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## **A TWO-PHASE SOLUTION APPROACH FOR A MANUFACTURING-DISTRIBUTION PROBLEM WITH REWORK, OUTSOURCING, AND MULTI-SHIPMENT POLICY**

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*In a recent study, a manufacturing batch-size and end-product shipment problem with outsourcing, multi-shipment, and rework was investigated using mathematical modeling and derivatives in its solution procedure. This study demonstrates that a simplified two-phase algebraic approach can also solve the problem and decide the cost-minimization policies for batch-size and end-product shipments. Our proposed straightforward solution approach enables the practitioners in the production planning and controlling filed to comprehend and efficiently solve the best replenishing batch-size and shipment policies of this real problem.*

*Key words: production management, outsourcing, manufacturing-distribution problem, rework, two-phase solution approach*

### **INTRODUCTION**

Management of today's manufacturing firms makes every effort to obtain a competitive advantage by achieving various operating goals, such as maintaining higher product quality, efficient and timely delivery, and reduced total production-inventory-shipment costs. In production planning and scheduling, when a short supply of in-house capacity exists, adoption of a tentative outsourcing option can often help to level the fabrication workloads and smooth the production plan. A recent study [1], a manufacturing batch-size and end-product shipment problem with outsourcing, multi-shipment, and rework, was investigated using mathematical modeling and derivatives in its solution procedure. This study demonstrates that an alternative two-phase algebraic approach can also solve the problem and decide the cost-minimization policies for batch-size and end-product shipments. The following studies were closely related to our studied subject. Cachon and Harker [2] presented and examined a competitive model between two corporations facing the scale economies, particularly in the declining unit cost of demand. Two competitive services providers that have time- and price-sensitive markets were examined, and each has individual fixed ordering cost and price-sensitive customers. Their result indicated that the service provider who has lower cost might possess a larger market share, higher price, and outsourcing is a favorite option. Gray et al. [3] explored the effects of quality and cost on a manufacturer's leaning to outsource, aiming the gap between production strategy and company boundaries, and constructing a model that links the product quality and cost priorities to a manufacturer who intends to outsource fabrication. Empirical analyses were conducted based on real data to conclude that insourcing de-

cision-making and outsourcing cost did play an integral role. But the product quality priority did not. Their findings offered theoretical insights to future studies, especially on the improvement of managerial decision-making in outsourcing production. Other studies related to diverse characteristics of outsourcing policies and their influences can refer to [4-10]. Wherein outsourcing decisions, production outsourcing, and managerial incentives and implication were examined in [4,5]. The bargaining perception on outsourcing strategy, key successful perspectives for production outsourcing, and in-house capability relating to competition in supply chains were explored [6-8]. Mathematical modeling for single-item and multi-product replenishing decisions with partial outsourcing options and quality reassurance issues were investigated [9-10].

Also, in most production settings due to varied unforeseen factors, producing defects is inevitable. Sometimes, the reworking of defective products can be done with extra reworked costs [11-27]. Two-stage transfer lines comprising scraps, allocations of service-level and stock-level in a multiproduct inventory system, and determining the operating policies for the repairable production systems were studied [11-14]. Applying Taguchi-simulated annealing approach, using the d-q based controlling analysis and production flexibility consideration, implementing lean manufacturing SMEs, handling of the system generator in hardware design, and developing a reliability, availability, and maintainability (RAM) model for quality control and improvements were investigated [15-20]. Replacement policies for failures/damages [21-22], Single-machine batch processing, production control using fuzzy neural network, and fault-tolerant system simulation for quality improvement and performance enhancement were studied [23-27]. Furthermore, in real supply

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chain environments, a multi-shipment policy is practically adopted for transporting end products. Hill [28] considered a fabrication model where the purchased raw material was manufactured into a product and transported to customers under a fixed-quantity fixed time intervals discipline. The author aimed to minimize total fabrication, holding, and procurement costs by deciding the optimal manufacturing and procurement schedule. Additional works [29-36] were also related to diverse aspects of multi-delivery policies in supply-chain environments. In a recent paper, a lot-size and finished stocks shipping problems with outsourcing, multi-shipment strategy, and rework [1] were studied using mathematical modeling and derivatives approaches. This study demonstrates that without applying derivatives, a simplified two-phase algebraic approach (Grubbstrom and Erdem [37]) can be used to decide the cost-minimization batch-size and end-product shipments for a manufacturing- distribution problem featuring outsourcing, multi-shipment, and rework [1].

**MATERIALS AND METHODS**

**The description of problem**

Consider annual demand  $\lambda$  of a particular manufactured product must be met. A  $\pi$  (where  $0 < \pi < 1$ ) proportion of each replenishment batch is outsourced, and the  $(1 - \pi)$  proportion of the batch is manufactured by an in-house facility, at a fabrication rate  $P$  [1]. A random defective rate  $x$  is associated with the in-house process. So, the production rate of the nonconforming  $d = Px$ , and to prevent shortage occurrences,  $P - d - \lambda > 0$  is assumed. All nonconforming stocks are repaired through the rework process each replenishment cycle when regular manufacturing task ends. The annual reworking rate is  $P_1$ . The outsourced portion is quality assured by the outsourcer and to be received right before the beginning of delivery time  $t_{3\pi}$  (Fig. 1).

In delivery time  $t_{3\pi}$ ,  $n$  installments (with fixed quantity) of

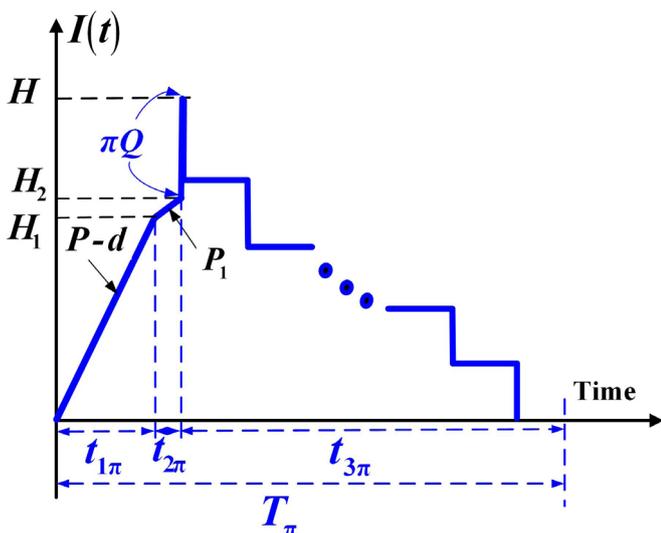


Figure 1: Status of finished items in the proposed model

the batch are shipped to buyers at fixed time intervals. Appendix A shows an extra notation of the present work. Total cost in a cycle  $TC(Q, n)$ , comprises in-house set-up, variable manufacturing, and rework costs, variable and fixed outsourcing costs, variable and fixed shipping costs, holding costs at both producer's and customer's sides. Substitute relating parameters  $K_\pi$  and  $C_m$ , and apply the expected values to the random variable  $x$ , and spending extra efforts of derivations,  $E[TCU(Q, n)]$  is gained as follows [1]:

$$\begin{aligned}
 E[TCU(Q, n)] &= \\
 &= \frac{\lambda(1 + \beta_1)K}{Q} + \frac{\lambda(K + nK_1)}{Q} \\
 &+ \lambda[(1 + \beta_2)C\pi + (1 - \pi)(C + C_R E[x]) + C_T] \\
 &+ \frac{(h_1 - h)Q}{2} \left( \frac{\lambda E[x]^2 (1 - \pi)^2}{P_1} \right) + \frac{h_2 \lambda (1 - \pi) Q}{2} \left( \frac{1}{P} + \frac{E[x]}{P_1} \right) \quad (1) \\
 &+ \frac{hQ}{2} \left( 1 - \frac{\lambda(1 - \pi)\pi}{P} + \frac{\lambda(1 - \pi)E[x](1 - 2\pi)}{P_1} \right) \\
 &+ \frac{(h_2 - h)Q}{2n} \left[ 1 - \lambda(1 - \pi) \left( \frac{1}{P} + \frac{E[x]}{P_1} \right) \right]
 \end{aligned}$$

**The proposed two-phase solution approach**

**Phase 1: find  $n^*$**

Since  $E[TCU(Q, n)]$  includes terms of constant,  $Q^{-1}$ ,  $nQ^{-1}$ ,  $Q$ , and  $Qn^{-1}$ , let  $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ , and  $\gamma_4$  denote the following:

$$\gamma_0 = \lambda[(1 + \beta_2)C\pi + (1 - \pi)(C + C_R E[x]) + C_T] \quad (2)$$

$$\gamma_1 = \lambda[(1 + \beta_1)K + K]; \quad \gamma_2 = \lambda K_1 \quad (3)$$

$$\begin{aligned}
 \gamma_3 &= \frac{(h_1 - h)}{2} \left( \frac{\lambda E[x]^2 (1 - \pi)^2}{P_1} \right) \\
 &+ \frac{h}{2} \left( 1 - \frac{\lambda(1 - \pi)\pi}{P} + \frac{\lambda(1 - \pi)E[x](1 - 2\pi)}{P_1} \right) \quad (4) \\
 &+ \frac{h_2 \lambda (1 - \pi)}{2} \left( \frac{1}{P} + \frac{E[x]}{P_1} \right)
 \end{aligned}$$

$$\gamma_4 = \frac{(h_2 - h)}{2} \left[ 1 - \lambda(1 - \pi) \left( \frac{1}{P} + \frac{E[x]}{P_1} \right) \right] \quad (5)$$

Then,  $E[TCU(Q, n)]$  is rearranged as follows:

$$\begin{aligned}
 E[TCU(Q, n)] &= \\
 &= \gamma_0 + \gamma_1 Q^{-1} + \gamma_2 (nQ^{-1}) + \gamma_3 Q + \gamma_4 (Qn^{-1}) \\
 &= \gamma_0 + (\sqrt{\gamma_1} - \sqrt{\gamma_3} Q)^2 Q^{-1} \\
 &+ (\sqrt{\gamma_2} - \sqrt{\gamma_4} n^{-1} Q)^2 (nQ^{-1}) + 2\sqrt{\gamma_1 \gamma_3} + 2\sqrt{\gamma_2 \gamma_4}
 \end{aligned} \quad (6)$$

Suppose both the 2<sup>nd</sup> and 3<sup>rd</sup> terms of Eq. (6) equal to zeros, then  $E[TCU(Q, n)]$  is minimized. That is

$$Q = \sqrt{\frac{\gamma_1}{\gamma_3}} \text{ and } n = \frac{\sqrt{\gamma_4} Q}{\sqrt{\gamma_2}} = \sqrt{\frac{\gamma_4 \gamma_1}{\gamma_2 \gamma_3}} \quad (7)$$

Substitute Eqs. (2) to (5) in Eq. (7),  $n^*$  is gained as follows:

$$n^* = \sqrt[4]{K_1 \left( \frac{(2 + \beta_1)K(h_2 - h) \left( 1 - \frac{\lambda(1 - \pi)}{P} - \frac{\lambda E[x](1 - \pi)}{P_1} \right)}{\left( \frac{(h_1 - h)\lambda E[x]^2(1 - \pi)^2}{P_1} + h_2\lambda(1 - \pi) \left( \frac{1}{P} + \frac{E[x]}{P_1} \right) \right)} + h \left( 1 - \frac{\lambda\pi(1 - \pi)}{P} + \frac{\lambda E[x](1 - \pi)(1 - 2\pi)}{P_1} \right) \right)} \quad (8)$$

**Phase 2: find  $Q^*$**

Since  $n^*$  is found, we now treat  $E[TCU(Q, n)]$  as the following single decision variable function:

$$E[TCU(Q, n)] = \gamma_0 + \gamma_5 Q^{-1} + \gamma_6 Q \quad (9)$$

$$= \gamma_0 + (\sqrt{\gamma_5} - \sqrt{\gamma_6}Q)^2 Q^{-1} + 2\sqrt{\gamma_5}\sqrt{\gamma_6}$$

where  $\gamma_5 = (\gamma_1 + \gamma_2 n)$  and  $\gamma_6 = (\gamma_3 + \gamma_4 n^{-1})$ . Suppose the second of Eq. (9) equals to zero, then  $E[TCU(Q, n)]$  is minimized. So,

$$Q^* = \frac{\sqrt{\gamma_5}}{\sqrt{\gamma_6}} \quad (10)$$

Substitute Eqs. (2) to (5) and (9) in Eq. (10),  $Q^*$  is determined as follows:

$$Q^* = \sqrt[4]{\frac{2[(2 + \beta_1)K + nK_1]\lambda}{\left( \frac{(h_1 - h)\lambda(1 - \pi)^2 E[x]^2}{P_1} + h_2\lambda(1 - \pi) \left( \frac{1}{P} + \frac{E[x]}{P_1} \right) \right)} + h \left[ 1 - \frac{\lambda(1 - \pi)\pi}{P} + \frac{\lambda(1 - \pi)E[x](1 - 2\pi)}{P_1} \right]} + \frac{(h_2 - h)}{n} \left[ 1 - \lambda(1 - \pi) \left( \frac{1}{P} + \frac{E[x]}{P_1} \right) \right]} \quad (11)$$

**DISCUSSIONS AND CONCLUSIONS**

This study presents a simplified two-phase algebraic approach rather than a conventional differential calculus method to solve a manufacturing-distribution problem featuring rework, outsourcing, and multi-shipment policy [1]. We successfully decide on the cost-minimization policies for batch-size and end-product shipments, as shown Eqs. (8) and (11) above. These results are identical to those derived in Chiu et al. [1] by using the differential calculus. In additions, by applying the same procedure as in [1] to first find the integer value of  $n^*$  and its corresponding  $Q^*$ , we also obtain the following simplified formula for calculating  $E[TCU(Q^*, n^*)]$  (refer to Eqs (1) or (6)).

$$E[TCU(Q, n)] = \gamma_0 + 2\sqrt{\gamma_1\gamma_3} + 2\sqrt{\gamma_2\gamma_4} \quad (12)$$

**CONCLUSIONS**

In this study, we demonstrate that without applying derivatives, a simplified two-phase algebraic approach can also be used to decide the cost-minimization batch-size and end-product shipments for a manufacturing-distribution problem featuring outsourcing, multi-shipment, and rework [1]. Our proposed straightforward solution approach enables the practitioners in the production planning and controlling filed to comprehend and efficiently solve the best replenishing batch-size and shipment policies of this real problem without the need to reference to differential calculus.

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### Appendix

- $T_{\pi}$  = replenishing cycle length,  
 $t_{1\pi}$  = in-house fabrication uptime,  
 $t_{2\pi}$  = in-house reworking time,  
 $t_{3\pi}$  = delivery time,  
 $Q$  = replenishing batch size,  
 $K$  = in-house fabrication setup cost,  
 $C$  = unit manufacturing cost,  
 $h$  = unit holding cost,  
 $C_R$  = unit rework cost,  
 $K_{\pi}$  = fixed outsourcing cost,  
 $C_{\pi}$  = unit outsourcing cost,  
 $\beta_1$  = the connecting factor between  $K_{\pi}$  and  $K$ , where  $K_{\pi} = K(1 + \beta_1)$  and  $(-1 \leq \beta_1 \leq 0)$ ,  
 $\beta_2$  = the linking factor between  $C_{\pi}$  and  $C$ , where  $C_{\pi} = C(1 + \beta_2)$  and  $\beta_2 > 0$ ,  
 $K_1$  = fixed delivery cost per shipment,  
 $C_T$  = unit delivery cost,  
 $n$  = number of shipments in a cycle,  
 $h_2$  = customer's unit holding cost,  
 $TC(Q, n)$  = total fabrication-shipment cost in a cycle,  
 $E[TCU(Q, n)]$  = expected annual fabrication- shipment cost.

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