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# MATHEMATICAL MODELLING OF THE EFFECT OF HEAT FLUXES FROM EXTERNAL SOURCES ON THE SURFACE OF SPACECRAFT

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*Modelling the extraneous heat exchange of spacecraft using solar radiation simulation facility and simulators of the planetary radiation field in several cases is an intractable problem not only in technical but also in methodological terms. For some technical reasons, solar radiation simulator is stationary. Consequently, to reproduce a possible change in the orientation of the test object relative to the solar radiation flux, it is necessary to equip the thermal vacuum unit with devices that allow the test object to be rotated at least about two axes. In this paper, a mathematical model and a method for solving the problem of heat transfer in a multilayer structure of screen-vacuum thermal insulation under the influence of solar radiation is proposed. A method is proposed for the numerical solution of a normal system of nonlinear differential equations using the linearisation of nonlinear terms. Various results of numerical modelling were obtained, which indicate the adequacy of the proposed mathematical model. It has been revealed that high-inertia thermal insulation of sufficient thickness is required to stabilise the thermal state inside the spacecraft.*

*Key words: non-stationary radiant fields, simulation technology, heat transfer, parallel plates, differential equation*

## INTRODUCTION

Modelling the extraneous heat exchange of spacecraft (SC) using solar radiation simulation facility and simulators of the planetary radiation field in a number of cases is an intractable problem not only in technical but also in methodological terms. The difficulties are caused primarily by the fact that it is often necessary to recreate in an experimental setup the radiant fields, non-stationary in time and space, formed simultaneously by the Sun and a planet, for example, the Earth [1-3]. For some technical reasons, the solar simulator is stationary [4]. Consequently, in order to reproduce a possible change in the orientation of the test object relative to the solar radiation flux, it is necessary to equip the thermal vacuum unit with devices that allow the test object to be rotated at least about two axes. In addition, with a stationary imitator of the Sun, the simulated planetary radiation field must also change its orientation, which can be achieved either by rotating the entire planetary radiation simulator or by using a special and very complex simulator (existing only in scientific developments) with modules equipped with drives [5].

Thus, the experimental study of the thermal state of the spacecraft under conditions as close as possible to the full-scale conditions is fraught with great difficulties, despite the fact that the imitation technique makes it possible to reproduce in the experimental setup both the solar radiation field and the planetary radiation field (by the

accepted radiation models) separately. But the technical organisation of the joint work of the simulation systems and the test object, which is necessary during testing, is often an insoluble problem [6, 7]. Therefore, approximate methods and means of mathematical modelling of external thermal loads (including the thermal effect of planets) are becoming important. In this paper, a mathematical model and a method for solving the problem of heat transfer in a multilayer structure of screen vacuum thermal insulation under the influence of solar radiation is proposed. Thus, the problem is posed of non-stationary temperature distribution over the thickness of the screen vacuum spacecraft thermal insulation and internal thermal insulation, which is reduced to solving a normal system of nonlinear differential equations with the thermal conductivity equation in thermal insulation with a constraint imposed on the temperature of the inner surface of the screen vacuum thermal insulation.

Similar problems of heat transfer in structural elements of aircraft with complex heat transfer were considered by V.F. Formalev, S.A. Kolesnik, and others [8-10]. In the paper [3], a complex mathematical model of screen-vacuum insulation was proposed and a study was carried out to select various parameters. In this paper, a method is used that eliminates the nonlinearity caused by taking into account radiant heat fluxes by exact solutions of the system of finite-difference equations [11]. Differential equations of the fourth degree relative to the temperature of the screen vacuum insulation plate on the calcu-

lated time layer are linearised using exact solutions of a nonlinear system of algebraic equations, after which the sweep method is applied. This approach makes it possible to completely eliminate the instability of the numerical solution, which occurs when linearising radiant heat fluxes at a known time level and the algorithm is iterative [12-14].

The purpose of the article is to solve the problem of heat transfer in a multilayer structure of screen-vacuum thermal insulation under the influence of solar radiation is proposed. The main objectives of this paper are: to consider the problem of heat transfer in a pack of thermally thin plates; to make a mathematical model for study the thermal state of a spacecraft; calculate heat transfer in screen-vacuum thermal insulation.

### MATERIALS AND METHODS

Let us consider the problem of heat transfer in a pack of thermally thin plates, which are separated by airless layers, under the influence of solar radiation in outer space with a heat-inertial layer (Fig. 1). Since the heat exchange between the plates occurs under the action of radiation with re-radiation and non-stationary accumulation of thermal energy in the plates under the influence of the volumetric heat capacity, this method of thermal protection, characteristic of spacecraft, is called screen vacuum thermal insulation [15, 16].

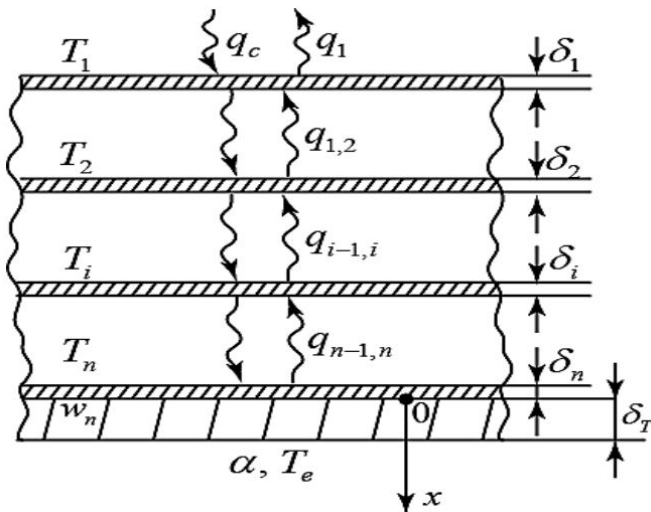


Figure 1: Screen vacuum thermal insulation of several layers

Heat transfer by radiation between two parallel plates with degrees of emissivity  $\epsilon_1, \epsilon_2$  and absolute temperatures  $T_1, T_2$  is characterised by a reduced degree of emissivity  $\bar{\epsilon}_{1,2}$  (Eq. 1) [17]:

$$\bar{\epsilon}_{1,2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \quad (1)$$

In accordance with this, for a plate with a number  $i, i=2, n-1$ , surrounded by two plates with numbers  $i-1$  and  $i+1$

the balance of heat will be (Eq. 2) [18, 19]:

$$(c\rho\delta)_i \cdot \frac{\partial T_i}{\partial t} = \sigma \bar{\epsilon}_{i-1,i} (T_{i-1}^4 - T_i^4) - \sigma \bar{\epsilon}_{i,i+1} (T_i^4 - T_{i+1}^4) \quad (2)$$

where  $c, \rho, \delta$  – heat capacity,  $\frac{J}{kg \cdot K}$  density,  $\frac{kg}{m^3}$ , and thickness,  $m$  of  $i$ -th plate,  $t$  – time,  $C, T$  – temperature,

$K, \sigma = 5.7 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4}$  – Stefan-Boltzmann constant.

For all plates, the following normal system of nonlinear ordinary differential equations can be written (Eqs. 3-7) [20-22]:

$$(c\rho\delta)_1 \frac{dT_1}{dt} = q_c - \epsilon_1 \sigma T_1^4 - \bar{\epsilon}_{1,2} \sigma (T_1^4 - T_2^4) \quad (3)$$

$$(c\rho\delta)_1 \frac{dT_2}{dt} = \bar{\epsilon}_{1,2} \sigma (T_1^4 - T_2^4) - \bar{\epsilon}_{2,3} \sigma (T_2^4 - T_3^4) \quad (4)$$

$$(c\rho\delta)_i \frac{dT_i}{dt} = \bar{\epsilon}_{i-1,i} \sigma (T_{i-1}^4 - T_i^4) - \bar{\epsilon}_{i,i+1} \sigma (T_i^4 - T_{i+1}^4) \quad (5)$$

$$(c\rho\delta)_{n-1} \frac{dT_{n-1}}{dt} = \bar{\epsilon}_{n-2,n-1} \sigma (T_{n-2}^4 - T_{n-1}^4) - \bar{\epsilon}_{n-1,n} \sigma (T_{n-1}^4 - T_n^4) \quad (6)$$

$$(c\rho\delta)_n \frac{dT_n}{dt} = \bar{\epsilon}_{n-1,n} \sigma (T_{n-1}^4 - T_n^4) - \epsilon_n \sigma T_n^4 - \alpha (T_n - T_e) \quad (7)$$

where  $q_c$  – the density of the solar heat flux, equal  $1420 \text{ W/m}^2$ ,  $\epsilon_1$  – the emissivity of the outer surface facing the Sun,  $\epsilon_n$  – the emissivity of the inner surface facing the inner area of the spacecraft,  $\alpha, T_e$  – heat transfer coefficient and effective air temperature on the inner surface.

For a heat insulator with thickness  $\delta_T$  with thermophysical characteristics  $\lambda_T, c_T, \rho_T$  instead of the last equation in system (3-7), the relations should be given (Eqs. 8-10) [23-25]:

$$(c\rho\delta)_n \frac{\partial T_n}{\partial t} = \bar{\epsilon}_{n-1,n} \sigma (T_{n-1}^4 - T_n^4) - \lambda_T \frac{\partial T}{\partial x} \Big|_{x=0} \quad (8)$$

$$T_n = T_{wn}, \quad x=0, \quad t>0,$$

$$c_T \rho_T \frac{\partial T}{\partial t} = \lambda_T \frac{\partial^2 T}{\partial x^2} \quad x \in (0, \delta_T), \quad t>0, \quad (9)$$

$$\alpha (T_e - T|_{x=\delta_T}) + \lambda_T \frac{\partial T}{\partial x} \Big|_{x=\delta_T} \quad x=\delta_T, \quad t>0, \quad (10)$$

where  $\alpha$  – heat transfer coefficient at the boundary  $x=\delta_T$ ,  $W/(m^2K)$ ;  $T_e$  – air temperature inside the spacecraft;  $c_T, \lambda_T, \rho_T$  – heat capacity, thermal conductivity and density of the heat insulator material;  $x$  – coordinate measured from the boundary of the last plate;  $\delta_T$  – heat insulator thickness,  $m$ . Boundary conditions (8) – conditions for the continuity of heat fluxes and temperatures at the interface “Plate – thermal insulation”.

The numerical solution of system (3-7) can be carried out using the method of linearisation of radiant heat flux-

es by replacing the fourth power of temperature with the product of the first power of temperature at the upper time layer and the third power of temperature at the lower time level (explicitly) [26, 27], that is (Eq. 11):

$$T_i^4 \approx T_i^{k+1} (T_i^k)^3 \quad (11)$$

where the temperature values  $T_i^k$  are known.

### RESULTS AND DISCUSSION

According to the proposed mathematical model, the results of calculations of heat transfer in screen-vacuum thermal insulation with internal insulation with a thickness of  $\delta_r=0.02$  m were obtained. In different versions, the thickness of the plates, the number of plates in a pack separated by airless space, the exposure time of the structure, the emissivity of the plates, and thermal insulation were varied. At the inner boundary, the heat transfer coefficient  $\alpha$  was taken as equal to  $5 \text{ W}/(\text{m}^2\text{K})$  and the effective air temperature  $T_e$  equal to  $295 \text{ K}$ . The plate thicknesses  $\delta$  were taken as  $0.0002 \text{ m}$  and  $0.0005 \text{ m}$ . The emissivity of all surfaces was assumed constant from the set  $\epsilon=0.1; 0.3; 0.5$ . The initial temperature was taken in the vicinity of absolute zero  $T^0=(1\div 5)\text{K}$ ,  $q_c=1420\text{W}/\text{m}^2$ . The radiation treatment was taken equal to  $90 \text{ min}$ . The heat capacity  $c$  and density  $\rho$  of the plates were  $900 \text{ J}/(\text{kgK})$  and  $2700 \text{ kg}/\text{m}^3$  respectively [28-29]. Heat insulator characteristic  $c_r\rho_r=36750 \text{ J}/(\text{m}^3\text{K})$ .

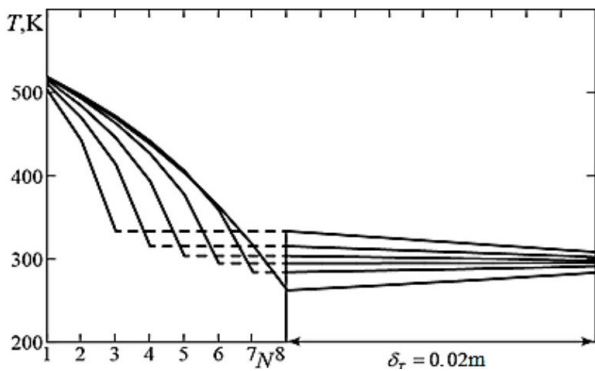


Figure 2: Temperature distribution over the thickness of the screen vacuum thermal insulation and heat insulating material, depending on the number of plates  $N$ ,  $\delta=0.0002\text{m}$

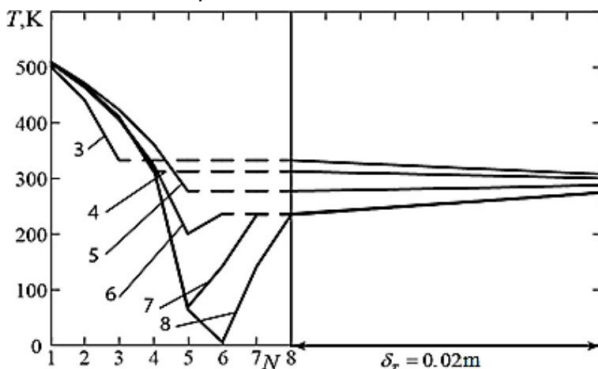


Figure 3: Temperature distribution over the thickness of the screen vacuum thermal insulation and heat insulating material, depending on the number of plates  $N$ ,  $\delta=0.0005\text{m}$

Figures 2 and 3 show the dependences of the temperature distribution in the screen vacuum thermal insulation with a heat-insulating material with a different number of plates (2-8) with plate thicknesses of  $0.0002 \text{ m}$  (Fig. 2) and  $0.0005 \text{ m}$  (Fig. 3). It has been revealed that the temperature at the inner boundary of the heat insulating material weakly depends on the number of plates and is located in the vicinity of  $300 \text{ K}$ , which is acceptable for life support.

Among the many factors that require attention, the problem of protecting the main part of the spacecraft from overheating in the dense layers of the atmosphere occupies a large place [30, 31]. Adequacy of thermal protection is a determining condition both to avoid the destruction of the load-bearing structure and to ensure the optimal thermal regime in the area of the payload, in which case their normal functioning is maintained [32].

This problem is solved by making the main fairing from heat-resistant materials and applying a heat-protective coating on its outer surface. To calculate the parameters of heat-protective coatings, information on aerodynamic heat load on the main part is required, the value of which is traditionally determined by mathematical modeling based on temperature transducers installed under heat protection of the load-bearing structure, taking into account turbulent-laminar flow transition in the boundary layer [33]. However, the ambiguity of the data on the moment of change of the aerodynamic regime in flight causes differences in the calculated values of the heat transfer coefficients [34]. The use of means of measuring the actual values of heat flux coming to the outer surface of the spacecraft and heat flux allows not only to monitor the adequacy of thermal protection in real-time but also to optimize the choice of heat protection.

### CONCLUSIONS

A mathematical model has been compiled to study the thermal state of a spacecraft with thermal protection with screen vacuum thermal insulation and a heat-insulating material under the influence of solar radiation. A method is proposed, in which the nonlinearity caused by taking into account radiant heat fluxes is completely eliminated by obtaining exact solutions to the system of finite-difference equations and further using an effective sweep method. This made it possible to completely eliminate the instability of the nonlinear system of differential equations that arises when using the finite-difference method. Various results of numerical modelling were obtained, which showed the adequacy of the proposed mathematical model. It is shown that high-inertia thermal insulating material of sufficient thickness is required to stabilise the thermal state inside the spacecraft.

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