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Eduard I. Starovoitov
Belarusian State University of Transport, Department of Building Mechanics, Gomel, Republic of Belarus

Denis V. Leonenko
Belarusian State University of Transport, Department of Building Mechanics, Gomel, Republic of Belarus

Alexander A. Orekhov
Moscow Aviation Institute (National Research University), Institute of General Engineering Education, Moscow, Russian Federation

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DYNAMIC BEHAVIOR OF THIN-WALLED ELEMENTS OF AIRCRAFT MADE OF COMPOSITE MATERIALS, EXCITED BY HEAT SHOCK

Eduard I. Starovoitov[1], Denis V. Leonenko[1], Alexander A. Orekhov[2]*

[1]Belarusian State University of Transport, Department of Building Mechanics, Gomel, Republic of Belarus
[2]Moscow Aviation Institute (National Research University), Institute of General Engineering Education, Moscow, Russian Federation

To improve the performance characteristics of modern aerospace systems, research is conducted and expensive programs are being carried out to provide for reducing the weight of the aircraft structure through the use of new, more promising materials, which include the so-called composite materials. Special attention is paid to the dynamic behaviour of composite structures under the influence of high-intensity heat fluxes of various physical nature. The paper considers the dynamic behaviour of composite structures of modern aerospace systems under the influence of high-intensity heat fluxes. As an example, the axisymmetric transverse vibrations of a composite circular plate connected to an elastic base, excited by thermal shock, are investigated. The plate material is modelled with a three-layer composite. To describe the kinematics of an asymmetric plate pack, the hypotheses of a broken normal are accepted. In thin bearing layers, Kirchhoff's hypotheses are valid. In a relatively thick lightweight core, the normal does not change its length, remains rectilinear, but rotates through some additional angle. The base reaction is described by the Winkler model. The statement of the initial-boundary value problem is given. The analytical solution is obtained as a series expansion in terms of eigenfunctions. Its numerical parametric analysis is carried out.

Key words: composite round plate, vibration, heat flux, elastic base

INTRODUCTION

For the first time, a three-layer structure was used in the construction of a railway bridge in 1845 by the English engineer Robert Stephenson with the assistance of William Fairbairn. In the 1940s, the first aircraft with three-layer load-bearing elements began to appear. At the present day, similar structures have found their application in aerospace and transport engineering, construction, production and transportation of hydrocarbons. All this led to the demand for the sandwich-type structural elements, including three-layer.

Sandwich rods, plates, and shells are usually made from two thin, high-strength load-bearing layers designed to resist external loading and a relatively thick core. Fillers are polymer, composite, cellular, etc. They connect the load-bearing layers, increase the structure bending stiffness, and serve as protection against thermo-radiation effects. During operation, these structures have proven to be the most rational from material consumption under given strength constraints.

The introduction of sandwich structures in technology required the creation of appropriate mathematical models of deformation and the intensive development of calculation methods. As a result, during this time in the mechanics of deformable solid bodies, a whole direction has appeared, associated with the study of the stress-strain state of sandwich plates, rods and shells. To improve the performance characteristics of modern aerospace systems, research is conducted and expensive programs are being carried out that provides for reducing the aircraft structure weight through the use of new, more promising materials. Particular attention is paid to the dynamic behaviour of composite structures under the influence of high-intensity heat fluxes of various physical nature.

At present time, the formation of a general theory of quasi-static and dynamic deformations of sandwich structures of transport technology has not yet been completed and is being intensively developed. For them, mathematical models of deformation under complex thermo-force, thermo-radiation loads are created. The problems of strength, stability, dynamic behaviour are considered. The formulations and solutions of the initial-boundary value problems about forced vibrations of sandwich rods under the local and impulsive forces are given in the [1]. The deformation is taken in accordance with the kinematic broken line hypotheses, the solution is constructed as a series expansion in terms of the system of orthonormal eigenfunctions. Papers [2, 3] are devoted to the study of free and forced vibrations of two-layer circular metal-polymer plates. The polymer layer is assumed to be rather thick; its deformation corresponds to Timoshenko's hypothesis of the straightness and normal incompressibility. The general theory of the quasi-static loading of physically nonlinear sandwich structural ele-

*ed.starovoitov5722-9@murdoch.in

ments was developed in the [4]. Oscillations of circular three-layer plates under the action of distributed types of local loads (uniform, sinusoidal, parabolic) were considered in the [5]. Papers [6-8] are devoted to the study of various aspects of technological high-speed processes.

The theory of quasi-static variable loading of elastoplastic three-layer rods and shells was developed in papers [9, 10]. The proposed method for solving the corresponding problems of the theory of small elastoplastic deformations uses the hypothesis of the similarity of the deformation curves on an arbitrary half-cycle with the curve under loading from the natural state. The properties of cyclic hardening and softening of layer materials are taken into account. Analytical solutions and their numerical analysis are presented. In papers [11, 12], the effect of temperature on the mechanical properties of materials and sandwich structures is investigated. Approximation formulas are proposed for the plasticity functions of metals and physical nonlinearity of polymer materials under isothermal and thermal force loading [13-15]. The experimental constants included in these formulas, obtained as a result of processing experimental ones for some structural materials, are given. Publications [16-18] are devoted to the influence of neutron irradiation on the deformation of elastoplastic three-layer rods. In this paper, vibrations of an elastic circular sandwich plate connected to the Winkler base, caused by the action of high-intensity heat flux (thermal shock) are considered.

MATHERIALS AND METHODS

The problem and its solution will be formulated in a cylindrical coordinate system r, φ, z. Let us take the median plane of the core as the coordinate one, the z axis is directed perpendicularly upward to layer 1 (Fig. 1). For thin outer bearing layers with a thickness $h1 \neq h2$, Kirchhoff's hypotheses are accepted; for a thick lightweight (not working in the tangential direction) core ($h3 = 2c$), the hypothesis of straightness and incompressibility of the deformed normal is valid. External load projections on the vertical and radial coordinate axes $q = q(r, t)$ and $p = p(r, t)$. On the plate contour, it is assumed that there is a rigid diaphragm that prevents the relative layers displacement ($\psi = 0$ at $r = r0$). Plate radius – $r1$.

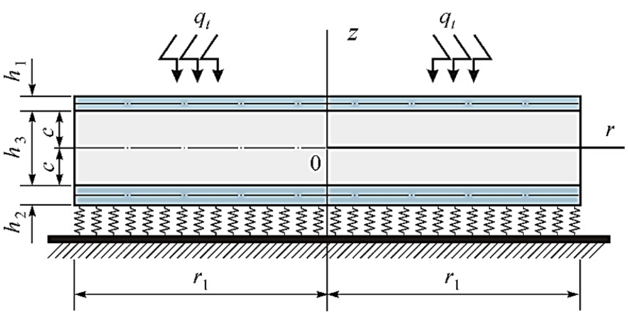

Figure 1: Design model of considered plate

Due to the problem symmetry, tangential displacements in the layers are absent, and the plate deflection $w(r, t)$, the relative shear in the core $\psi(r, t)$ and the radial displacement of the coordinate surface $u(r, t)$ do not depend on the coordinate φ. In what follows, we consider these functions as required. h_k denotes the relative thickness of the k-th layer. It is assumed that in the process of deformation, the soil base exhibits elastic properties. The relationship between its reaction q_R and plate deflection is described by the Winkler model (Eq. 1):

$$q_R = \kappa_0 w, \tag{1}$$

where κ_0 – bed coefficient of elastic foundation.

Let us assume that at the initial moment of time an axisymmetric load of intensity $q(r, t)$ is applied to the outer surface of the first layer ($z = c + h1$), and the heat flux q_t is supplied. Bottom surface (Eq. 2) and the plate contour is assumed to be thermally insulated:

$$z = -c - h_2 \tag{2}$$

Under the indicated conditions of heat transfer, the unsteady one-dimensional temperature field $T(z, t)$ satisfies the differential heat conduction equation (Eq. 3):

$$T'_{zz} = \frac{\dot{T}}{d_n} \tag{3}$$

At the initial T = 0 (t = 0) and boundary conditions on the outer planes of the plate (Eqs. 4-5):

$$\lambda_1 T'_z = -q_t \text{ at } z = c + h_1,\ T'_z = 0 \text{ at } z = -c - h_2, \tag{4}$$

$$d_n = \lambda_k/(C_k \rho_k), \tag{5}$$

where d_n – thermal diffusivity of the k-th layer; λ_k, C_k – coefficients of thermal conductivity and heat capacity; ρ_k – material density.

The characteristics λ_k, C_k, ρ_k vary discontinuously over the thickness of the three-layer pack. Therefore, in the exact formulation of the problem of temperature field determination, (Eq. 3) must be solved inside each homogeneous region (layer) separately, setting additional conditions for heat transfer and equality of temperatures on the surfaces splicing. For simplicity, the thermophysical characteristics were averaged over the plate thickness, and the problem was reduced to determining the temperature field in a homogeneous plate with modified characteristics (Eq. 6):

$$\begin{gathered} d = \lambda/C,\ \lambda = \textstyle\sum_{k=1}^{3} \lambda_k h_k /H,\ C = \sum_{k=1}^{3} \rho_k C_k h_k /H, \\ s = z/H,\ H = h_1 + h_2 + h_3,\ \tau = td/H^2, \end{gathered} \tag{6}$$

The problem of determining the temperature field in the plate in this case follows from (3) after replacing dk by d and λ_k by λ. Its solution allows the inhomogeneous temperature field $T(r, t)$, measured from a certain initial temperature T_0, to be calculated by the formula [9] (Eq. 7):

$$\begin{aligned} T = \frac{q_t H}{\lambda} \Big\{ & \tau + \frac{1}{2}\left(s + \frac{c + h_2}{H}\right)^2 - \frac{1}{6} \\ & - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} cos\left[\pi n \left(s + \frac{c + h_2}{H}\right)\right] e^{-n^2\pi^2\tau} \Big\} \end{aligned} \tag{7}$$

Using the hypothesis of the straightness of core normal (Eq. 8) after integration, we obtain expressions for the radial displacements in the layers $u_r^{(k)}$ in terms of the required functions (Eqs. 9-11):

$$2\varepsilon_{rz}^{(3)} = u_r^{(3)},_z + w,_r = \psi, \tag{8}$$

$$u_r^{(1)} = u + c\psi - zw,_r \ (c \le z \le c + h_1), \tag{9}$$

$$u_r^{(3)} = u + z\psi - zw,_r \ (-c \le z \le c), \tag{10}$$

$$u_r^{(2)} = u - c\psi - zw,_r \ (-c - h_2 \le z \le -c), \tag{11}$$

where z – fibre coordinate, $(u + c\psi)$ – the amount of upper bearing layer displacement due to core deformation, $(u - c\psi)$ – lower bearing layer displacement, the comma in the subscript indicates the operation of differentiation by the coordinate following it.

Small deformations in layers follow from displacements (9-11) and Cauchy relations [9]. The physical equations of thermoelasticity are taken in the form $(\alpha=r,\varphi;k=1,2,3)$ (Eqs. 12-13):

$$s_\alpha^{(k)} = 2G_k Э_\alpha^{(k)},\ s_{rz}^{(3)} = 2G_3 Э_{rz}^{(3)}, \tag{12}$$

$$\sigma^{(k)} = 3K_k(\varepsilon^{(k)} - \alpha_{0k}T_k), \tag{13}$$

where $s_\alpha^{(k)}$, $Э_\alpha^{(k)}$ – deviators, and $\sigma^{(k)}$, $\varepsilon^{(k)}$ – spherical parts of stress and strain tensors; G_k, K_k – thermally independent shear moduli and volumetric deformation of layer materials, T_k – temperature in the k-th layer, calculated by the formula (7), α_{0k} – linear thermal expansion coefficient.

RESULTS AND DISCUSSION

The system of differential equations describing the transverse vibrations of solid circular sandwich plates without taking into account the compression and inertia of rotation of the normal in the layers is derived from the Lagrange variational principle with allowance for inertial forces [19, 20] (Eqs. 14-16):

$$u = b_1 w,_r + C_1 r, \tag{14}$$

$$\psi = b_2 w,_r + C_3 r, \tag{15}$$

$$L_3(w,_r) + \kappa^4 w + M^4 \ddot{w} = 0, \tag{16}$$

where (Eq. 17):

$$M_0 = (\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3) r_1^2, \tag{17}$$

Coefficients b_i are expressed through the geometric and elastic characteristics of the layers (Eqs.18-29):

$$b_1 = \frac{a_3 a_4 - a_2 a_5}{a_1 a_4 - a_2^2}, \tag{18}$$

$$b_2 = \frac{a_1 a_5 - a_2 a_3}{a_1 a_4 - a_2^2}, \tag{19}$$

$$\kappa^4 = \kappa_0 D, \tag{20}$$

$$M^4 = M_0 D, \tag{21}$$

$$D = \frac{a_1(a_1 a_4 - a_2^2)}{(a_1 a_6 - a_3^2)(a_1 a_4 - a_2^2) - (a_1 a_5 - a_2 a_3)^2}, \tag{22}$$

$$a_1 = \sum_{k=1}^{3} h_k K_k^+, \tag{23}$$

$$a_2 = c(h_1 K_1^+ - h_2 K_2^+), \tag{24}$$

$$a_3 = h_1\left(c + \tfrac{1}{2}h_1\right)K_1^+ - h_2\left(c + \tfrac{1}{2}h_2\right)K_2^+, \tag{25}$$

$$a_4 = c^2\left(h_1 K_1^+ + h_2 K_2^+ + \tfrac{2}{3}cK_3^+\right), \tag{26}$$

$$a_5 = c\left[h_1\left(c + \tfrac{1}{2}h_1\right)K_1^+ + h_2\left(c + \tfrac{1}{2}h_2\right)K_2^+ + \tfrac{2}{3}c^2 K_3^+\right], \tag{27}$$

$$\begin{aligned} a_6 = {} & h_1\left(c^2 + ch_1 + \frac{1}{3}h_1^2\right)K_1^+ + \\ & + h_2\left(c^2 + ch_2 + \tfrac{1}{3}h_2^2\right)K_2^+ + \tfrac{2}{3}c^3 K_3^+, \end{aligned} \tag{28}$$

$$K_k^+ \equiv K_k + \tfrac{4}{3}G_k, \tag{29}$$

Differential operators L_2, L_3 (Eq. 30-31):

$$L_2(g) \equiv g,_{rr} + \frac{g,_r}{r} - \frac{g}{r^2}, \tag{30}$$

$$L_3(g) \equiv g,_{rrr} + \frac{2g,_{rr}}{r} - \frac{g,_r}{r^2} + \frac{g}{r^3}, \tag{31}$$

The initial conditions of motion are assumed to be uniform $w(r,0)=0$; $\dot{w}(r,0)=0$; $T(z,0)=0$. In comparison with the problem of isothermal vibrations [21-23], changes will occur in the accepted boundary conditions for the hinge support of the plate contour. They add a "temperature" moment M_t, due to the volumetric thermal deformation in each layer (Eq. 32):

$$\theta_{tk} = \alpha_{0k} T(z,t) \tag{32}$$

Thus, for hinged support, the conditions on the contour must be met $(r = r1)$ (Eqs. 33-34):

$$u = \psi = w = 0, \tag{33}$$

$$M_r = \sum_{k=1}^{3} \int_{h_k} \sigma_r^{(k)} z dz = 0, \tag{34}$$

where M_r – internal radial moment (Eq. 35):

$$M_r = a_3 u,_r + a_5 \psi,_r - a_6 w,_{rr} - a_{60} w,_r / r - M_t \tag{35}$$

Using the formula for temperature (7), we obtain the expression for the moment M_t (Eq. 36):

$$M_t = \sum_{k=1}^{3} M_{kt} = 3\sum_{k=1}^{3} \alpha_{0k}\left(K_k - \frac{2}{3}G_k\right)\int_{h_k} Tzdz \tag{36}$$

where (Eqs. 37-39):

$$\begin{aligned} M_{1t} = {} & \frac{3q_t H\alpha_{01}}{\lambda}\left(K_1 - \frac{2}{3}G_1\right)\left[h_1\left(c + \frac{h_1}{2}\right)\left(\tau - \frac{1}{6}\right) + \frac{1}{2H^2} \times \right. \\ & \times \left(\frac{(h_1 + c)^4 - c^4}{4} + 2h_1(c + h_2)(c^2 + ch_1 + \frac{1}{3}h_1^2) + \right. \\ & \left. + h_1(c + \frac{1}{2}h_1)(c + h_2)^2\right) - \frac{2H}{\pi^3} \times \sum_{n=1}^{\infty} \frac{(-1)^n \exp(-n^2\pi^2\tau)}{n^3} \times \\ & \left. \times \left(\frac{H}{\pi n}\left((-1)^n - cos\frac{\pi n(2c + h_2)}{H}\right) - c\, sin\frac{\pi n(2c + h_2)}{H}\right)\right] \end{aligned} \tag{37}$$

$$\begin{aligned} M_{2t} = {} & \frac{3q_t H\alpha_{02}}{\lambda}\left(K_2 - \frac{2}{3}G_2\right)\left[-h_2\left(c + \frac{h_2}{2}\right)\left(\tau - \frac{1}{6}\right) + \frac{1}{2H^2}\left(\frac{c^4 - (h_2 + c)^4}{4} + \right.\right. \\ & \left. + 2h_2(c + h_2)(c^2 + ch_2 + \frac{1}{3}h_2^2) - h_2(c + \frac{1}{2}h_2)(c + h_2)^2\right) - \\ & \left. - \frac{2H}{\pi^3}\sum_{n=1}^{\infty} \frac{(-1)^n \exp(-n^2\pi^2\tau)}{n^3}\left(\frac{H}{\pi n}\left(cos\frac{\pi n h_2}{H} - 1\right) - c\, sin\frac{\pi n h_2}{H}\right)\right], \end{aligned} \tag{38}$$

$$M_{3t}=\frac{3q_tH\alpha_{03}}{\lambda}\left(K_3-\frac{2}{3}G_3\right)\left[\frac{2c^3(c+h_2)}{3H^2}-\frac{2H}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n\exp(-n^2\pi^2\tau)}{n^3}\times\right.$$
$$\left.\times\left(\frac{H}{\pi n}\left(\cos\frac{\pi n(2c+h_2)}{H}-\cos\frac{\pi nh_2}{H}\right)+c\left(\sin\frac{\pi n(2c+h_2)}{H}+\sin\frac{\pi nh_2}{H}\right)\right)\right].\quad(39)$$

From the first two conditions (9-11) at the boundary it follows (Eqs. 40-41), which allows to write out two boundary conditions for deflection on the plate contour ($r = r_1$) (Eq. 42):

$$C_1=-\frac{b_1}{r_1}w,_r|_{r=1},\quad(40)$$

$$C_3=-\frac{b_2}{r_1}w,_r\Big|_{r=1},\quad(41)$$

$$w=0;\ a_7w,_{rr}+a_8w,_r+M_t=0,\quad(42)$$

where (43-45):

$$a_7=a_6-a_3b_1-a_5b_2,\quad(43)$$

$$a_8=a_{60}+a_3b_1+a_5b_2,\quad(44)$$

$$a_{60}=h_1\left(c^2+ch_1+\frac{1}{3}h_1^2\right)K_1^-+$$
$$+h_2\left(c^2+ch_2+\frac{1}{3}h_2^2\right)K_2^-+\frac{2}{3}c^3K_3^-,\quad(45)$$

The solution of the last of the equations of system (14-16) is represented as the sum of the quasi-static deflection ws and the dynamic part w_d (Eq. 46):

$$w=w_s+w_d,\quad(46)$$

Quasi-static deflection satisfies the equation (Eq. 47):

$$L_3(w_{s},_r)=0,\quad(47)$$

Under boundary conditions on the contour (Eq. 48):

$$w_s=0;\ a_7w_{s},_{rr}+a_8w_{s},_r+M_t=0,\quad(48)$$

In the case under consideration, for a solid plate, it has the form (Eq. 49):

$$w_s=\frac{M_tr_1^2}{2(a_7+a_8)}\left[1-\left(\frac{r}{r_1}\right)^2\right],\quad(49)$$

Substituting solution (46) into the general equation for the deflection (14-26) *w(r, 0)=0; w ̇(r, 0)=0; T(z, 0)=0* into the initial and boundary conditions (42) and taking into account the quasi-static deflection (49), we obtain a closed initial-boundary value problem for determining the dynamic part of the deflection w_d. The partial differential equation for its definition is inhomogeneous (Eq. 50):

$$L_3(w_{d},_r)+\kappa^4w_d+M^4\ddot{w}_d=-\frac{r_1^2(\kappa^4M_t+M^4\ddot{M}_t)}{2(a_7+a_8)}\left[1-\left(\frac{r}{r_1}\right)^2\right]\quad(50)$$

where (Eq. 51-54):

$$\ddot{M}_t=\sum_{k=1}^{3}\ddot{M}_{kt},\quad(51)$$

$$\ddot{M}_{1t}=-\frac{6q_t\alpha_{01}a^2\pi}{\lambda H^2}\left(K_1-\frac{2}{3}G_1\right)\times\sum_{n=1}^{\infty}(-1)^n\exp(-n^2\pi^2\tau)\times$$
$$\times\left(\frac{H}{\pi}\left((-1)^n-\cos\frac{\pi n(2c+h_2)}{H}\right)-nc\sin\frac{\pi n(2c+h_2)}{H}\right),\quad(52)$$

$$\ddot{M}_{2t}=-\frac{6q_t\alpha_{02}a^2\pi}{\lambda H^2}\left(K_2-\frac{2}{3}G_2\right)\sum_{n=1}^{\infty}(-1)^n\exp(-n^2\pi^2\tau)\times$$
$$\times\left(\frac{H}{\pi}\left(\cos\frac{\pi nh_2}{H}-1\right)-nc\sin\frac{\pi nh_2}{H}\right),\quad(53)$$

$$\ddot{M}_{3t}=-\frac{6q_t\alpha_{03}a^2\pi}{\lambda H^2}\left(K_3-\frac{2}{3}G_3\right)\sum_{n=1}^{\infty}(-1)^n\exp(-n^2\pi^2\tau)\times$$
$$\times\left(\frac{H}{\pi}\left(\cos\frac{\pi n(2c+h_2)}{H}-\cos\frac{\pi nh_2}{H}\right)+nc\left(\sin\frac{\pi n(2c+h_2)}{H}+\sin\frac{\pi nh_2}{H}\right)\right),\quad(50)$$

In the initial conditions of motion, nonzero "initial" deflection and speed appear ($t = 0$) (Eqs. 55-56):

$$w_d=-\frac{1-r^2}{2(a_7+a_8)}M_t(0),\quad(55)$$

$$\dot{w}_d=-\frac{r_1^2\dot{M}_t(0)}{2(a_7+a_8)}\left[1-\left(\frac{r}{r_1}\right)^2\right],\quad(56)$$

The value $M_t(0)$ follows from (36) at $t = 0$ (Eqs. 57-60):

$$\dot{M}_t(0)=\sum_{k=1}^{3}\dot{M}_{kt}(0),\quad(57)$$

$$\dot{M}_{1t}(0)=\frac{3q_t\alpha_{01}a}{\lambda H}\left(K_1-\frac{2}{3}G_1\right)\left[h_1\left(c+\frac{h_1}{2}\right)+\frac{2H}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\times\right.$$
$$\left.\times\left(\frac{H}{\pi n}\left((-1)^n-\cos\frac{\pi n(2c+h_2)}{H}\right)-c\sin\frac{\pi n(2c+h_2)}{H}\right)\right],\quad(58)$$

$$\dot{M}_{2t}(0)=\frac{3q_t\alpha_{02}a}{\lambda H}\left(K_2-\frac{2}{3}G_2\right)\left[-h_2\left(c+\frac{h_2}{2}\right)+\frac{2H}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\times\right.$$
$$\left.\times\left(\frac{H}{\pi n}\left(\cos\frac{\pi nh_2}{H}-1\right)-c\sin\frac{\pi nh_2}{H}\right)\right],\quad(59)$$

$$\dot{M}_{3t}(0)=\frac{6q_t\alpha_{03}a}{\lambda\pi}\left(K_3-\frac{2}{3}G_3\right)\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\times$$
$$\times\left(\frac{H}{\pi n}\left(\cos\frac{\pi n(2c+h_2)}{H}-\cos\frac{\pi nh_2}{H}\right)+\right.$$
$$\left.+c\left(\sin\frac{\pi n(2c+h_2)}{H}+\sin\frac{\pi nh_2}{H}\right)\right)\quad(60)$$

Boundary conditions become homogeneous (Eq. 61):

$$w_d=0,\ a_7w_{d},_{rr}+\frac{a_8}{r_1}w_{d},_r=0,\ (r=r_1),\quad(61)$$

The solution of the homogeneous differential equation corresponding to equation (50) is taken in the form (Eq. 62):

$$w_d^0=v(r)(A\cos(\omega t)+B\sin(\omega t)),\quad(62)$$

After substituting expression (62) into the indicated homogeneous equation, we obtain a differential equation for determining the function *v(r)* (Eq. 63):

$$v,_{rrrr}+\frac{2}{r}v,_{rrr}-\frac{1}{r^2}v,_{rr}+\frac{1}{r^3}v,_r-(\beta^4-\kappa^4)v=0,\quad(63)$$

where $\beta^4 = \omega^2M^4$.

The solution to the equation (63), taking into account the boundedness at the origin, is known [9] (Eq. 64):

$$v=C_5J_0(\lambda r)+C_6I_0(\lambda r),\quad(64)$$

where J_0, I_0 – zero-order Bessel functions of the first kind (Eq. 65):

$$\lambda^4=\beta^4-\kappa^4,\quad(65)$$

Substituting (64) into boundary conditions (55) and requiring the nontriviality of the solution of the resulting system of equations with respect to the unknown constants of integration C_5, C_6, we obtain a transcendental equation for determining the eigenvalues λ_n (Eq. 66):

$$J_0(\lambda r_1)\left[a_7\left(\lambda I_0(\lambda r_1)-\frac{I_1(\lambda r_1)}{r_1}\right)+\frac{a_8}{r_1}I_1(\lambda r_1)\right]+ \\ +I_0(\lambda r_1)\left[a_7\left(\lambda J_0(\lambda r_1)-\frac{J_1(\lambda r_1)}{r_1}\right)+\frac{a_8}{r_1}J_1(\lambda r_1)\right]=0, \quad (66)$$

The eigenvalues for the considered hingedly supported plate D16T-PTFE-D16T, calculated with an accuracy of 0.001, are summarised in the Table 1.

Table 1: Eigenvalues for the D16T-PTFE-D16T pack

Item no	Eigenvalue λ_n	Item no	Eigenvalue λ_n
0	3.367	10	31.891
1	6.456	11	34.820
2	9.258	12	37.798
3	12.134	13	40.831
4	14.946	14	43.915
5	17.754	15	46.043
6	20.559	16	50.207
7	23.351	17	53.392
8	26.168	18	56.589
9	29.011	19	59.782

The frequencies of natural vibrations of the plate can then be determined from the (Eq. 67):

$$\omega_n^2=\frac{\beta_n^4}{M^4}=\frac{\beta_n^4}{M_0 D},\ \beta_n^4=\lambda_n^4+\kappa^4, \quad (67)$$

As a result, to describe the dynamic part of the deflection of the investigated circular sandwich plate, the fundamental orthonormal system of eigenfunctions obtained for isothermal vibrations [24, 25] is used (Eq. 68):

$$v_n(\lambda_n,r)\equiv\frac{1}{d_n}\left[J_0(\lambda_n r)-\frac{J_0(\lambda_n)}{I_0(\lambda_n)}I_0(\lambda_n r)\right], \quad (68)$$

where d_n – normalisation coefficients determined from the requirement that the system of functions be orthonormal (Eq. 69):

$$d_n^2=\int_0^{r_1}\left[J_0(\lambda_n r)-\frac{J_0(\lambda_n r_1)}{I_0(\lambda_n r_1)}I_0(\lambda_n r)\right]^2 r dr= \\ =\frac{r_1^2}{2}[J_0^2(\lambda_n r_1)+J_1^2(\lambda_n r_1)]-\frac{r_1 J_0(\lambda_n r_1)}{\lambda_n I_0(\lambda_n r_1)}\times \\ \times[J_1(\lambda_n r_1)I_0(\lambda_n r_1)+J_0(\lambda_n r_1)I_1(\lambda_n r_1)]+ \\ +\frac{r_1^2 J_0^2(\lambda_n r_1)}{2I_0^2(\lambda_n r_1)}[I_0^2(\lambda_n r_1)-I_1^2(\lambda_n r_1)], \quad (69)$$

The desired deflection w_d, which satisfies the inhomogeneous equation (50), can be represented as a series expansion in the system of functions (68) (Eq. 70):

$$w_d=\sum_{n=0}^{\infty}v_n T_n(t), \quad (70)$$

Substituting it into the vibration equation (50), initial and boundary conditions (55-56), (61), multiplying the terms of the equation by the value $rv_n dr$ and integrating over the radius of the plate from zero to one, due to the orthonormality of the system of eigenfunctions v_n, we obtain to determine unknown function $T_n(t)$ of equation (Eq. 71):

$$\ddot{T}_n+\omega_n^2 T_n=-\frac{r_1^2(\kappa^4 M_t\ +M^4\ddot{M}_t)I(\lambda_n)}{2(a_7+a_8)M^4}, \quad (71)$$

Under initial conditions (t = 0) (Eq. 72):

$$T_n=0,\ \dot{T}_n=-\frac{r_1^2 I(\lambda_n)}{2(a_7+a_8)}\dot{M}_t(0), \quad (72)$$

Here through $I(\beta_n)$ the following integral is denoted(Eq. 73):

$$I(\lambda_n)=\int_0^{r_1}\left[1-\left(\frac{r}{r_1}\right)^2\right]rv_n\,dr= \\ =\frac{2}{d_n\lambda_n^2}\left(J_2(\lambda_n r_1)-\frac{J_0(\lambda_n r_1)}{I_0(\lambda_n r_1)}I_2(\lambda_n r_1)\right), \quad (73)$$

The solution of the problem (71) will be following(Eq. 74):

$$T_n(t)=A_n\cos(\omega_n t)+B_n\sin(\omega_n t)- \\ -\frac{r_1^2 I(\lambda_n)}{2(a_7+a_8)M^4\omega_n}\int_0^t\left(\kappa^4 M_t(\tau)+M^4\ddot{M}_t(\tau)\right)\times \\ \times\sin[\omega_n(t-\tau)]\,d\tau \quad (74)$$

where (Eq. 75):

$$A_n=0, B_n=-\frac{r_1^2 I(\lambda_n)}{2(a_7+a_8)\omega_n}\dot{M}_t(0), \quad (75)$$

The quasi-static deflection (55-56) is expanded in a series in terms of the system of eigenfunctions (68) (Eq. 76):

$$w_s=\frac{M_t r_1^2}{2(a_7+a_8)}\sum_{n=0}^{\infty}I(\lambda_n)v_n, \quad (76)$$

where $I(\lambda_n)$ – is given by (73).

The full dynamic deflection of a circular sandwich plate is obtained by summing (70) and (76), after which the radial displacement and shear follow from the relations (14-16) (Eqs. 77-79):

$$w=\sum_{n=0}^{\infty}v_n\left(T_n+\frac{r_1^2 M_t I(\lambda_n)}{2(a_7+a_8)}\right), \quad (77)$$

$$u=b_1\sum_{n=0}^{\infty}v_{n,r}\left(T_n+\frac{r_1^2 M_t I(\lambda_n)}{2(a_7+a_8)}\right)+C_1 r, \quad (78)$$

$$\psi=b_2\sum_{n=0}^{\infty}v_{n,r}\left(T_n+\frac{r_1^2 M_t I(\lambda_n)}{2(a_7+a_8)}\right)+C_3 r, \quad (79)$$

Here the function T_n is defined by formula (73), the moment M_t by (36), the integration constants (Eq. 80):

$$\{C_1,C_3\}=\{b_1,b_2\}\sum_{n=0}^{\infty}\frac{\lambda_n}{d_n}\left[J_1(r_1\lambda_n)+\right. \\ \left.+\frac{J_0(r_1\lambda_n)}{I_0(r_1\lambda_n)}I_1(r_1\lambda_n)\right]\left[T_n+\frac{r_1^2 M_t I(\lambda_n)}{2(a_7+a_8)}\right] \quad (80)$$

Thus, the functions defining the transverse vibrations of a sandwich circular plate due to thermal shock are determined by the formulas (77-79). Numerical results are given for a plate with layers made of D16T-PTFE-4-D16T materials at a heat flux intensity q_t = 0.2·10^6 and layer thicknesses h_3 = 10; h_2 = 10; h_1 = 0.2 m. Thermophysical and elastic characteristics of materials are given in [23]. In this case, the temperature in the upper bearing layer reached values of 540 *K* (Fig. 2). The curve number corresponds to different points in time: 1 – t_1 = 1.5 *sec.*; t_2 =2 *sec.*

When calculating the temperature field, we neglect the heat spent on heating the outer metal layer (due to its thinness and low heat capacity). Its temperature is taken to be equal to the temperature of the filler at their gluing point: $T^{(1)} = T^{(3)}(c, t)$ [26-28]. The temperature of the second bearing layer is also assumed to be equal to the temperature of the core at the place of their gluing $T^{(2)} = T^{(3)}(-c, t)$. Assuming the plate is thermally insulated along the contour, the temperature field in the filler can be calculated by the formula (7), assuming the specific thermal conductivity $\lambda = \lambda_3$ and the heat capacity $c_0 = c_3$.

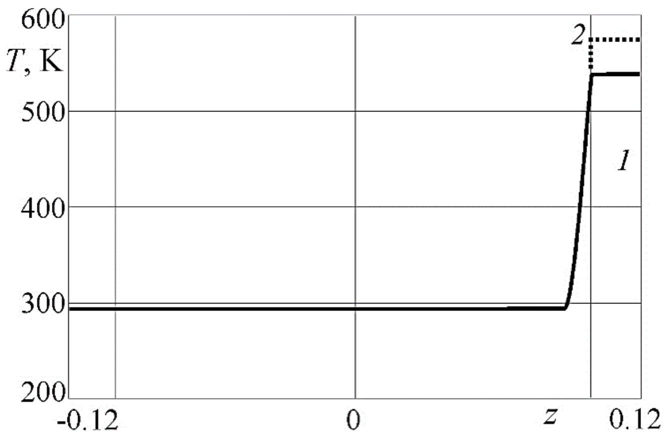

Figure 2: Temperature distribution in the cross section of the plate

Let us consider the coefficient k_n, which characterises the ratio of the dynamic deflection component w_{dn} (70) to the quasi-static component w_{sn} (76) (Eq. 81):

$$k_n = \frac{w_{dn}}{w_{sn}}, \tag{81}$$

Using expression (73) for the functions T_n, we obtain (Eq. 82-83):

$$k_n = \frac{\nu_n T_n}{M_t I(\beta_n)\nu_n / [2(a_7 + a_8)]} = \frac{f_n(\omega_n, t)}{M_t}, \tag{82}$$

$$f_n(\omega_n,\ t) = -M_t(0)\cos(\omega_n t) - \frac{\dot{M}_t(0)}{\omega_n}\sin(\omega_n t) - \\ - \frac{1}{\omega_n}\int_0^t \ddot{M}_t(\tau)\sin[\omega_n(t-\tau)]d\tau, \tag{83}$$

where the moment M_t and its derivatives are determined by relations (36), (50), (55-56).

Figure 3 shows the change in parameter k_n (the ratio of the dynamic component to the quasi-static component of the deflection at the centre of the plate) as a function of time for the first two frequencies ω_0 (a) and ω_1 (b), respectively. Here 1 – plate not connected to an elastic base, 2 – plate on an average stiffness foundation κ_0=10^8 Pa/m. The main bursts of oscillations are observed at the initial moments. Further, the dynamic component decays relative to the static one, which continues to grow over time.

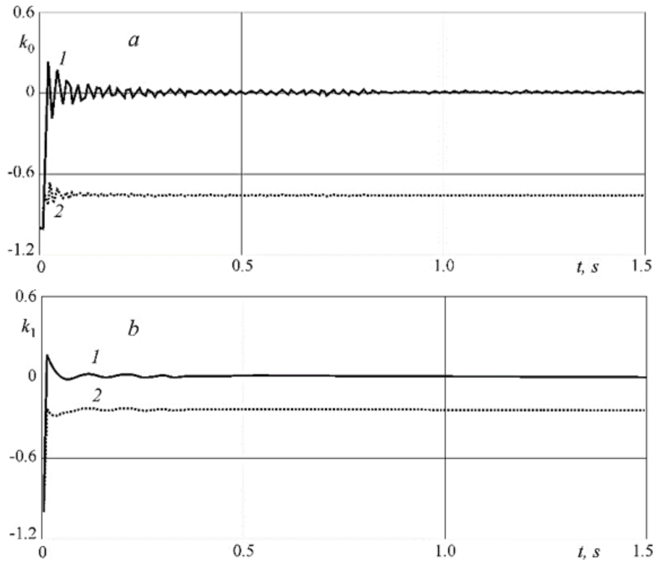

Figure 3: Change of parameter k_n depending on time

CONCLUSIONS

When studying the dynamic behaviour of three-layer structural elements used in aircraft, it is necessary to accept kinematic hypotheses for each layer separately, which makes it possible to significantly refine the frequencies and amplitudes of oscillations. In a relatively thick core, lateral shear deformation must be considered. In this case, the inertia of rotation of the normal in the layers can be ignored. The interaction of a structure with an elastic foundation is described with an accuracy sufficient for engineering practice by the Winkler model. The high-intensity effect of the temperature field leads to the appearance of axisymmetric natural vibrations of plates with a hinged support contour. To study the temperature field in a sandwich plate, we can use the solution of the heat conduction equation with its thermophysical characteristics averaged over the thickness.

The solution to the initial-boundary value problem of transverse vibrations of an elastic sandwich plate arising as a result of thermal shock is constructed as a series expansion in terms of a system of orthonormal eigenfunctions. A transcendental equation for the eigenvalues is given and the first 20 of them are obtained, which makes it possible to fully describe the dynamic deflection, relative shear and radial displacement in the core of the considered plate. Numerical results have shown that the quasi-static component of the deflection increases with time along with the duration of the thermal effect. The amplitude of the dynamic component remains unchanged. Thus, the formulation of the initial-boundary value problem proposed in the paper, the proposed method for its solution, the analytical and numerical results obtained make it possible to calculate the dynamic displacements in three-layer circular plates connected with an elastic foundation under thermal shock.

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