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A DECISION MODEL FOR A QUALITY-ASSURED EPQ-BASED INTRA-SUPPLY CHAIN SYSTEM CONSIDERING OVERTIME OPTION

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In order to gain competitive advantages, managers of present-day supply chain systems need to achieve critical operation goals, such as keeping flexible fabrication schedule, retaining product quality, maintaining timely delivery, and lowering overall operating costs. Inspired by these factors, this study developed a decision model for managers to investigate the joint impacts of quality-assured issues, overtime, and multi-shipment plan on optimal fabrication-shipment policy and on diverse system parameters of the economic production quantity (EPQ)-based supply chain system. An imperfect fabrication process producing perfect quality, repairable, and scrap items is assumed in the proposed system, along with assumptions of a flexible overtime schedule to partially expedite fabrication rate and a discontinuous multi-delivery policy for distributing end products. With a help from mathematical modeling and optimization method, the closed-form optimal fabrication-shipment policy is derived. A numerical example was employed to demonstrate applicability of our result and to expose critical managerial information of the system for supporting decision-making.

Key words: Quality assurance, Flexibility, Optimization of supply chains, Overtime, Multi-delivery, Fabrication-shipment policy

INTRODUCTION

In order to stay competitive in unstable business environment, managers of current supply chain systems often seek ways to keep flexible fabrication schedule, retain product quality, maintain timely delivery, and lower their overall operating costs. Hence, in past decades, researchers have paid extensive attentions to matters of imperfect quality product/production in various aspects of fabrication systems [1-8]. At times, the defect items produced are rework-able at extra repairing cost, so the product quality can be further assured [9-15].

For the reason of shortening fabrication cycle length or smoothing machine loadings, production managers would constantly consider a flexible overtime schedule or implement expedite manufacturing rate to increase output. Kunreuther [16] developed the planning-horizon algorithm to decide the optimal production plan for period 1, with assumptions of deterministic demand, linear overtime and inventory costs, and no shortages permitted. The algorithm was further extended to k periods for deciding the optimal fabrication plan, and the result showed that future demands for k periods must be known in advance, and k is decided by the wage difference ratio between overtime and regular time. Utilizing the concept of k and relating algorithms, planning horizons were decided for both storage and backlogging permitted situations. Dixon et al. [17] explored the lot sizing and replenishment timing for a fabrication system under the time-varying known demands and with no shortages permitted. Their flexible fabrication options include both regular time and overtime, and the capacities can vary with time. The aim was to determine optimal batch size per replenishment cycle, so that the sum of setup, variable fabrication, holding, and overtime expenses could be minimized. A heuristic algorithm was proposed and tested on several large sets of problems, and demonstrated its excellent performance to these problems. Robinson and Sahin [18] extended classic EPQ model by examining extra required fixed charges (e.g., the cleanup or inspection costs) for each fabrication cycle. They proposed two different problems with mathematical modeling and optimization procedures, featuring separate regular time production restrictions, setup costs, and availability of overtime fabrication capacity. Experimental examples were provided to show that their algorithms outperformed conventional models. Merzifonluoğlu et al. [19] proposed deterministic profit maximizing production models with outsourcing and overtime options, for deciding optimal demand and internal capacity levels. These models could help determine vendor's optimal price, fabrication, inventory, outsourcing, overtime, and internal capacity levels. The polyhedral properties and dynamic programming techniques were utilized to derive the optimal solutions. Extra studies [20-21] also investigated systems with diverse overtime policies. With the similar objectives of expediting production cycle times, papers that explored manufacturing systems with various features of adjustable/variable production rate or outsourcing option can be referred to [22-27].

Moreover, various intra-supply chain systems undoubtedly exist in current transnational enterprises, wherein, the operating objective is to minimize total system cost consisting of the fabrication-inventory cost, products'

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transportation cost, and end items' holding cost at the sales offices. Schwarz [28] presented heuristics for solving a deterministic inventory problem with one- warehouse, N-retailer, and with continuous review. As a result, solutions and a few properties to the problem are gained and suggested. Hahm and Yano [29] simultaneously decided the frequencies of fabrication and distribution that keep overall operating costs to minimum. As a result, they derived the optimal operating policies and confirmed that the ratio between fabrication and distribution intervals have to be integer, and most favorably to have synchronized fabrication and distribution. Sarmah et al. [30] believed that an important means for reducing supply-chain cost so as to gain competitive advantage is by coordinating two different business entities. Therefore, they reviewed literature relating to buyer-vendor coordination models, in particular, those subjects regarding quantity discounts as coordination device. As a result, different models were classified and various critical issues were pointed out for future study. Other studies [31-38] also investigated supply chain systems with different features. This present study aims at offering managers a decision model to help them accomplish critical operation goals, such as keeping flexible fabrication schedule, retaining product quality and timely delivery, and lowering total operating expenses.

MATERIALS AND METHODS

Problem description and modeling

The proposed decision model, an EPQ-based intra-supply chain system, considers overtime option, quality assurance and multi-delivery policy. The objectives are to simultaneously decide the optimal refilling lot size and frequency of shipment that keep the expected system cost minimum, and study impacts of overtime and quality-assured issues on the proposed intra-supply chain system. Assume that demand rate of a particular product is λ items per year, and an overtime option is incorporated into the fabrication system to expedite production process. Such an overtime option changes the production/throughput rate to P_A per year, where $P_A = (1 + \alpha_1)P$. Notation α_1 denotes the increase percentage of production/output rate due to overtime option, and P denotes the standard (unadjusted) production rate. Consequent to this overtime option, the setup cost K_A and unit fabrication cost C_A increase, where $K_A = (1 + \alpha_2)K$ and $C_A = (1 + \alpha_3)C$. Symbols α_2 and α_3 stand for the percentages in cost increase, and K and C represent the standard setup and unit costs, respectively. It is further assumed the manufacturing process is imperfect, x proportion of defect items may be generated at a rate d_A (i.e., $d_A = P_A x$). All nonconforming products are screened, and a θ portion is identified as scrap, and the others are considered rework-able. The latter will be reworked in the end of ordinary fabrication, at annual rate of P_{1A} (See Fig. 1),

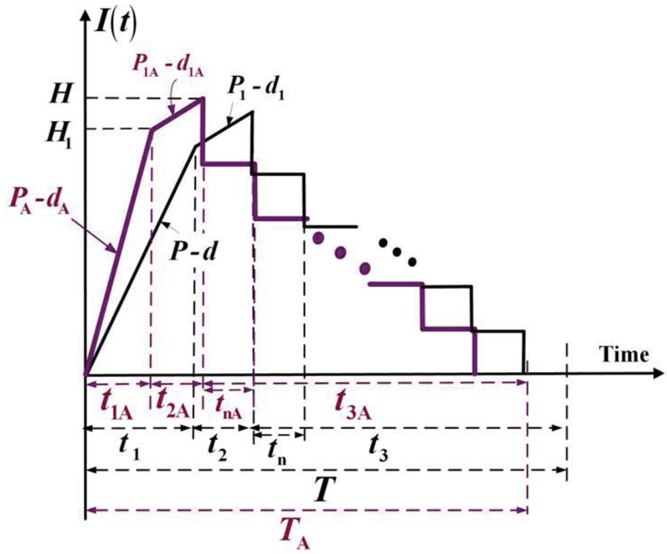

Figure 1: On-hand level of perfect quality stocks in the proposed system (in purple) as compared to the system without implementing overtime (in black)

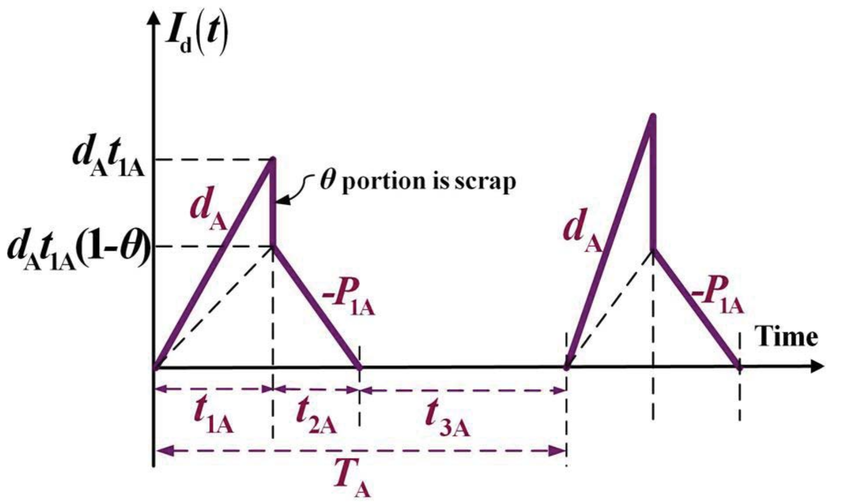

Figure 2: The on-hand level of defect stocks in the proposed system

where $P_{1A} = (1 + \alpha_1)P_1$ and P_1 denotes standard rework rate.

The on-hand level of defect items in the proposed system is shown in Fig. 2. The proposed system does not permit stock-out situation, so $(P_A - d_A - \lambda)$ must be greater than zero (as shown in Fig.1). Also, the imperfect rework process is assumed, a θ_1 proportion of reworked products fails and is scrapped during t_{2A}. On-hand level of scrap products generated during the uptime and rework time is exhibited in Fig. 3. Definitions of additional system variables are given in Appendix - A.

When rework process finishes, shipping of finished stocks begins in t_{3A}, n fixed-quantity installments of the finished lot are transported to sales units, at a fixed-interval of time t_{nA}. Figure 4 illustrates production units' on-hand inventory status in delivery time t_{3A}, and Figure 5 shows sales units' on-hand stock status in a given replenishment cycle, respectively.

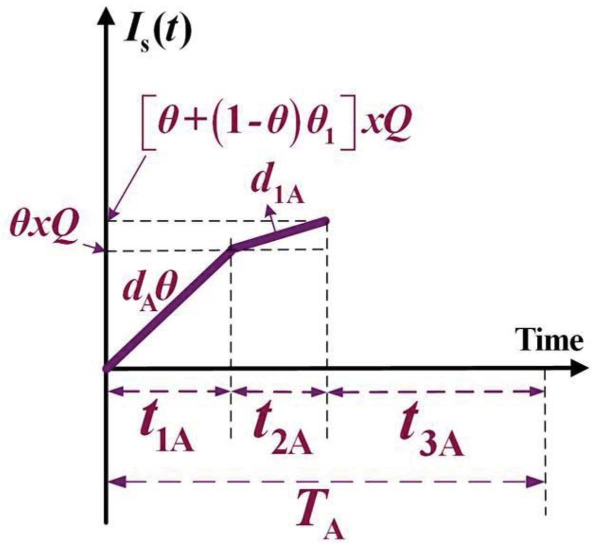

Figure 3: The on-hand level of scraps in the proposed system

Formulations

According to the assumptions and description of the proposed system, one obtains fabrication lot size Q and the refilling cycle time T_A as follows:

$$Q = P_A t_{1A} \tag{1}$$

$$T_A = t_{1A} + t_{2A} + t_{3A} \tag{2}$$

From Fig. 1, fabrication uptime t_{1A}, rework time t_{2A}, and shipping time t_{3A} are expressed as follows:

$$t_{1A} = \frac{Q}{P_A} = \frac{H_1}{P_A - d_A} = \frac{H_1}{P_A - P_A x} = \frac{H_1}{\left[(1+\alpha_1)P\right](1-x)} \tag{3}$$

$$t_{2A} = \frac{(1-\theta)xQ}{P_{1A}} = \frac{(1-\theta)xQ}{(1+\alpha_1)P_1} \tag{4}$$

$$t_{3A} = T_A - t_{1A} - t_{2A} = \frac{Q(1-\varphi x)}{\lambda} - \frac{Q}{(1+\alpha_1)P} - \frac{xQ(1-\theta)}{(1+\alpha_1)P_1} \tag{5}$$

Also, the stock levels H_1 and H can be observed from Fig. 1 as

$$H_1 = (P_A - d_A)t_{1A} = \left[(1+\alpha_1)P\right](1-x)t_{1A} \tag{6}$$

$$H = H_1 + (P_{1A} - d_{1A})t_{2A} = H_1 + (P_{1A} - \theta_1 P_{1A})t_{2A} = \\ = H_1 + \left[(1-\theta_1)P_{1A}\right]t_{2A} \tag{7}$$

Total defect items generated in uptime are $d_A t_{1A}$ or xQ (see Fig. 2) and among them total number of scrap items are $\theta(d_A t_{1A})$ or $\theta(xQ)$. In rework process, total number of scrap items is $\theta_1(P_{1A}t_{2A})$ or $\theta_1[(1 - \theta)xQ]$. Hence, total scrap items produced in the proposed system are as follows (see Fig. 3).

$$\theta(xQ) + \theta_1\left[(1-\theta)xQ\right] = \left[\theta + \theta_1(1-\theta)\right](xQ) = \varphi(xQ) \tag{8}$$

Total holding inventories in the delivery time (see Fig. 4) are as follows:

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_{3A} = \left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]Ht_{3A} = \left(\frac{n-1}{2n}\right)Ht_{3A} \tag{9}$$

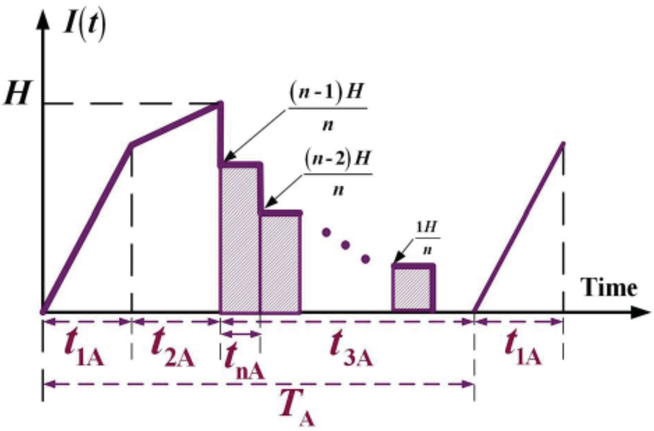

Figure 4: Production units' on-hand inventory status in delivery time t_{3A}

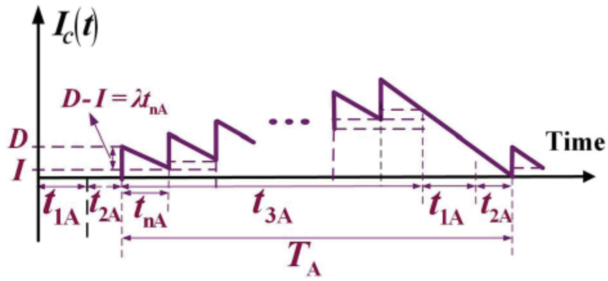

Figure 5: Sales units' on-hand stock status in a cycle

In Fig. 5, the following equations can been observed according to our assumption:

$$D = \frac{H}{n} \tag{10}$$

$$t_{nA} = \frac{t_{3A}}{n} \tag{11}$$

$$I = D - (\lambda t_{nA}) \tag{12}$$

Total inventories at the customer end in a given cycle are:

$$nt_{nA}\left(D-\frac{\lambda t_{nA}}{2}\right)+\frac{n(n-1)}{2}It_{nA}+\frac{nI}{2}\left(t_{1A}+t_{2A}\right)=\frac{1}{2}\left[\frac{Ht_{3A}}{n}+\left(H-\lambda t_{3A}\right)T_A\right] \tag{13}$$

TC(Q, n), consists of variable manufacturing cost, setup, disposal, and rework costs, variable and fixed shipping costs, holding costs of perfect stocks, reworked items, and stocks in sales units. *TC(Q, n)* becomes

$$\begin{aligned}TC(Q,\ n)&=C_AQ+K_A+C_S[\varphi xQ]+C_R[(1-\theta)xQ]+C_T[(1-\varphi x)Q]+nK_1\\&+h\left[\frac{H_1+(xP_A)t_{1A}}{2}(t_{1A})+\frac{H+H_1}{2}(t_{2A})\right]+h\left(\frac{n-1}{2n}\right)Ht_{3A}+h_1\frac{P_{1A}t_{2A}}{2}(t_{2A})\\&+\frac{h_2}{2}\left[\frac{Ht_{3A}}{n}+T_A\left(H-\lambda t_{3A}\right)\right]\end{aligned} \tag{14}$$

Substituting overtime related variables with standard variables (*K*, *C*, *P*, and *P1*), *TC(Q, n)* becomes

$$\begin{aligned}TC(Q,\ n)&=[(1+\alpha_2)K]+[(1+\alpha_3)C]Q+C_R[(1-\theta)xQ]+C_S[xQ\varphi]\\&+nK_1+C_T[(1-\varphi x)Q]+h\left[\frac{H_1+x[(1+\alpha_1)P]t_{1A}}{2}(t_{1A})+\frac{H_1+H}{2}(t_{2A})\right]\\&+h\left(\frac{n-1}{2n}\right)Ht_{3A}+h_1\frac{[(1+\alpha_1)P_1]t_{2A}}{2}(t_{2A})+\frac{h_2}{2}\left[\frac{Ht_{3A}}{n}+T_A\left(H-\lambda t_{3A}\right)\right]\end{aligned} \tag{15}$$

Take randomness of defect rate into consideration by using the expected values of *x*, substitute relating equations (1) to (7) in Eq. (15), along with extra deriving efforts, E[*TCU(Q, n)*] can be found as follows:

$$\begin{aligned}E[TCU(Q,\ n)]&=\frac{E[TC(Q,\ n)]}{E[T_A]}=\lambda\{[(1+\alpha_3)C]E_0+C_R(1-\theta)E_1+C_S\varphi E_1\}+\frac{[(1+\alpha_2)K]\lambda}{Q}E_0\\&+\frac{nK_1\lambda}{Q}E_0+C_T\lambda+\frac{hQ}{2}(1-E[x]\varphi)+\frac{hQ\lambda\varphi E_1}{2P(1+\alpha_1)}+\frac{\lambda Q(1-\theta)E_2}{2P_1(1+\alpha_1)}[h_1(1-\theta)-h]\\&+\frac{hQ\lambda(1-\theta)E_1}{2P_1(1+\alpha_1)}+Q(h_2-h)\left(\frac{1}{2n}\right)E_3+\frac{h_2Q}{2}\left[\frac{\lambda}{P(1+\alpha_1)}+\frac{\lambda(1-\theta)E[x]}{P_1(1+\alpha_1)}\right]\end{aligned} \tag{16}$$

where

$$E_0=\frac{1}{1-\varphi E[x]};\ E_1=\frac{E[x]}{1-\varphi E[x]};\ E_2=\frac{E[x]^2}{1-\varphi E[x]};\ E_3=\left[(1-E[x]\varphi)-\frac{\lambda}{P(1+\alpha_1)}-\frac{\lambda(1-\theta)E[x]}{P_1(1+\alpha_1)}\right] \tag{17}$$

RESULTS AND DISCUSSION

Optimal lot size & frequency of deliveries

The following Hessian matrix equations [39] are applied to help determine whether or not E*[TCU(Q, n)*] is convex:

$$[Q\quad n]\cdot\begin{pmatrix}\frac{\partial^2E[TCU(Q,n)]}{\partial Q^2} & \frac{\partial^2E[TCU(Q,n)]}{\partial Q\partial n}\\ \frac{\partial^2E[TCU(Q,n)]}{\partial Q\partial n} & \frac{\partial^2E[TCU(Q,n)]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}>0 \tag{18}$$

The related partial derivatives of Eq. (16) can be found as follows:

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q}=-\frac{\left[(1+\alpha_2)K\right]\lambda}{Q^2}E_0-\frac{nK_1\lambda}{Q^2}E_0+\frac{h}{2}(1-\varphi E[x])+\frac{h\lambda\varphi}{2(1+\alpha_1)P}E_1+\frac{h\lambda(1-\theta)E_1}{2P_1(1+\alpha_1)} \\ +\frac{\lambda(1-\theta)E_2}{2P_1(1+\alpha_1)}\left[h_1(1-\theta)-h\right]+\left(h_2-h\right)\left(\frac{1}{2n}\right)E_3+\frac{h_2}{2}\left[\frac{\lambda}{P(1+\alpha_1)}+\frac{\lambda(1-\theta)E[x]}{P_1(1+\alpha_1)}\right] \tag{19}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2}=\frac{2[(1+\alpha_2)K]\lambda}{Q^3}E_0+\frac{2nK_1\lambda}{Q^3}E_0 \tag{20}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial n}=\frac{K_1\lambda}{Q}E_0-Q\left(h_2-h\right)\left(\frac{1}{2n^2}\right)E_3 \tag{21}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2}=\left(h_2-h\right)Q\left(\frac{1}{n^3}\right)E_3 \tag{22}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n}=-\frac{K_1\lambda}{Q^2}E_0-\left(h_2-h\right)\left(\frac{1}{2n^2}\right)E_3 \tag{23}$$

Substituting equations (20), (22) and (23) in Eq. (18), one has

$$\left[Q \quad n\right]\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} \\ \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q \\ n\end{bmatrix}=\frac{2\left[\left(1+\alpha_2\right)K\right]\lambda}{Q}E_0>0 \tag{24}$$

Eq. (24) results positive, because α_2, K, λ, Q, and E_0 are positives. So, *E[TCU(Q, n)]* is convex for all Q and *n* > 0. By setting the first derivatives of *E[TCU(Q, n)]* with respect to *n* and Q equal to zeros (i.e., Eqs. (25) and (26)), and solve the linear system, we can get *n** and *Q** as expressed in Eqs. (27) and (28).

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q}=-\frac{\left[(1+\alpha_2)K\right]\lambda}{Q^2}E_0-\frac{nK_1\lambda}{Q^2}E_0+\frac{h}{2}(1-\varphi E[x])+\frac{h\lambda\varphi}{2(1+\alpha_1)P}E_1 \\ +\frac{\lambda(1-\theta)}{2(1+\alpha_1)P_1}\left[h_1(1-\theta)-h\right]E_2+\frac{h\lambda(1-\theta)}{2(1+\alpha_1)P_1}E_1+\left(h_2-h\right)\left(\frac{1}{2n}\right)E_3 \\ +\frac{h_2}{2}\left[\frac{\lambda}{(1+\alpha_1)P}+\frac{\lambda(1-\theta)E[x]}{(1+\alpha_1)P_1}\right]=0 \tag{25}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial n}=\frac{K_1\lambda}{Q}E_0-\left(h_2-h\right)Q\left(\frac{1}{2n^2}\right)E_3=0 \tag{26}$$

$$n^*=\sqrt{\frac{(1+\alpha_2)K\left(h_2-h\right)\left(1-\varphi E[x]\right)E_3}{K_1\left[\begin{array}{l}h\left(1-\varphi E[x]\right)^2+\frac{h\lambda E[x]}{(1+\alpha_1)}\left[\frac{\varphi}{P}+\frac{(1-\theta)}{P_1}\right]+\frac{\lambda(1-\theta)\left[h_1(1-\theta)-h\right]E[x]^2}{(1+\alpha_1)P_1} \\ +\frac{h_2\lambda\left(1-\varphi E[x]\right)}{(1+\alpha_1)}\left[\frac{1}{P}+\frac{(1-\theta)E[x]}{P_1}\right]\end{array}\right]}} \tag{27}$$

$$Q^*=\sqrt{\frac{2\left[(1+\alpha_2)K+nK_1\right]\lambda}{\begin{array}{l}h\left(1-\varphi E[x]\right)^2++\frac{h\lambda E[x]}{(1+\alpha_1)}\left[\frac{\varphi}{P}+\frac{(1-\theta)}{P_1}\right]+\frac{\lambda(1-\theta)\left[h_1(1-\theta)-h\right]E[x]^2}{(1+\alpha_1)P_1} \\ +\frac{h_2\lambda\left(1-\varphi E[x]\right)}{(1+\alpha_1)}\left[\frac{1}{P}+\frac{(1-\theta)E[x]}{P_1}\right]+\left(h_2-h\right)\left(\frac{1}{n}\right)(1-\varphi E[x])E_3\end{array}}} \tag{28}$$

The computational result of Eq. (27) is a real number, however, the number of deliveries n should be an integer. So, we examine two adjacent integers to n by letting n^+ be the smallest integer, greater than or equal to n; and n^- denote the largest integer, less than or equal to n. Then, apply Eq. (28) with n^+ and n^- to obtain corresponding Q^+ and Q^-, respectively. Lastly, apply Eq. (16) with (Q^+, n^+) and (Q^-, n^-), and choose the one that gives the minimum cost as the optimal (Q^*, n^*) policy.

Numerical demonstration

A numerical illustration is given here to exhibit applicability of our result. The following are the assumed values for system parameters: λ = 4,000; K = \$5,000; h = \$30; C = \$100; x = [0, 0.2]; P = 20,000; α_1 = 0.5; α_2= 0.1 (we assume $\alpha_2 = 0.2(\alpha_1)$); α_3 = 0.25 (we assume $\alpha_3 = 0.5(\alpha_1)$); K_A=\$5,500; P_A = 30,000; C_A = \$125, C_R = \$60; h_1 = \$40; θ = 0.1; θ_1 = 0.1; φ = 0.19 (since $\varphi = [\theta + (1 - \theta)\theta_1]$); C_S = \$20; K_1 = \$800; C_T = \$0.5; h_2 = \$80.

Computational results of Eqs. (27), (28), and (16) give Q^* = 1,046, n* = 3, and E[$TCU(Q^*, n^*)$] = \$596,820. Effect of variations in fabrication lot size on diverse system cost variables is depicted in Figure 6. It indicates that as Q increases, both holding cost and defect products' reworking cost rises significantly; but the fabrication setup cost declines notably.

Joint impacts of variations in nonconforming rate x and overall scrap rate φ on *E[TCU(Q, n)]* are exhibited in Figure 7. It shows that as both x and φ increases, *E[TCU(Q, n)]* climbs significantly, due to the increase of total quality costs (i.e., both rework and scrap costs).

From further analytical results of the proposed EPQ-based intra-supply chain system, we obtain the following critical system information (as displayed in Table 1 & Table B-1 (in Appendix – B)):

From Table 1, it indicates as the ratio P_A /P goes up; unit fabrication cost ratio C_A /C, lot size, variable fabrication cost, and E[TCU(Q, n)] all notably increase. Joint effects of P_A / P ratio and Q on *E[TCU(Q, n)]* are analyzed and presented in Figure 8.

It is also noted from Table 1 that as the ratio C_A /C goes up, the variable fabrication cost increase accordingly. Effect of variations in C_A /C ratio on variable fabrication cost is explored and illustrated in Fig. 9. The analytical result (as in our example) indicates that at C_A /C = 1.25, variable fabrication cost rises 20.51%.

From Table B-1, it reveals that as P_A / P ratio increases; uptime t_{1A} , reworking time t_{2A}, and machine utilization all decrease accordingly. Impact of differences in P_A /P ratio on utilization is shown in Figure 10. The analytical result (in our example) indicates that at P_A /P = 1.50, machine utilization drops 18.48%.

Table 2 shows that as P_A /P ratio goes up, Q^* rises and the optimal operating policy (Q^*, n^*) changes accordingly. Further analysis on the quality assurance reveals the behavior of *E[TCU(Q, n)]* with respect to the nonconforming rate and overall scrap rate as demonstrated in Figure 11. Unsurprisingly, it indicates that as x increases, *E[TCU(Q, n)]* climbs accordingly; and as φ goes up, *E[TCU(Q, n)]* rises notably.

CONCLUSIONS

In the present study, a decision model is built to explore the joint effects of overtime in fabrication, quality assurance issues, and multiple deliveries on optimal fabrication- shipment policy and diverse parameters of the proposed EPQ-based system. Hessian matrix equations help us derive a closed-form optimal policy. Numerical demonstration helps illustrate applicability of our results and expose critical managerial information (see Figures 6 to 11 & Tables 1 and 2) for supporting decision making.

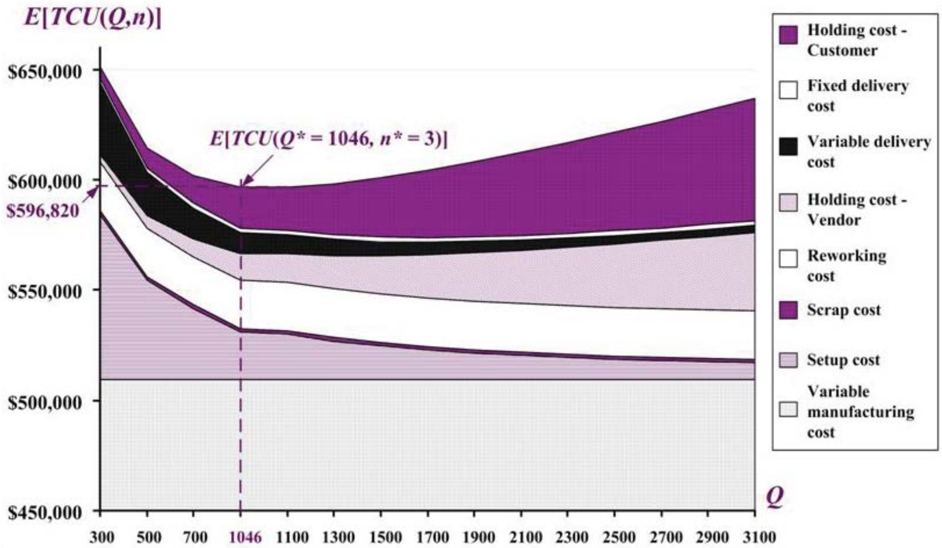

Figure 6: Effect of variations in lot size on diverse system cost parameters

Table 1: Effects of adjusted factors of overtime and its corresponding cost on optimal lot size, variable cost, expected system cost, and its increase %

$1+\alpha_1$ (P_A / P)	$1+\alpha_3$ (C_A / C)	Q*	Variable fabrication cost	E[TCU(Q,n)]	System cost increase %
1.0	1	869	$407,747	$495,253	-
1.1	1.05	885	$428,135	$515,415	4.07%
1.2	1.10	900	$448,522	$535,673	8.16%
1.3	1.15	915	$468,909	$556,006	12.27%
1.4	1.20	928	$489,297	$576,397	16.38%
1.5	1.25	1046	$509,684	$596,820	20.51%
1.6	1.30	1060	$530,071	$617,165	24.62%
1.7	1.35	1074	$550,459	$637,550	28.73%
1.8	1.40	1086	$570,846	$657,969	32.86%
1.9	1.45	1099	$591,233	$678,417	36.98%
2.0	1.50	1110	$611,621	$698,889	41.12%
2.1	1.55	1122	$632,008	$719,381	45.26%
2.2	1.60	1132	$652,396	$739,892	49.40%
2.3	1.65	1143	$672,783	$760,417	53.54%
2.4	1.70	1153	$693,170	$780,956	57.69%
2.5	1.75	1163	$713,558	$801,506	61.84%
2.6	1.80	1173	$733,945	$822,066	65.99%
2.7	1.85	1183	$754,332	$842,636	70.14%
2.8	1.90	1192	$774,720	$863,212	74.30%
2.9	1.95	1202	$795,107	$883,796	78.45%
3.0	2.00	1211	$815,494	$904,386	82.61%

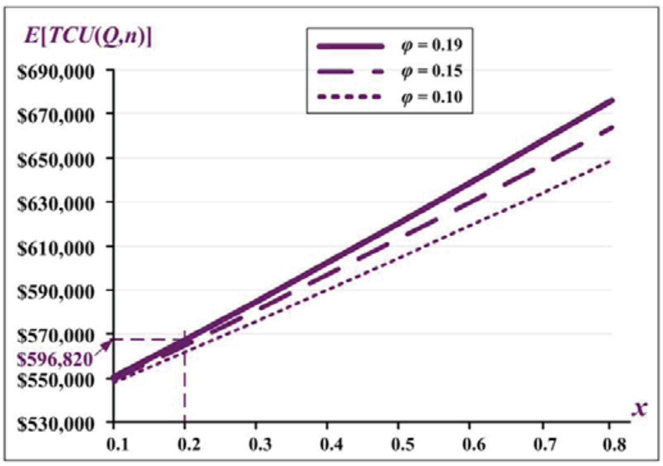

Figure 7: Joint impacts of variations in nonconforming and scrap rates on E[TCU(Q,n)]

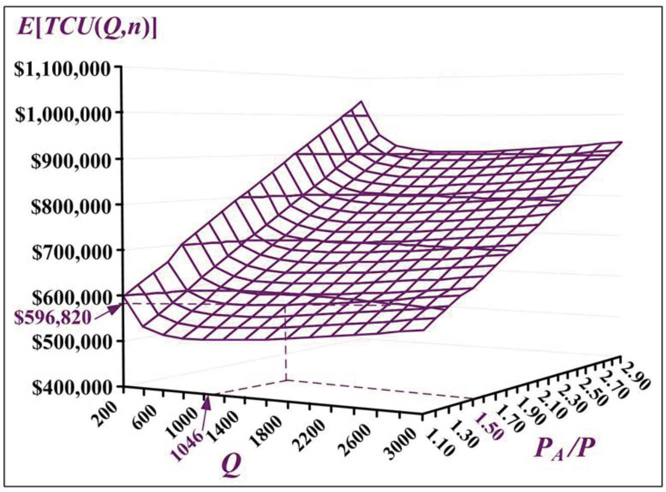

Figure 8: Joint influences of P_A /P ratio and Q on E[TCU(Q, n)]

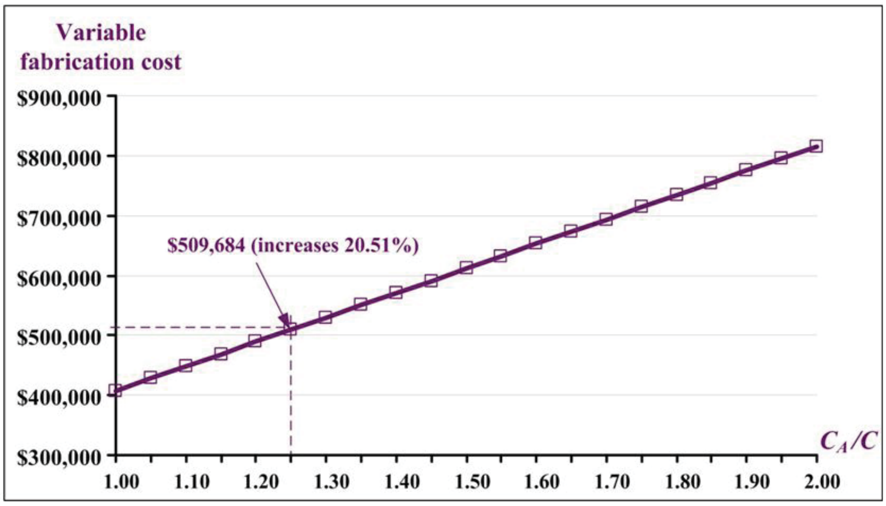

Figure 9: Effect of variations in C_A /C ratio on variable fabrication cost

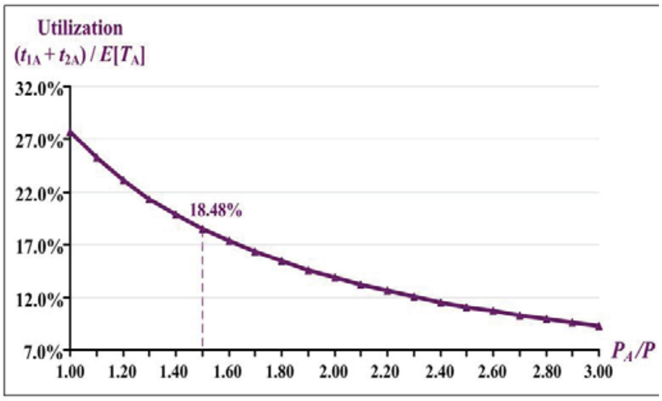

Figure 10: Effect of variations in P_A / P ratios on the machine utilization

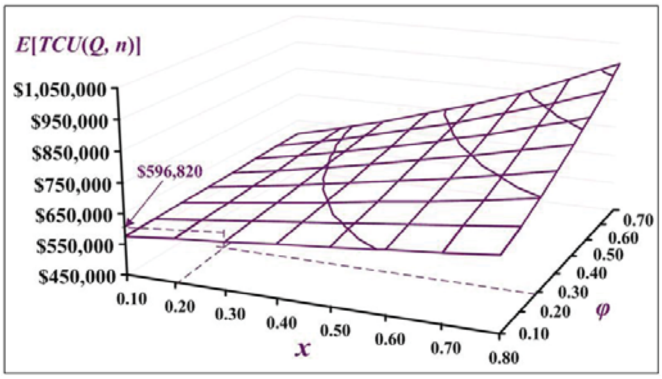

Figure 11: Behavior of E[TCU(Q, n)] with respect to nonconforming rate x and scrap rate φ

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Appendix - A

T_A = cycle time,

t_{1A} = uptime,

t_{2A} = rework time,

t_{3A} = delivery time,

Q = lot size,

n = number of deliveries,

t_{nA} = time interval between shipments,

H_1 = inventory level when uptime ends,

H = inventory level when rework time ends,

h = holding cost/item/year,

h_1 = holding cost/reworked item/year,

C_R = rework cost/item,

C_S = disposal cost per scrap item,

φ = overall scrap rate ($\varphi = [\theta + (1 - \theta)\theta_1]$),

d_{1A} = production rate of scrap items during rework,

K_1 = fixed cost per delivery,

C_T = delivery cost per item,

D = number of products per delivery,

I = number of leftover products after demand is satisfied in each time interval tnA,

h_2 = holding cost per item stored in sales units,

T = cycle time for a regular system (without overtime plan),

t_1 = uptime of regular system,

t_2 = rework time of regular system,

t_3 = shipping time of regular system,

t_n = time interval between shipments in regular system,

d = production rate of nonconforming item in regular system,

d_1 = production rate of scrap item during rework in regular system,

$I(t)$ = on-hand level of finished stocks at time t,

$I_d(t)$ = on-hand level of defect items at time t,

$I_S(t)$ = on-hand level of scrapped items at time t,

$I_c(t)$ = sales units' on-hand stock level at time t,

$TC(Q, n)$ = total cost per cycle for the proposed system,

$E[T_A]$ = expected cycle time,

$E[TCU(Q, n)]$ = expected annual system cost.

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Appendix - B

Table B-1: Effect of adjusted factors of overtime on optimal lot-size, number of deliveries, uptime, rework time, cycle time, utilization, and its decline %

$1+\alpha_1$ (P_A/P)	Q^*	n^*	t_{1A}	t_{2A}	$E[T_A]$	Utilization $(t_{1A}+t_{2A})/E[T_A]$	Utilization decrease %
1.0	869	2	0.0434	0.0156	0.2131	27.73%	-
1.1	885	2	0.0402	0.0145	0.2171	25.21%	-9.09%
1.2	900	2	0.0375	0.0135	0.2208	23.11%	-16.67%
1.3	915	2	0.0352	0.0127	0.2243	21.33%	-23.08%
1.4	928	2	0.0331	0.0119	0.2275	19.80%	-28.57%
1.5	1046	3	0.0349	0.0126	0.2566	18.48%	-33.33%
1.6	1060	3	0.0331	0.0119	0.2601	17.33%	-37.50%
1.7	1074	3	0.0316	0.0114	0.2633	16.31%	-41.18%
1.8	1086	3	0.0302	0.0109	0.2664	15.40%	-44.44%
1.9	1099	3	0.0289	0.0104	0.2694	14.59%	-47.37%
2.0	1110	3	0.0278	0.0100	0.2723	13.86%	-50.00%
2.1	1122	3	0.0267	0.0096	0.2751	13.20%	-52.38%
2.2	1132	3	0.0257	0.0093	0.2777	12.60%	-54.55%
2.3	1143	3	0.0248	0.0089	0.2803	12.06%	-56.52%
2.4	1153	3	0.0240	0.0087	0.2829	11.55%	-58.33%
2.5	1163	3	0.0233	0.0084	0.2853	11.09%	-60.00%
2.6	1173	3	0.0226	0.0081	0.2878	10.66%	-61.54%
2.7	1183	3	0.0219	0.0079	0.2901	10.27%	-62.96%
2.8	1192	3	0.0213	0.0077	0.2924	9.90%	-64.29%
2.9	1202	3	0.0207	0.0075	0.2947	9.56%	-65.52%
3.0	1211	3	0.0202	0.0073	0.2969	9.24%	-66.67%

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