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MATHEMATICAL ANALYSIS OF A PARKING SYSTEM FOR TELEMETRY APPLICATIONS

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In this paper we are using the waiting system theory and we make a mathematical analysis, to find the optimal solution for a smart parking system. We apply to the smart parking system the theory of one waiting system with many points of service, and for the compatible parking space. More specifically, according to the waiting system theory , we made a count of customers so we can see the number of customers trying to find a parking space. Then we mentioned the customers who arrive in the system either according to a known space or otherwise, at "random" mathematical times. In the "random" times that customers have come to the system, we have been helped by the distribution of Poisson. Thus, we have clearly seen the time of customer service as well as their positions in the system. In the end, we analyzed the models of Poisson distribution where each separately explains the cases of customers in the system and with mathematical equations we arrived at a right outcome. It is necessary to notice that, the following proof is a mathematical example to understand the proper use of a smart parking by using the Waiting system theory and Poisson distribution.

Key words: Mathematic Analysis, Poisson distribution, Waiting system, parking, Telemetry

INTRODUCTION

Finding an economical and easy parking in a congested country like Greece is really difficult. So, we had an idea to find a solution for that fact. We decided to analyze with mathematical precision a smart parking system for telemetry applications and a right parking space for our country. Our mathematical analysis based on the waiting system theory for the source of costumers , the arrivals of the costumers in the parking lot, so that we could arrive at the distribution of Poisson for a clear conclusion.

With the help of mathematical equations we ended up in two customer cases, the automated parking system and the compatible parking system. What we understood was that with the proper cost of operation and the average of customers an automated parking system is much better for the right balance and a parking space than the compatible parking system.

METHOD

According to the textbook of Georgios Kostaras, who is teaching at the University of Patras in the postgraduate section entitled "Computer Mathematics and Decisions" ,waiting systems are ordinary in service systems where demand for a service can't be met sometimes directly by the capacity of the system that provides the service. Knowing the functional characteristics of service and queue queues can result in spectacular improvements in performance. The performance of the system is evaluated on the basis of the values of some key indicators (performance indices - functionality measures), such as the average wait time of a customer in the waiting system, the total average stay time of a client in the system, the average number of customers in the waiting system, the average number of customers in the system, the employment rate of the service or service locations, etc. The purpose of the study of a service system is to minimize its operating costs provided the prices of the performance indices As the system meet minimum standards.

Waiting System Features

Customer Source: The population from which customers arrive is considered to be either infinite (practically very large), such as bank customers, cars at toll stations, etc., or finite, as for example in the case of the machines of a factory waiting for repair. On more waiting systems, unless we specifically refer to a finite population, we will assume that the population from which the system's "clients" are inexperienced.

Arrivals in the System: In each waiting system there are "customers" who come for service. By the generic term "customer" we mean the persons, objects or events that enter the system for service. Arrivals in a queue system are characterized by the following key features:

Arrival Distribution: "Customers" arrive at the system either at a known and constant rate (eg a semi-finished product at a workstation exactly every 15 minutes) or else, as in most cases, at "random" times eg patients on call). Arrivals are considered random when they are independent of one another (no one is affected by an earlier one) and their timing can't be predicted exactly. In

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this case, the Average arrivals rate is the average number of arrivals per unit of time (eg "customers" per hour). In queue theory, the random variable "number of arrivals per unit time" can often be approximated by the Poisson distribution. If this is done then the average value of Poisson corresponds to the average of arrivals per unit of time, denoted by λ and is the average rate of arrivals per unit of time. For example, if in a service system the arrival process follows the Poisson distribution and arrives on an average of 10 customers per hour, then λ = 10 and this is the average Poisson allocation for the arrivals process. If we show X the arrivals that are likely to be made in one hour (ie in the time unit), then X is as mentioned, a random variable and the probability to get it a certain value of x (x is a given number) is given by the relationship:

$$P\{X = x\} = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}, x = 0,1,2,3,... \quad 1)$$

Note that when the average rate of arrivals is λ (= 10 persons / hour as mentioned above), then it is reasonable to assume that between two consecutive arrivals, time is averaged equal to 1 / λ (= 1/10 6 minutes in the example).

Service Time: The time required to serve the customer may be fixed (eg in an automatic car wash where exactly 10 minutes are required for each vehicle, at a processing station in an industry where exactly three seconds are required to fit a component) or, as is the case with most sleep queue systems, it is volatile due to various factors. For many sleep queue systems, we can assume that service time follows the exponential distribution, with an average of 1 / μ. For example, if a cashier in a bank is able to serve an average of 15 people per hour, then we say that the average service rate is μ = 15 people / hour and logically 1 / μ = 1/15 hours is the average service time (i.e. in this case is 1 / μ = 1/15 hours = 4 minutes). If T is the time required for a service, then T is a random variable and the probability that this time is less than or equal to a given value of t is given by:

$$P(T \leq t) = 1 - e^{-\frac{1}{\mu}t} \quad 2)$$

where μ, as mentioned, denotes the average number of customers served in the time unit.

Service locations: For a waiting customer, there may be more than one parallel service (eg bank, toll, treasury, etc.). In this case, the customer is served by the first available service location. Also, for the full service of the client, it is necessary to have it successively in more than one service place, ie it is served in successive phases (eg the processing of a job requiring multi-stage approvals).

Waiting system Function: The queue is formed by 'customers' waiting to be serviced. The way in which a queued customer is selected to serve is one of the main features of queuing systems and is called discipline. The methods applied are mainly:

- FIFO (First In First Out): Customers are served on a turn-by-turn basis.
- LIFO (Last In First Out): Customers are serviced inversely in turn-by-turn order.
- Random Selection: Customers are randomly selected by queued.
- Priorities: Customers are divided into categories with different priorities. First, customers with the highest priority are selected. Among customers with the same priority class, the one who is most likely to wait for the longest time (eg disabled, elderly, first served)

Another interesting feature concerning the queue is its capacity. The capacity of the queue can be infinite (practically, whoever comes in may stay) or finite (when someone comes in after having all the queues occupied, he can't enter the system).

Model symbology: Depending on the operating characteristics of a queue system, we also have a different model of its analysis.

We are having a handy five-symbol symbol having the general form "A / B / s / k / N", where the symbols represent the following:

- A: Location for the Customer Input Allocation symbol. Possible symbol for position A is M, representing the Poisson process. Other symbols are G, which means general or any distribution, and D means a deterministic entry process, that is to say, with a known Deterministic.
- B: location for the service time allocation symbol. The same symbols are used as in case A.
- s: position for the number of parallel service locations.
- k: position for service capacity when queue positions are limited. K is the number of queues together with service locations.
- N: position for the number of customers at source when finite.

Before proceeding to quote the models, we note that when we have a Poisson distribution with an average arrival rate equal to λ, then the time between successive arrivals follows an exponential distribution with an average value of 1 / λ. Similarly, when the service time follows an exponential distribution with an average of 1 / μ, then the number of customers served in a time interval follows a Poisson distribution with an average value of μ. We have already reported some data on this as a further example, let us assume that we have a central computing system in which the work entry process follows a Poisson distribution with an average λ = 20 jobs per minute. Then, the time between successive arrivals follows an exponential distribution with an average value of 1 / λ = 1/20 minutes, that is, 3 seconds on average. This is very logical, since when we have a system that arrives on an average of 20 jobs a minute, we can actually assume we have an average of one work per 3 seconds. Be careful! This does not mean that and in fact we always have a job to arrive every 3 seconds, just this is the average behavior of the entry process. Similarly, let us have the computer system (a service facility) that can handle an average job in 2 seconds following an exponential distribution for the service time. Then it is 1 / μ = 1/30 minutes (= 2 seconds) and the number of jobs served per minute follows the Poisson distribution, with an average μ = 30 operations per minute.

It is also important to refer to the concept of steady state. A system is in a balanced state when its behavior does not depend on the initial conditions at its start-up. That is, a service system reaches equilibrium when a reasonable period of time has elapsed since its initial condition, during which the effect of the starting conditions is eliminated. The period required for the system to not depend on the initial starting conditions and converge to equilibrium is called the transition period (warm period). The models mentioned below and the type we use consider the system to be in balance.

MATH

Note: We are considering the parking as a service center without looking at its internal features. This explains the long service time of the parking system.

Model M / M / 1

It is used to study a queuing system where the following applies: The customer arrival process follows the Poisson allocation with an average arrival rate of λ per unit of time. The service process follows a Poisson distribution with an average number of customers served per unit time equal to μ. That is, the service time follows the exponential distribution with an average value of 1 / μ. There is a service station. The number of customers at source is infinite (too large), customers are served with FIFO discipline, they form a queue that has infinite capacity and does not leave as long as the tail is. The fundamental relationship that must be in place for a state of equilibrium can be found.

Model M / M / s

It is used to study a queuing system where the following applies: The customer arrival process follows the Poisson allocation with an average of arrivals λ per unit of time. There are more than one parallel service (s> 1). The service time in each location follows the exponential distribution with an average number of customers served at each location, per unit of time. The number of customers at source is practically infinite, customers are served on a FIFO basis, they form a queue that has infinite capacity and does not leave as long as the queue is and is served by the first available service unit. The fundamental relationship that must be in place for a balance can be found.

Determination of capacity of service systems

The objective of analyzing waiting systems is usually to determine the system's capacity, that is, the number of service locations, for which the overall expected variable cost of the system is minimized. This cost for an enterprise consists of two sub-components, the cost of waiting for customers and the cost of providing the service. When the capacity of the system increases with additional service locations, the average customer service time in the system decreases, this reducing the cost of waiting for customers. In this case, the cost of providing the service is increased due to the addition of service locations. On the other hand, when system capacity is reduced, the cost of customer staying in the system increases due to an increase in their stay time, but at the same time reduces service costs due to a lower number of service locations. So the question is how we can balance between the waiting costs of customers and the cost of service on the part of the company by designing the service system in the most appropriate manner.

Configuring cost-effectiveness

The total variable cost of operation of the system, which we denote by TC (Total Cost), is the sum of two individual cost elements, the cost of waiting for customers, which we denote by WC (Waiting Cost) and the service cost of the system, which we denote with SC (Service Cost). Let us denote the cost of waiting a customer in the time unit. Note that when we refer to standby costs, we basically mean the total time of the customer's stay in the system. Consequently, once we have denoted the average total time of a customer's stay in the system, the expected cost of waiting the customer is in the unit of time. However, this quantity refers to a single customer and does not reflect the total cost of waiting for customers. In order to calculate the total expected customer cost, we multiply this quantity by the average rate of arrivals of customers per unit of time, i.e. by λ. So, we have that:

$$WC = c_w W\lambda = c_w L \qquad 3)$$

As you can see, the wait time of the customers in the unit per time, WC, ultimately results from the product of the customer's waiting time in the unit of time with the average number of customers in the system. At this point, we consider it worthwhile to refer in more detail to the cost of a customer in the time unit, that is to say. Its discretion depends mainly on whether the customer is part of the service (internal customer) or not (external customer). If the client is internal to the system, e.g. owned vehicles of the company waiting to be loaded / unloaded, machines that remain unused due to damage and are not repaired immediately, craftsmen expecting equipment or spare parts to repair engine damage, semi-finished products "waiting" for their shipment to the next production phase, etc., then the estimate of the customer's cost of waiting is relatively easy.

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Table 1 : Mathematical formulas for Model M/M/1 and Model M/M/s

	M/M/1	M/M/s
Ls : average number of customers in service	$L_s = \frac{\lambda}{\mu}$	$L_s = \frac{\lambda}{\mu}$
ρ: Degree of service of the service system	$\rho = \frac{\lambda}{\mu}$	$\rho = \frac{\lambda}{s\mu}$
Lq: average number of customers in waiting	$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$	$L_q = \frac{(\lambda/\mu)^s \lambda\mu}{(s-1)!(s\mu-\lambda)^2} \cdot P_0$
L: average number of customers in the system as a whole	$L = \frac{\lambda}{\mu-\lambda}$ or $L = L_q + L_s = L_q + \frac{\lambda}{\mu}$	$L = L_q + L_s = L_q + \frac{\lambda}{\mu}$
Wq: average wait time for a customer in the waiting system	$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$ or $W_q = \frac{L_q}{\lambda}$	$W_q = \frac{L_q}{\lambda}$
W: average stay time of a client in the system	$W = \frac{1}{\mu-\lambda}$ or $W = \frac{L}{\lambda}$ or $W = W_q + \frac{1}{\mu}$	$W = \frac{L}{\lambda}$ or $W = W_q + \frac{1}{\mu}$
P0: the probability that there is no client in the system or the equivalent of the time that all the positions are inactive	$P_0 = 1 - \frac{\lambda}{\mu}$	$P_0 = \frac{1}{\sum_{n=0}^{s-1}\left[\frac{(\lambda/\mu)^n}{n!}\right] + \frac{(\lambda/\mu)^s}{s!} \cdot \left(\frac{s\mu}{s\mu-\lambda}\right)}$
Pw: the probability that a customer who arrives in the system will have to wait	$P_W = 1 - P_0$	$P_w = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{s\mu}{s\mu-\lambda}\right) P_0$
Pn: probability of n customers in the system	$P_n = \left(\frac{\lambda}{\mu}\right)^n \cdot P_0$	$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n \cdot P_0, & n \le s \\ \frac{1}{s! s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n \cdot P_0, & n > s \end{cases}$
Pn>k: the likelihood of there being more than one customer in the system	$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1}$	$P_{n>k} = 1 - (P_0 + P_1 + \dots P_k)$

On the contrary, when customers are not part of the service system (external customers), such as supermarket customers, maintenance or repair vehicles in a workshop, patients in outpatient clinics, etc., the cost estimate from customer expectation is more difficult. In these cases of external customers, actual standby costs may vary considerably from customer to customer.

With regard to service costs, things are simpler. We have seen that the key factor determining the service cost is the number of service locations, i.e. s. If it represents the service cost of one position in the unit of time, then the service cost SC for offered capacity s is:

$$SC = c_s \cdot s \quad 4)$$

Using the relations (3) and (4) we get the relation (5) that gives the total expected variable cost of operation of the system to the unit of time:

$$TC = WC + SC = c_w L + c_s s \qquad 5)$$

To identify the optimum capacity of a service system, you need to specify TC for different s values and select that s, which corresponds to the lowest total cost of operation. Since the behavior of the "function" TC is convex it is easy to identify the only minimum. In relation (6) the values of the coefficients are estimates and in particular it is more difficult to calculate than. Therefore, the total TC cost is an approximation as accurate as the estimates of the above values are accurate. Also, if we consider that the customer's time at the service site is not included in the cost, we can use a model based only on waiting queue rather than the total time spent in the system. Then, it is enough to replace in relation (6) with it, so we get the relation:

$$TC = WC + SC = c_w L_q + c_s s \qquad 6)$$

One company belongs to the automated parking area, operating in the following way A:

Peak customers demanding service, arrive on average every 1.6 minutes with the time inserted between successive arrivals following the exponential distribution. At peak times the automated system keeps open 5 evenly parking with a common waiting system and FIFO discipline. Based on historical financial analysis, it is estimated that when customers are in the queue of either waiting or servicing customers, they incur a cost to the company of € 3 per hour (per customer). One of the five uniforms takes an average of 6 minutes to complete a customer service, following an expansive service time allocation. The relative (variable) hourly cost for the company to maintain an open parking space is € 45.

If the company operated the five (5) parking spaces as if they were each in accordance with mode B:

There can be nothing to change about the way customers arrive, which continues to be an average customer per 1.6 minutes with exponential distribution. One of these five parking spaces will have their own waiting system with FIFO discipline .. It is noted that from the data available there is known that the percentage of clients coming in that category that come to each conventional parking space accounts for 20% customers. The average customer service time is 10 minutes (exponential service time allocation). The cost of waiting - customer stay and for company B remains the same as in case A regardless of the category to which it belongs, that is, € 3 per hour (per customer). The same as before, the service cost is still € 45 per hour.

RESULTS

Prerequisites for Current State A:

Customers arrive by Poisson process at an average rate of λ = 37.5 clients / hour, after the average time between successive arrivals is 1.6 minutes (60 / 1.6 = 37.5). We have five uniform parking spaces, each having the same service rate (Poisson) at an average rate of at least one, which for each parking lot is equal to 10 customers per hour (after an average service time of 6 minutes). Because λ / sμ = 37,5 / (5x 10) = 37,5 / 50 = 0,75 <1 there is convergence in equilibrium mode and we can proceed to the calculations.

So, for the current A mode, in equilibrium mode, using the formulas for the M / M / s system, for λ = 37.5, μ = 10 and s = 5, we get the following system M / M / s, for λ = 37.5, μ = 10 and s = 5, we get the following:

First we calculate the probability P0 that is needed to calculate the LqA to follow. So it is :

$$P_0 = \frac{1}{\left[\frac{\left(\frac{37{,}5}{10}\right)^0}{0!}\right] + \left[\frac{\left(\frac{37{,}5}{10}\right)^1}{1!}\right] + \left[\frac{\left(\frac{37{,}5}{10}\right)^2}{2!}\right] + \left[\frac{\left(\frac{37{,}5}{10}\right)^3}{3!}\right] + \left[\frac{\left(\frac{37{,}5}{10}\right)^4}{4!}\right] + \frac{\left(\frac{37{,}5}{10}\right)^5}{5!}\cdot\left(\frac{5\cdot 10}{5\cdot 10 - 37{,}5}\right)}$$

= 0.018681 (i.e., about 1.87%).

• Average waiting system length:

$$L_{qA} = \frac{(\lambda/\mu)^s \lambda\mu}{(s-1)!(s\mu-\lambda)^2}\cdot P_0 =$$

$$\frac{\left(\frac{37{,}5}{10}\right)^5 \cdot 37{,}5\cdot 10}{(5-1)!(5\cdot 10-37{,}5)^2}\cdot P_0 = 1{,}385367 \text{ customers}$$

• Average number of clients in the system:

$$L_A = L_{qA} + \frac{\lambda}{\mu} = 1{,}385367 + 37{,}5/10 = 5{,}135367 \text{ customers.}$$

• = 1,385367 + 37,5/10 = 5,135367 customers.

To calculate the total operating cost for A mode we have:

cw = € 3 per hour, cs = € 45 per hour, LA = 5,135367 and s = 5. So, TCA = WCA + SCA = cw x LA + cs x s, maxTC = 3 x 5,135367 + 45 x 5 = 240,4061 € per hour.

Prerequisites for the proposed mode B:

First of all, customers of all parking lots continue to arrive at an average average rate of λ = 37.5 per hour. However, according to the scenario, 20% of them are customers of a conventional parking lot, so we actually split the initial average arrival rate in five directions, those of the class of a conventional one, which is λ1 = λ x 0.20 = 37,5 x 0,20 = 7,5 customers per hour and that of the customers of the other categories that are λ2 = λ

x 0,80 = 37,5 x 0,80 = 30 customers per hour. Obviously λ1 + λ2 = λ = 37.5.

So we have a service system consisting of five parallel, simultaneously operating subsystems, with the same queue each, but with a common source of customers and a common total arrival rate of λ = 37.5, but split into five queues, each of which has the same arrival rate. Each subsystem is M / M / 1 type with λ1 = 7,5 and μ1 = 8 (after an average of 7,5 minutes is required for each customer in this category). In the first case, λ / μ1 = 7,5 / 8 <1. Consequently, there is convergence in equilibrium in the five parallel subsystems of the B mode of operation so we can continue with the calculations.

Thus, for the B1 subsystem in equilibrium mode, using the formulas for the M / M / 1 system, for λ1 = 7,5 and μ1 = 8, we get the following:

• Average waiting subsystem length

$$\text{B1: } L_{q1} = \frac{\lambda_1^2}{\mu_1(\mu_1 - \lambda_1)} = \frac{(7{,}5)^2}{8(8-7{,}5)} = 14{,}0625 \text{ customers.}$$

• Average number of customers, subsystem B1:

$$L_1 = \frac{\lambda_1}{\mu_1 - \lambda_1} = L_{q1} + \frac{\lambda_1}{\mu_1} 14{,}0625 + \frac{7{,}5}{8} = 15 \text{ customers.}$$

15 customers.

While, to calculate the total operating cost for B mode, we have:

• For Subsystem B1:

cw = € 3 per hour, cs = 45 € per hour, L1 = 15 and s = 1. So, TCB1 = WCB1 + SCB1 = cw x L1 + cs x s, hence TCB1 = 3 x 15 + 45 x 1 = 90 € per hour.

For all five, the same car park TCB = 5 × 90 = 450

Therefore, the total expected operating cost for Mode B is greater than the average cost of Mode A of approximately (approximately) 450-240,4061 = 209,539 € per hour.

But also in terms of service:

For A: Average queue length 1.385367

Average number of customers in system 5.135367

For B:

Average queue length = 5 × Average queue length of subsystem B1 = 5 × 14,0625 = 70,315

Average number of clients in the system = 5x2 Average number of customers, subsystem B1 = 5 × 15 = 75

Therefore, the ratio θ

$$\frac{medium\_range\_waiting\_system}{medium\_range\_at\_the\_system} \times 100$$

%, showing the expected payout ratio

System A is:

$$\theta_A = \frac{1{,}385367}{5{,}135367} \times 100 = 26{,}67698139 \ \%$$

and system B is :

$$\theta_B = \frac{70{,}135}{75} \times 100 = 93{,}51333333 \ \%$$

θB= 93,51333333 %

that is θA <θB.

It is overwhelming, therefore, with the superiority of the A mode of operation (automated system) versus the B mode of operation (conventional system).

CONCLUSION

The result of this mathematical analysis shows that the superiority of the automated parking system by the conventional parking system is overwhelming. This happens because the operating cost of the automated system is much less than the operating cost of the conventional system, the average service time of each car is smaller than the automated system by the conventional system, so the people served are more of the same time and the waiting hours are less. Thus time and money are served.

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