JAES | JOURNAL OF APPLIED ENGINEERING SCIENCE | www.engineeringscience.rs

ISTRAŽIVANJA I PROJEKTOVANJA ZA PRIVREDU

Indexed by

Scopus

DOAJ DIRECTORY OF OPEN ACCESS JOURNALS

Crossref

ROAD

KoBSON

SCINDEKS Srpski citatni indeks

Google Scholar

SOLUTION OF INVERSE NON-STATIONARY BOUNDARY VALUE PROBLEMS OF DIFFRACTION OF PLANE PRESSURE WAVE ON CONVEX SURFACES BASED ON ANALYTICAL SOLUTION

Olga V. Egorova

Moscow Aviation Institute (National Research University), Department of Resistance of Materials Dynamics and Strength of Machines, Moscow, Russian Federation

Ye Ko Kyaw

Defence Services Academy (D.S.A), Department of Mathematics, Pyin Oo Lwin, Myanmar

Key words: *unsteady dynamics, fluid deformation, transitional surface functions, the inverse boundary value problems*

doi:10.5937/jaes0-28051

Cite article:

Egorova, V. O., & Kyaw, K. Y. [2020]. Solution of inverse non-stationary boundary value problems of diffraction of plane pressure wave on convex surfaces based on analytical solution. *Journal of Applied Engineering Science*, 18(4), 676 - 680.

Online aceess *of full paper is available at: www.engineeringscience.rs/browse-issues*

Original Scientific Paper

Istraživanja i projektovanja za privredu
Journal of Applied Engineering Science

doi:10.5937/jaes0-28051 Paper number: 18(2020)4, 744, 676-680

SOLUTION OF INVERSE NON-STATIONARY BOUNDARY VALUE PROBLEMS OF DIFFRACTION OF PLANE PRESSURE WAVE ON CONVEX SURFACES BASED ON ANALYTICAL SOLUTION

Olga V. Egorova[1], Ye Ko Kyaw[2]*

[1]Moscow Aviation Institute (National Research University), Department of Resistance of Materials Dynamics and Strength of Machines, Moscow, Russian Federation

[2]Defence Services Academy (D.S.A), Department of Mathematics, Pyin Oo Lwin, Myanmar

Research in the field of unsteady interaction of shock waves propagating in continuous media with various deformable barriers are of considerable scientific interest, since so far there are only a few scientific works dealing with solving problems of this class only for the simplest special cases. In this work, on the basis of analytical solution, we study the inverse non-stationary boundary-value problem of diffraction of plain pressure wave on convex surface in form of parabolic cylinder immersed in liquid and exposed to plane acoustic pressure wave. The purpose of the work is to construct approximate models for the interaction of an acoustic wave in an ideal fluid with an undeformable obstacle, which may allow obtaining fundamental solutions in a closed form, formulating initial-boundary value problems of the motion of elastic shells taking into account the influence of external environment in form of integral relationships based on the constructed fundamental solutions, and developing methods for their solutions. The inverse boundary problem for determining the pressure jump (amplitude pressure) was also solved. In the inverse problem, the amplitude pressure is determined from the measured pressure in reflected and incident waves on the surface of the body using the least squares method. The experimental technique described in this work can be used to study diffraction by complex obstacles. Such measurements can be beneficial, for example, for monitoring the results of numerical simulations.

Key words: unsteady dynamics, fluid deformation, transitional surface functions, the inverse boundary value problems

INTRODUCTION

One of the most pressing problems of modern mechanics is the study of unsteady interaction of shock waves propagating in continuous media with various deformable barriers. Research in this area is of considerable interest both from the point of view of developing mathematical methods for solving initial boundary-value problems of mechanics, and for a number of technical applications, in particular, the calculation of thin-walled structural elements loaded by shock waves in a liquid.

At this point we study the inverse non-stationary boundary-value problems of diffraction of plane pressure wave on convex surfaces immersed in liquid and exposed to acoustic shock waves. As an example, we study the diffraction of direct pressure of plane wave on a convex surface in the form of parabolic cylinder. To determine the hydrodynamic pressure acting on an obstacle, we used a transition function, built on the basis and hypothesis of a thin layer [1-3]. In this case, approximate models of interaction of a wave in fluid with a rigid obstacle, which allows obtaining fundamental solutions in closed form, were built. The diffraction of weak shock waves in liquid was studied on the basis of approximate models [4].

During the study of various problems of continuum mechanics, two main approaches to the statement of problems naturally arise – direct and inverse [5-7]. A lot of works have been devoted to various problems of continuum mechanics, both direct and inverse [8-10]. In this work, we consider a method for solving the boundary inverse problem of determining the amplitude pressure. Numerous computational experiments have been carried out in which the experimental pressure values were determined from the solution of the direct problem with the addition of error.

MATERIALS AND METHODS

When stating and solving diffraction problems for an unsteady direct pressure of plane wave on a hard obstacle, the parameters of the incident wave are often not known and it is difficult to measure them in field and bench experiments [11, 12]. At the same time, the technique of measuring pressure on the surface of an obstacle is significantly developed [13]. A problem arises: by measuring the pressure on the surface of the body, to determine the parameters of the incident wave. The leading method in this research is the method of solving the boundary inverse problem of determining the amplitude pressure. By measuring the pressure on the surface of the body at spatio-temporal points using the analytical solution (least squares method), the amplitude pressure value is determined. Numerous computational experiments have been

* ov-egorova@nuos.pro

carried out in which the experimental pressure values were determined from the solution of the direct problem with the addition of error. In this case, the accuracy in the obtained values does not exceed the accuracy in the experimental data.

The mathematical apparatus developed in the work are the transition functions – fundamental solutions to the unsteady initial-boundary-value problem of diffraction of an acoustic medium on a smooth convex surface. In particular, a transition function is used, built on the basis of the hypothesis of thin layer [14-16]. The use of transition functions provides a transition from solving the associated non-stationary problem of joint movement of the acoustic medium and the deformable obstacle to solving the problem only for the obstacle, the mathematical model of which takes into account interaction with the environment in form of integral relations [17]. The cores of integral terms of the equations of motion of the obstacle were formed on the basis of transition functions of the diffraction problem.

Therefore, the dimension of the problem was reduced, which makes it possible to significantly simplify the numerical solution on the basis of the finite element or finite difference approach, and in some important particular cases, construct analytical solutions and estimate the accuracy introduced by the accepted hypotheses. The mathematical formulation of direct problem has the following form [18] (Eqs. 1-3):

$$\frac{\partial^2 \varphi}{\partial \tau^2} = \Delta\varphi \tag{1}$$

$$p = -\frac{\partial \varphi}{\partial \tau}, v = grad\varphi \tag{2}$$

$$\varphi|_{\tau=0} = \left.\frac{\partial \varphi}{\partial \tau}\right|_{\tau=0} = 0 \tag{3}$$

where φ is the velocity potential in acoustic medium, p is the pressure in the reflected and incident waves, v is the velocity vector of the acoustic medium, Δ is the Laplace operator.

Then, the problem is solved by determining the pressure at the boundary of the body in a dimensionless form [19-21]. Furthermore, all linear dimensions are assigned to the focal distance α, velocities to the speed of sound in an acoustic medium c_0, quantities having the dimension of pressure to the complex $\rho_0 c_0^2$, time τ to tc_0/α [22-24]. The pressures p_1 in the reflected wave can be found using the transition function $G(x^i,\tau)$ constructed in the framework of thin layer hypothesis (an asterisk denotes the convolution operation in time τ) (Eqs. 4-5):

$$p_1(\xi^1,\tau) = \frac{\partial \phi_*(\xi^1,0,\tau)}{\partial n} *|G_p(\xi^1,\tau) \tag{4}$$

$$G_p(\xi^1,\tau) = -\left.\frac{\partial G(x^i,\tau)}{\partial \tau}\right|_G \tag{5}$$

At this, the influence function $G(x^i,\tau)$ satisfies the following initial-boundary-value problem (Eqs. 6-8):

$$\frac{\partial^2 G}{\partial \tau^2} = \Delta_\xi G \tag{6}$$

$$G|_{\tau=0} = \left.\frac{\partial G}{\partial \tau}\right|_{\tau=0} = 0 \tag{7}$$

$$\left.\frac{\partial G}{\partial n}\right|_\Gamma = \delta(\tau), G(r,\tau) = O(1) \text{ at } r \to \infty \tag{8}$$

where $\delta(\tau)$ is the Dirac delta function, * is the time convolution operation.

The transition function of the effect $G_0(\xi^1,\tau)$ on the surface of the obstacle F is found by the operational method and has the form [2] (Eqs. 9-11); at $r\to\infty$, where F_0 ([a], [b, c], z) is the generalized hypergeometric function.

$$G_0(\xi,\tau) = -H(\tau)R(z) \tag{9}$$

$$R(z) = -z + F_2\left(\left[-\frac{1}{2}\right],\left[\frac{1}{2},1\right],-\frac{z^2}{4}\right) \tag{10}$$

$$z = \frac{k(\xi)\tau}{2} \tag{11}$$

In this case, the expressions for the pressure in reflected and radiated waves, taking into account (Eqs. 8-10), can be presented in form (Eq. 12):

$$p_1(\xi^1,\tau) = -\int_0^\tau \frac{\partial \varphi_*(\xi^1,0,\tau-t)}{\partial \xi^2} G_p(\xi^1,t)\,dt \tag{12}$$

RESULTS AND DISCUSSION

Diffraction of plane wave of pressure on convex surfaces

Let us consider the problem of diffraction of plane step pressure wave at a rigid motionless curvilinear obstacle [25]. A direct plane acoustic wave with front, at the initial moment of time τ=0, touches at a point A (Fig. 1) the surface of parabolic cylinder with a guide G, with a focal distance a>0 in Cartesian rectangular coordinate system Ox^1x^2, which is defined as follows (Eqs. 13-14):

$$G: x^1 = \frac{\xi^2}{2} \tag{13}$$

$$x^2 = \xi, \xi \in \mathbb{R} \tag{14}$$

where the linear size in (1.2.23) is the value a: L=a.

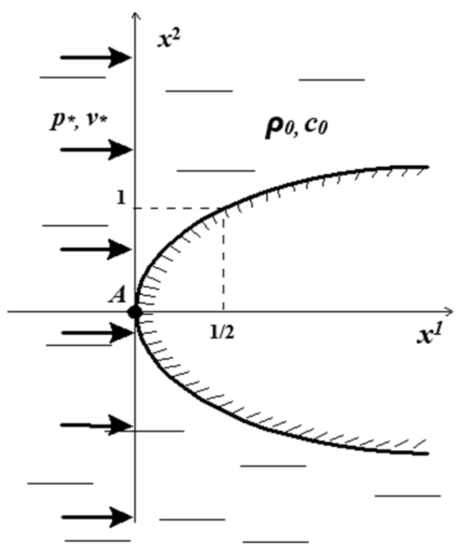

Figure 1: Calculation area

The pressure behind the wave front in the coordinate system Ox^i=*(i=1,2)* is set by the relation [6] (Eq. 15) or (Eq. 16):

$$p_*(x^i,\tau) = p_0 H(\tau - x^1) \tag{15}$$

$$p_*(\xi,\tau) = p_0 H\left(\tau - \frac{\xi^2}{2}\right) h \tag{16}$$

where p_0 is the amplitude pressure.

The main curvature is determined by the formula (16), where the average curvature takes the form *k(ξ)/2*, and the components of normal vector are given by expressions (17) for the case of plane problem (Eqs. 17-19) [26-28]:

$$k(\xi) = \frac{1}{(1+\xi^2)^{3/2}} \tag{17}$$

$$n_0^1 = \frac{1}{\sqrt{1+\xi^2}} \tag{18}$$

$$n_0^2 = -\frac{\xi}{\sqrt{1+\xi^2}} \tag{19}$$

The pressure of the reflected wave is determined by the equality [6] (Eq. 20):

$$p_1(\xi,\tau) = -p_0 n_0^1 G_0(\xi, \tau - \frac{\xi^2}{2}) \tag{20}$$

Figure 2 shows sections of the spatio-temporal total pressure (Eq. 21), upon action of a unit pressure jump p_0=1 by planes τ=*const*:

$$p(\xi,\tau) = p_*(\xi,\tau) + p_1(\xi,\tau) \tag{21}$$

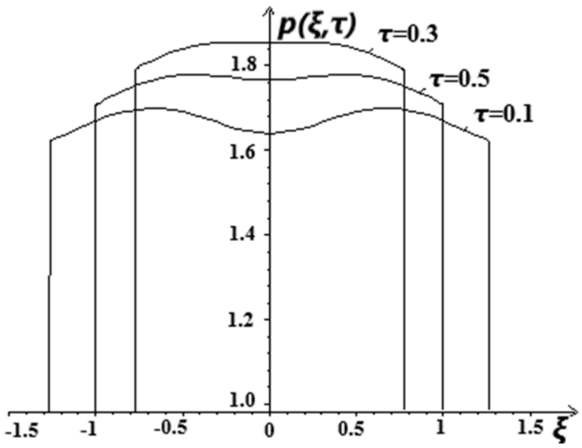

Figure 2: Distribution of overall pressure

Studying the inverse boundary value problems to determine the pressure jump

According to the experimental measurements in space-time points *(ξ_i,τ^k): i=1..I;k=1..K*, the overall pressure *p(ξ,τ)* on the surface of the parabolic cylinder is necessary to determine the value of the amplitude of pressure (jump) p_0. From (14) and (20) we get (Eq. 22):

$$\begin{aligned} p(\xi,\tau) &= p_*(\xi,\tau) + p_1(\xi,\tau) \\ &= p_0 H\left(\tau - \frac{\xi^2}{2}\right) - p_0 n_0^1 G_0(\xi, \tau - \frac{\xi^2}{2}) \\ &= p_0\left(H\left(\tau - \frac{\xi^2}{2}\right) - n_0^1 G_0(\xi,\tau - \frac{\xi^2}{2})\right) = p_0 F(\xi,\tau) \end{aligned} \tag{22}$$

To determine p_0 using the least squares method, we compile the functional (Eq. 23):

$$S(p_0) = \sum_{i=1}^{I}\sum_{k=1}^{K}\left(\tilde{p}_i^k - p(\xi_i,\tau^k)\right)^2 \to 0 \tag{23}$$

where $\tilde{p}_i^k$ are the experimental values of the total pressure on the surface of parabolic cylinder.

Calculating the gradient from the functional (23) by the parameter p_0 and equating it to zero, we get, taking into account (22):

$$\frac{\partial S(p_0)}{\partial p_0} = \sum_{i=1}^{I}\sum_{k=1}^{K}\left(\tilde{p}_i^k - p_0 F(\xi_i,\tau^k)\right) F(\xi_i,\tau^k) = 0 \tag{24}$$

Then we express parameter p_0 from (24):

$$p_0 = \frac{\sum_{i=1}^{I}\sum_{k=1}^{K}\left(\tilde{p}_i^k\right) F(\xi_i,\tau^k)}{\sum_{i=1}^{I}\sum_{k=1}^{K} F(\xi_i,\tau^k)\, F(\xi_i,\tau^k)} \tag{25}$$

Formula (25) lets us calculate the value of the amplitude pressure p_0 with controlled accuracy, while the more experimental values we have, the higher is the accuracy of determining the parameter p_0.

Simulation using the computational experiment

To simulate the experimental values, we calculate the values of total pressure *p(ξ,τ)* according to formula (22) at and add a random relative error in the range of 10% and 20%: p_0=12.3 (Eqs. 26-28):

$$\tilde{p}_i^k = p(\xi_i,\tau^k) + \delta \tag{26}$$

$$\xi_i = \{\xi_1 = -0.5; \xi_2 = 0; \xi_3 = 0.5\}, i = \{1,2,3\} \tag{27}$$

$$\tau^k = \{\tau^k = 0.3; \tau^k = 1.0\}, k = \{1.2\} \tag{28}$$

Values $\tilde{p}_i^k$ are shown in Table 1.

Table 1: Experimental values

	$p(\xi_i,\tau^k)$	$\tilde{p}_i^k=p(\xi_i,\tau^k)+\delta_{10\%}$	$\tilde{p}_i^k=p(\xi_i,\tau^k)+\delta_{20\%}$
ξ_1=-0.5;τ^1=0.3	22.632	20.66594	21.29192
ξ_2=0;τ^1=0.3	22.878	21.26062	21.99243
ξ_3=0.5;τ^1=0.3	22.632	24.26192	25.98572
ξ_1=-0.5;τ^2=1.0	20.664	19.40511	21.29945
ξ_2=0;τ^2=1.0	20.172	18.76031	23.84168
ξ_3=0.5;τ^2=1.0	20.664	21.15547	23.29237

Having calculated the value p_0 using formula (26), we obtain:

- p_0=11.91049, with the accuracy of input data of 10% δ_1=3.16%;
- p_0=13.03037, with the accuracy of input data of 20% δ_2=5.94%.

Therefore, the accuracy of the obtained parameter does not exceed the accuracy in the experimental data.

CONCLUSIONS

Consequently, the problem of diffraction of direct pressure of plane wave on a convex surface in the form of a parabolic cylinder was studied. A fundamental solution of the problem of the acoustic wave diffraction pressure on a smooth convex obstacle in the form of a parabolic cylinder was constructed. An algorithm for solving the inverse problem of the boundary to determine the amplitude-stand pressures was offered. Based on the analytical solution, a calculation was made to determine the amplitude pressure. Computational experiments were performed in which the experimental values of pressure were determined from the direct problem solution with the addition of error.

For the inverse problem, the amplitude pressure was determined from experimental data (measured pressure in the reflected and incident waves on the surface of the body) using the least squares method. Computational experiments demonstrated that the amplitude pressure can be determined with controlled accuracy, despite the high (up to 20%) relative error in the experimental data.

ACKNOWLEDGMENTS

The work has been conducted with the financial support of the grant of the Russian Foundation for Basic Research, project code No 19-01-00675.

REFERENCES

1. Medvedsky, A.L., Rabinsky, L.N. (2007). The method of surface influence functions in non-stationary diffraction problems. MAI Publishing House, Moscow.
2. Zhavoronok, S.I., Kuprikov, M.Yu., Medvedskiy, A.L., Rabinskiy, L.N. (2010). Numerical-analytical methods for solving the problems of diffraction of acoustic waves on absolutely solid bodies and shells. FIZMATLIT, Moscow.
3. Zhavoronok, S.I. (2018). On the use of extended plate theories of Vekua-Amosov type for wave dispersion problems. International Journal for Computational Civil and Structural Engineering, vol. 14, no. 1, 36-48.
4. Durand, O., Jaouen, S., Soulard, L., Heuzé, O., Colombet, L. (2017). Comparative simulations of micro jetting using atomistic and continuous approaches in the presence of viscosity and surface tension. Journal of Applied Physics, vol. 122, no. 13, https://doi.org/10.1063/1.4994789.
5. Rabinskiy, L.N. (2019). Non-stationary problem of the plane oblique pressure wave diffraction on thin shell in the shape of parabolic cylinder. Periodico Tche Quimica, vol. 16, no. 32, 328-337.
6. Rabinskiy, L.N., Tushavina, O.V. (2019). Investigation of an elastic curvilinear cylindrical shell in the shape of a parabolic cylinder, taking into account thermal effects during laser sintering. Asia Life Sciences, no. 2, 977-991.
7. Zheng, S., Zhang, Y., Liu, J., Iglesias, G. (2019). Wave diffraction from multiple truncated cylinders of arbitrary cross sections. Applied Mathematical Modelling, vol. 77, 1425-1445, https://doi.org/10.1016/j.apm.2019.08.006.
8. Kozorez, D.A., Kruzhkov, D.M. (2019). Autonomous navigation of the space debris collector. INCAS Bulletin, no. 11, 89-104.
9. Evdokimenkov, V.N., Kozorez, D.A., Krasilshchikov, M.N. (2019). Development of pre-flight planning algorithms for the functional-program prototype of a distributed intellectual control system of unmanned flying vehicle groups. INCAS Bulletin, no. 11, 75-88.
10. Lamas-Pardo, M., Iglesias, G., Carral, L. (2015). A review of very large floating structures (VLFS) for coastal and offshore uses. Ocean Engineering, vol. 109, 677-690, https://doi.org/10.1016/j.oceaneng.2015.09.012.
11. Bulychev, N.A., Kazaryan, M.A., Bodryshev, V.V. (2019). Application of methods for analyzing images of nanoparticles in transmitted light to study their properties. SPIE, Tomsk.
12. Zheng, S., Zhang, Y. (2016). Wave radiation from a truncated cylinder in front of a vertical wall. Ocean Engineering, vol. 111, 602-614, https://doi.org/10.1016/j.oceaneng.2015.11.024.
13. Bulychev, N.A. (2019). On the hydrogen production during the discharge in a two-phase vapor-liquid flow. Bulletin of the Lebedev Physics Institute, vol. 46, no. 7, 219-221.
14. Li, Z.F., Wu, G.X., Ren, K. (2020). Wave diffraction by multiple arbitrary shaped cracks in an infinitely extended ice sheet of finite water depth. Journal of Fluid Mechanics, vol. 893, no. A14, https://doi.org/10.1017/jfm.2020.238.
15. Bulychev, N.A. (2019). Experimental studies on hydrogen production in plasma discharge in a liquid-phase medium flow. International Journal of Hydrogen Energy, vol. 44, no. 57, 29933-29936.
16. Antufiev, B.A., Egorova, O.V., Orekhov, A.A., Kuznetsova, E.L. (2018). Dynamics of a clamped ribbed plate under moving loads. Periodico Tche Quimica, vol. 15, no. 1, 368-376.
17. Antufiev, B.A., Egorova, O.V., Orekhov, A.A., Kuznetsova, E.L. (2018). Dynamics of thin-walled structure with high elongation ratio and discrete elastic supports on the rigid surface under moving loads. Periodico Tche Quimica, vol. 15, no. 1, 464-470.
18. Fang, X.-Q., Zhang, T.-F., Yang, S.P., Ning, Y., Li, B.-L. (2019). Elastic–slip interface effect on the diffraction of plane waves around a saturated lining structure in saturated soil. Engineering Analysis with Boundary Elements, vol. 107, 134-141, https://doi.org/10.1016/j.enganabound.2019.07.006.

19. Bulychev, N.A., Kazaryan, M.A., Ethiraj, A., Chaikov, L.L. (2018). Plasma discharge in liquid phase media under ultrasonic cavitation as a technique for synthesizing gaseous hydrogen. Bulletin of the Lebedev Physics Institute, vol. 45, no. 9, 263-266.
20. Formalev, V.F., Kolesnik, S.A., Kuznetsova, E.L. (2019). Effect of components of the thermal conductivity tensor of heat-protection material on the value of heat fluxes from the gasdynamic boundary layer. High Temperature, vol. 57, no. 1, 58-62.
21. Sultanov, K., Khusanov, B., Rikhsieva, B. (2020). Shear waves around an underground pipeline. IOP Conference Series: Materials Science and Engineering, vol. 869, no. 052016.
22. Dinzhos, R., Lysenkov, E., Fialko, N. (2015). Simulation of thermal conductivuty of polymer composites based on poly (methyl methacrylate) with different types of fillers. Eastern-European Journal of Enterprise Technologies, vol. 6, no. 11, 21-24.
23. Skvortsov, A.A., Pshonkin, D.E., Luk'yanov, M.N. (2018). Influence of constant magnetic fields on defect formation under conditions of heat shock in surface layers of silicon. Key Engineering Materials, vol. 771, 124-129.
24. Mykhalevskiy, D.M., Kychak, V.M. (2019). Development of information models for increasing the evaluation efficiency of wireless channel parameters of 802.11 standard. Latvian Journal of Physics and Technical Sciences, vol. 56, no. 5, 22-32.
25. Formalev, V.F., Kolesnik, S.A., Kuznetsova, E.L. (2019). Approximate analytical solution of the problem of conjugate heat transfer between the boundary layer and the anisotropic strip. Periodico Tche Quimica, vol. 16, no. 32, 572-582.
26. Zvorykin, A., Aleshko, S., Fialko, N., Maison, N., Meranova, N., Voitenko, A., Pioro, I. (2016). Computer simulation of flow and heat transfer in bare tubes at supercritical parameters. International Conference on Nuclear Engineering, Proceedings, ICONE, vol. 5, no. V005T15A023.
27. Ryndin, V.V. (2019). Statement of the second law of thermodynamics on the basis of the postulate of nonequilibrium. Periodico Tche Quimica, vol. 16, no. 32, 698-712.
28. Sultanov, K., Khusanov, B., Rikhsieva, B. (2020). Interaction of a rigid underground pipeline with elasticviscous-plastic soil. IOP Conference Series: Materials Science and Engineering, vol. 883, no. 012038.

Paper submitted: 19.08.2020.
Paper accepted: 19.10.2020.