

This section presents the description, formulation, and modeling of the proposed EPQ-based system with random scraps and adjustable fabrication rate. Notations used for the proposed system analysis are given in Table 1 below.

With the aim of shortening production cycle time, the proposed EPQ-based system considers an adjustable production rate P_A units per year to meet the annual demand rate λ units. The relationship between adjusted production rate and standard (unadjusted) production rate is $P_A = (1 + \alpha_1)P$, where α_1 denotes the adjusted proportion and P represents the standard rate. Unit production cost C_A and setup cost K_A rise accordingly. The relationships between these costs and the standard ones are as follows: $C_A = (1 + \alpha_3)C$ and $K_A = (1 + \alpha_2)K$, where α_2 and α_3 denote the increase percentages, and C and K are standard unit and setup costs, respectively.

The proposed system also assumes that in production uptime t_{1A} (Figure 1), an x proportion of defective items is randomly produced at a rate d_A (so $d_A = P_A x$), and all defective items will be scrapped in the end of production, at a cost of C_S per scrap item. The shortage situations are not permitted in the proposed system, so the following equation must satisfy: $P_A - d_A - \lambda > 0$.

Modeling and analysis

The maximal quantity of finished products in the end of uptime t_{1A} is

$$H = (P_A - d_A - \lambda)t_{1A} = (P_A - xP_A - \lambda)t_{1A} = \{(1-x)[(1+\alpha_1)P] - \lambda\}t_{1A} \quad (1)$$

The maximal quantity of scrap items in the proposed EPQ-based system is $d_A t_{1A}$ or xQ (Figure 2). Upon completion of the production process, depletion of on-hand finished products starts in downtime t_{2A}. The following formulas of cycle time, production uptime, and downtime can be obtained from Figures 1 and 2.

$$T_A = t_{1A} + t_{2A} = \frac{Q(1-x)}{\lambda} \quad (2)$$

$$t_{1A} = \frac{Q}{P_A} = \frac{H}{P_A - d_A - \lambda} = \frac{H}{(1-x)(1+\alpha_1)P - \lambda} \quad (3)$$

$$t_{2A} = \frac{H}{\lambda} = \frac{\{(1-x)[(1+\alpha_1)P] - \lambda\}t_{1A}}{\lambda} \quad (4)$$

Total system cost per cycle of the proposed EPQ system, TC(Q) consists of production setup cost, variable fabrication and disposal costs, and stock holding costs of finished and defective items in the production cycle, as follows:

$$TC(Q) = K_A + C_A Q + C_S(xQ) + h\left[\frac{H + d_A t_{1A}}{2}(t_{1A}) + \frac{H}{2}(t_{2A})\right] \quad (5)$$

Substitute K_A, C_A, P_A, and dA in Equation (5), $TC(Q)$ turns into

$$TC(Q) = [(1+\alpha_2)K] + [(1+\alpha_3)C]Q + C_S(xQ) + h\left\{\frac{H + x[(1+\alpha_1)P]t_{1A}}{2}(t_{1A}) + \frac{H}{2}(t_{2A})\right\} \quad (6)$$

Further replace H, t_{1A}, and t_{2A} with the right-hand side results from Equations. (1), (3) and (4) in Equation (6), and apply the expected values of x to cope with its randomness, and with additional derivations, $E[TCU(Q)]$ can be found as follows:

$$E[TCU(Q)] = \frac{E[TC(Q)]}{E[T_A]} = \lambda[(1+\alpha_3)CE_0 + C_S E_1] + \frac{[(1+\alpha_2)K]\lambda E_0}{Q} + \frac{hQ(1-E[x])}{2} - \frac{h\lambda Q(1-2E[x])E_0}{2(1+\alpha_1)P} \quad (7)$$

where

$$E_0 = \frac{1}{1-E[x]};\ E_1 = \frac{E[x]}{1-E[x]}$$

Table 1: List of notations used in the proposed EPQ system

Symbol	Description
λ	Product demand rate per unit time in the proposed EPQ system
P_A	Adjusted production rate of the proposed EPQ system
P	Standard production rate in the conventional EPQ system
α_1	Adjusted proportion of production rate
Q	Production batch size of the proposed EPQ system – the decision variable
K_A	Adjusted production setup cost per cycle in the proposed EPQ system
K	Standard setup cost per cycle in the conventional EPQ system
α_2	Adjusted proportion of setup cost versus the standard setup cost
C_A	Adjusted unit production cost in the proposed EPQ system
C	Standard unit production cost in the conventional EPQ system
α_3	Adjusted proportion of unit production cost versus the standard one
x	Random scrap rate in production process of the proposed EPQ system
d_A	Production rate of scrap items in the proposed EPQ system
h	Unit holding cost per year
C_S	Unit disposal cost per scrapped item
t_{1A}	Uptime of the proposed EPQ system with adjustable production rate
t_{2A}	Downtime of the proposed EPQ system
T_A	Cycle time of the proposed EPQ system
$E[T_A]$	The expected cycle time of the proposed EPQ system
TC(Q)	Total system cost per cycle of the proposed EPQ system
$E[TCU(Q)]$	The expected total system cost per unit time in the proposed EPQ system
$I(t)$	On-hand inventory level of perfect quality products at time t
$Id(t)$	On-hand inventory level of scrap items at time t
d	Production rate of scrap items in the traditional EPQ system
t_1	Uptime in the traditional EPQ system
t_2	Downtime in the traditional EPQ system
T	Cycle time in the traditional EPQ system

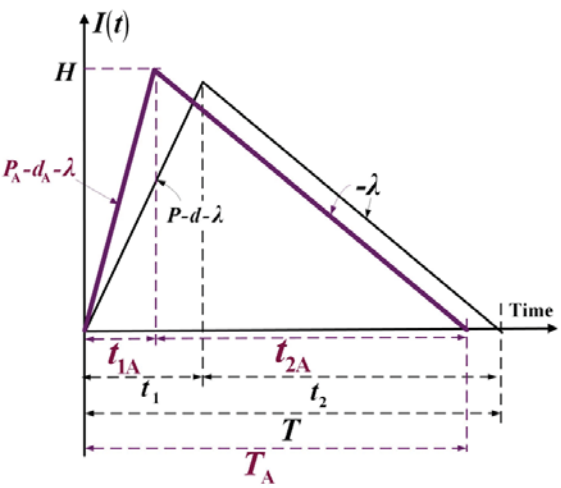

Figure 1: On-hand inventory level of finished products in the proposed EPQ-based system with an adjustable rate (in purple) compared to that in traditional EPQ model (in black)

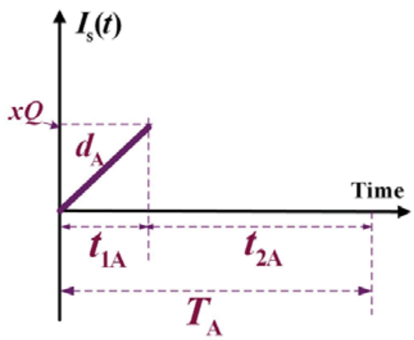

Figure 2: On-hand inventory level of scrap items in the proposed EPQ-based system

RESULTS AND DISCUSSION

Determining the optimal replenishment batch size

First, apply the first and second derivatives of $E[TCU(Q)]$ with respect to Q, and we find the following:

$$\frac{dE\left[TCU(Q)\right]}{dQ}=-\frac{\left[(1+\alpha_2)K\right]\lambda E_0}{Q^2}+\frac{h(1-E[x])}{2}-\frac{h\lambda\left(1-2E[x]\right)E_0}{2(1+\alpha_1)P} \qquad (8)$$

$$\frac{d^2E\left[TCU(Q)\right]}{dQ^2}=\frac{2\left[(1+\alpha_2)K\right]\lambda E_0}{Q^3} \qquad (9)$$

Since Equation (9) results positive (because α_2, K, λ, E_0, and Q are all positive), thus $E[TCU(Q)]$ is a convex function for all Q different from zero. To determine the optimal replenishment batch size, one can set Equation (8) equal to zero and solve for Q^* as follows:

$$\frac{dE\left[TCU(Q)\right]}{dQ}=-\frac{\left[(1+\alpha_2)K\right]\lambda E_0}{Q^2}+\frac{h(1-E[x])}{2}-\frac{h\lambda\left(1-2E[x]\right)E_0}{2(1+\alpha_1)P}=0 \qquad (10)$$

Therefore, Q^* can be obtained as follows:

$$Q^*=\sqrt{\frac{2(1+\alpha_2)K\lambda}{h\left(1-E[x]\right)^2-\frac{h\lambda\left(1-2E[x]\right)}{(1+\alpha_1)P}}} \qquad (11)$$

Verification of optimal batch size

Let α_1 and α_2 equal to zeros, the proposed system turns into traditional (unadjusted production rate) EPQ model with random scrap, Equation (11) becomes

$$Q^*=\sqrt{\frac{2K\lambda}{h\left(1-E[x]\right)^2-\frac{h\lambda\left(1-2E[x]\right)}{P}}} \qquad (12)$$

Further, if $x = 0$, the proposed system becomes a traditional EPQ model [28] as follows:

$$Q^*=\sqrt{\frac{2K\lambda}{h-\frac{h\lambda}{P}}}=\sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)}} \qquad (13)$$

Numerical example and sensitivity analysis

This subsection demonstrates the proposed model with the following values of system variables: annual demand rate of a specific product λ = 4000 units; standard production rate P = 20000 units; assuming adjusted production rate factor α_1 = 0.5; hence, $P_A = (1 + \alpha_1)P$ = 30000 units per unit time; random scrap rate of production process x (follows a uniform distribution) over the interval of [0, 0.2]; standard setup cost per cycle K = \$5,000; adjusted setup cost factor $\alpha_2 = 0.2(\alpha_1) = 0.1$; thus, $K_A = (1 + \alpha_2)K$ = \$5,500; standard unit production cost C = \$100; adjusted unit production cost $\alpha_3 = 0.5(\alpha_1) = 0.25$; hence, $C_A = (1 + \alpha_3)C$ = \$125; unit inventory holding cost per unit time h = \$30; and unit disposal cost C_S = \$20. Applying Equation (11), we obtain the optimal production batch size Q^* = 1444, and computing Equation (7), we also find the expected total system cost per unit time $E[TCU(Q^*)]$ = \$598,300. Variation in batch size Q effects on system cost $E[TCU(Q)]$ is depicted in Figure 3.

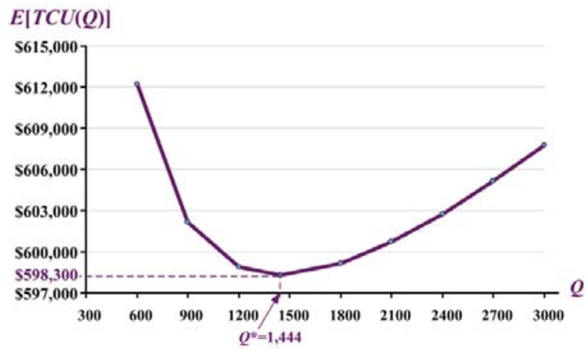

Figure 3: Variation in batch size Q effects on the expected system cost E[TCU(Q)]

Variation in random scrap rate effects on various cost components of $E[TCU(Q)]$ is illustrated in Figure 4. It can be seen that as random scrap rate x increases, variable production cost and scrap items' disposal cost go up significantly.

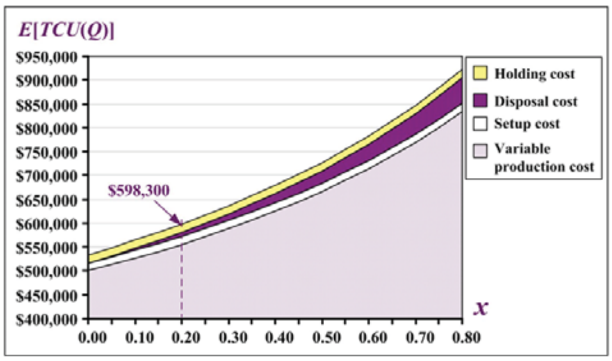

Figure 4: Variation in random scrap rate effects on various cost components of E[TCU(Q)]

Joint effects of the batch size Q and random scrap rate x on the expected total system cost $E[TCU(Q)]$ is shown in Figure 5. It is noted that as random scrap rate x increases, the expected total system cost $E[TCU(Q)]$ raises significantly.

Sensitivity analysis on the effect of production rate adjusted ratio P_A/P on the percentage of time that machine is used in a cycle (i.e., machine utilization) is depicted in Figure 6. It can be seen that at P_A/P = 1.5 (as assumed in this example), machine utilization decreases from 22.2% to 14.8% (i.e., a decrease of 33.33%); and as P_A/P declines, $t_{1A}/E[T_A]$ decreases accordingly. Detailed results from the sensitivity analyses are displayed in Table A-1 (see Appendix A).

Joint effects of random scrap rate x and production rate adjusted ratio P_A /P on the expected total system cost $E[TCU(Q)]$ is illustrated in Figure 7. It is noted that as random scrap rate x moves up, $E[TCU(Q)]$ raises; and as production rate adjusted ratio P_A /P goes higher, expected total system cost $E[TCU(Q)]$ increases, significantly.

Further analytical results (as shown in Table A-2) also reveal the effect of production rate adjusted ratio P_A /P on the optimal batch size (Figure 8), and the effect of unit production cost adjusted ratio C_A /C on the system's variable production cost (Figure 9).

From Figure 8, it can be seen that at P_A /P = 1.5 (as assumed in this example), Q^* = 1444; and as P_A /P goes up, optimal batch size Q^* increases significantly. It is also noted (from Figure 9) that at C_A /C = 1.25 (as assumed in this example), the variable production cost of the system is \$555,556 (see Table A-2), and as C_A /C moves up, system's variable production cost increase accordingly.

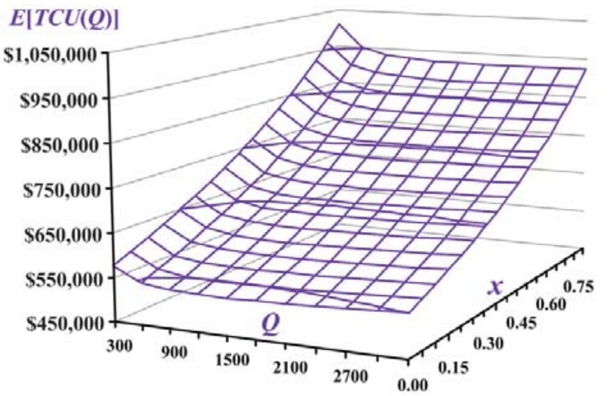

Figure 5: Joint effects of the batch size Q and random scrap rate x on the expected total system cost E[TCU(Q)]

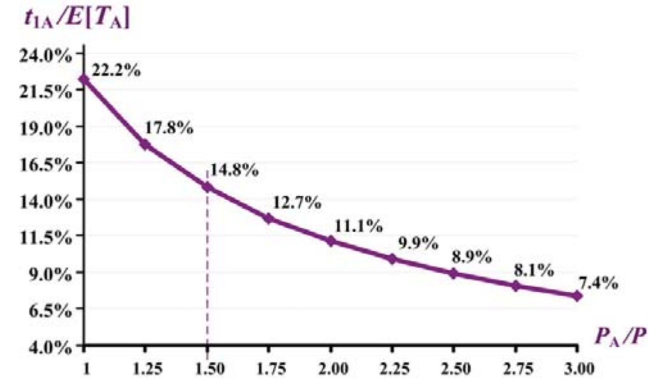

Figure 6: Effect of production rate adjusted ratio P_A /P on the percentage of time that machine is used in a cycle t_{1A}/$E[T_A]$

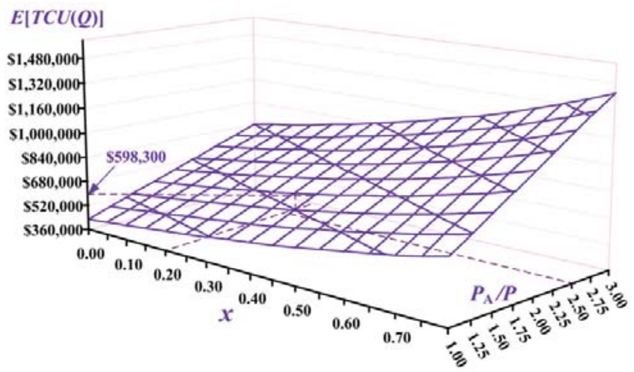

Figure 7: Joint effects of random scrap rate x and production rate adjusted ratio P_A /P on the expected total system cost E[TCU(Q)]

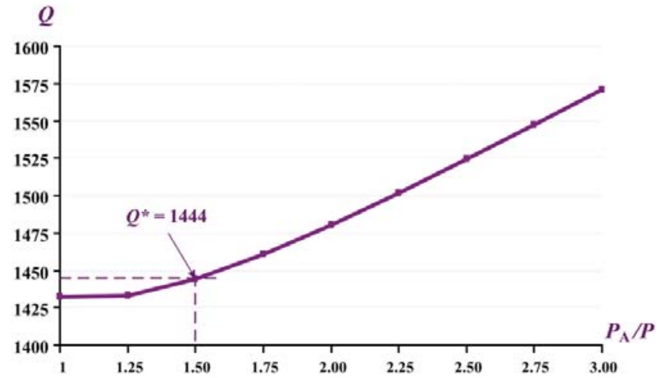

Figure 8: Effect of production rate adjusted ratio P_A /P on the optimal batch size

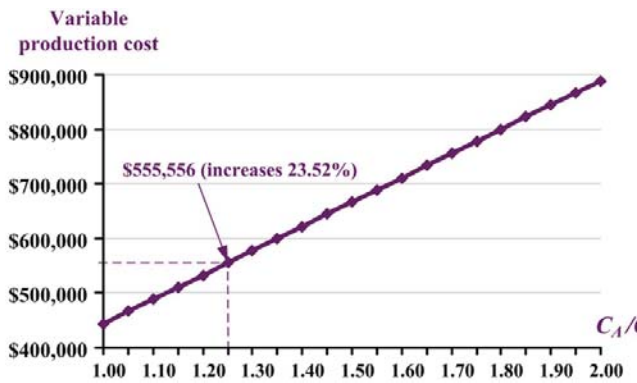

Figure 9: Effect of unit production cost adjusted ratio CA /C on system's variable production cost

CONCLUSION

With the aim of increasing capacity to smooth production planning and coping with existence of random scrap in real fabrication processes, this paper examines an economic production quantity (EPQ)-based inventory system with random scrap and adjustable production rate. An exact mathematical model to the problem is built to carefully portray, analyze, and solve the problem. Numerical example along with sensitivity analysis is provided to show the applicability of our obtained results.

Main contributions of this study include (1) for any given adjusted factors on production rate and cost variables, together with other system parameters, the proposed model can help determine the optimal batch size to the problem; (2) for any given random scrap rate of the production process, its effects on optimal batch size and on various system cost components can be obtained; (3) for any given production rate adjusted factor, its effects on machine utilization and optimal batch size can be determined; (4) for any given random scrap rate and production rate adjusted factor, their joint effects on the expected system cost can be revealed; and (5) for any given unit production cost adjusted factor, its effect on system's variable production cost can be obtained. These analytical outcomes indicated that without an in-depth exploration of the problem, many critical managerial decision-making related information (see Figures 3-9 and Tables A-1 to A-2 in Appendix - A) cannot be revealed. Exploring the effect of rework process on optimal batch size and system cost to the problem will be an interesting direction for future study.

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APPENDIX - A

Table A-1: Production rate adjusted factor effects on production uptime, batch size, the expected cycle time, and machine utilization and its decrease percentage

$1+\alpha_1$ (P_A / P)	Production uptime (t_{1A})	Q^*	$E[T_A]$	Machine utilization t_{1A} / $E[T_A]$	Utilization decrease %
1.0	0.0716	1432	0.3223	22.22%	-
1.1	0.0650	1431	0.3219	20.20%	-9.09%
1.2	0.0596	1432	0.3221	18.52%	-16.67%
1.3	0.0552	1434	0.3227	17.09%	-23.08%
1.4	0.0514	1439	0.3237	15.87%	-28.57%
1.5	0.0481	1444	0.3249	14.81%	-33.33%
1.6	0.0453	1450	0.3263	13.89%	-37.50%
1.7	0.0429	1457	0.3279	13.07%	-41.18%
1.8	0.0407	1465	0.3295	12.35%	-44.44%
1.9	0.0387	1472	0.3313	11.70%	-47.37%
2.0	0.0370	1480	0.3331	11.11%	-50.00%
2.1	0.0354	1489	0.3350	10.58%	-52.38%
2.2	0.0340	1497	0.3369	10.10%	-54.55%
2.3	0.0327	1506	0.3389	9.66%	-56.52%
2.4	0.0316	1515	0.3409	9.26%	-58.33%
2.5	0.0305	1524	0.3430	8.89%	-60.00%
2.6	0.0295	1533	0.3450	8.55%	-61.54%
2.7	0.0286	1543	0.3471	8.23%	-62.96%
2.8	0.0277	1552	0.3492	7.94%	-64.29%
2.9	0.0269	1561	0.3513	7.66%	-65.52%
3.0	0.0262	1571	0.3534	7.41%	-66.67%

Table A-2:Adjusted ratios of production rate and unit cost effect on optimal batch size, variable production cost, and the expected total system cost and its increase %

$1+\alpha_1$ (PA / P)	$1+\alpha_3$ (CA / C)	Q*	Variable Manufacturing cost	E[TCU(Q)]	Increase %
1.0	1	1432	$444,444	$484,365	
1.1	1.05	1486	$466,667	$507,245	4.72%
1.2	1.10	1472	$488,889	$530,067	9.44%
1.3	1.15	1461	$511,111	$552,844	14.14%
1.4	1.20	1452	$533,333	$575,586	18.83%
1.5	1.25	1444	$555,556	$598,300	23.52%
1.6	1.30	1437	$577,778	$620,990	28.21%
1.7	1.35	1431	$600,000	$643,660	32.89%
1.8	1.40	1426	$622,222	$666,314	37.56%
1.9	1.45	1422	$644,444	$688,953	42.24%
2.0	1.50	1417	$666,667	$711,580	46.91%
2.1	1.55	1414	$688,889	$734,196	51.58%
2.2	1.60	1410	$711,111	$756,802	56.25%
2.3	1.65	1407	$733,333	$779,399	60.91%
2.4	1.70	1405	$755,556	$801,989	65.58%
2.5	1.75	1402	$777,778	$824,571	70.24%
2.6	1.80	1400	$800,000	$847,147	74.90%
2.7	1.85	1398	$822,222	$869,716	79.56%
2.8	1.90	1396	$844,444	$892,280	84.22%
2.9	1.95	1394	$866,667	$914,839	88.87%
3.0	2.00	1392	$888,889	$937,393	93.53%

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