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doi:10.5937/jaes0-30656

Cite article:

Nguyen T.H., Le H.X. (2021) STRUCTURAL FUZZY RELIABILITY ANALYSIS USING THE CLASSICAL RELIABILITY THEORY, *Journal of Applied Engineering Science*, 19(4), 1074 - 1082, DOI:10.5937/ jaes0-30656

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doi:10.5937/jaes0-30656
Paper number: 19(2021)4, 888, 1074-1082

STRUCTURAL FUZZY RELIABILITY ANALYSIS USING THE CLASSICAL RELIABILITY THEORY

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This study is focused on a novel approach for calculating structural fuzzy reliability by using the classical reliability theory. In order to handle the structural fuzzy reliability problem, the formulae for establishing normal random variables equivalent to symmetric triangular fuzzy number are presented. From these equivalent random ones, the original problem is converted to the basic structural reliability problems, then the methods of the classical reliability theory should be applied to calculate. Moreover, this study proposes two notions in terms of central fuzzy reliability and standard deviation of fuzzy reliability as well as a calculation procedure to define them. Lastly, the ultimate fuzzy reliability of the proposed method is established and utilized to compare the allowable reliability in the design codes. Numerical results are supervised to verify the accuracy of the proposed method.

Key words: fuzzy reliability, probability theory, structural reliability, response surface method, energy methods, plastic bending of beams

INTRODUCTION

In engineering structures, most of the input data, such as load characteristics, material properties, boundary conditions, geometric dimensions, load-carrying capacities, contain non-deterministic quantities, which are described as uncertainty variables. In [1], uncertainties present in a structural system can be categorized as either aleatory or epistemic. If the input data are random parameters, the limit state function is a random parameter, is expressed as follows

$$M=R-S \quad (1)$$

where M is the limit state function;
R is the resistance function;
S is the load effect function.

The structural reliability is defined as follows [2, 3]

$$P_S = Prob\,(M>0) \quad (2)$$

Due to using the probability theory, which is the most complete theory, assessing structures by the reliability index is prescribed in the structural design code [4]. When the input data in structural systems are epistemic uncertainties, depending on how to describe the uncertainties, the structural reliability can be derived by the different approaches, as using either the fuzzy sets theory [5÷12] or the fuzzy random theory [13,14]. In this study, we only consider that the epistemic ones are represented as fuzzy numbers, which are always interested in the reality problems. In this case, the structural reliability had the different names, such as the safety possibility of structures [6], the fuzzy reliability index [7, 8], the fuzzy reliability [9, 10, 11]. The terminologies are named the fuzzy reliability later.This fuzzy reliability can be classified into three classes: the resistance function R and the load effect S are fuzzy numbers, the resistance R is a random parameter and the load effect S is a fuzzy number, the resistance R is a fuzzy number and the load effect S is a random parameter. These approaches [5÷12] for the fuzzy reliability are detailed analyzed below.

According to Dong et al. [5], the fuzzy failure is determined based on the two α -cuts M_α and R_α (Fig.1a) as follows

$$FP=\frac{1}{2}\left[T(\tilde{S}>\tilde{R})+T(\tilde{S}>\tilde{R})\right] \quad (3)$$

In Fig. 1a, the fuzzy failure (FF) and the fuzzy reliability (FR) are expressed as follows, respectively

$$FF=\frac{h}{2},\ FR=1-\frac{h}{2} \quad (4)$$

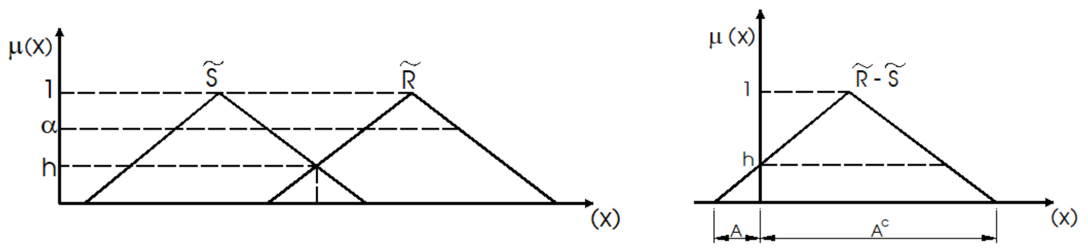

Figure 1: The method [5]
a. The interference model of the fuzzy resistance number $\tilde{R}$ and the fuzzy load effect number $\tilde{S}$
b. The subtraction of the fuzzy resistance number $\tilde{R}$ and the fuzzy load effect number $\tilde{S}$

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From formula (4), one realizes that the fuzzy failure is the average of the possibility measure and the necessity measure of event A (Fig. 1b) according to the possibility theory [15]. Hence, the method [6] is an approximation of the average, and the transformation from logical expression (3) to formula (4) is an intuitive formula. Besides, the effect of the spread of fuzzy numbers R˜ and S˜ aren't considered absolutely.

Differed from Dong et al. [5], Sherstha and Duckstein [6] considered directly $\tilde{M}$, which is the subtraction of the fuzzy resistance number and the fuzzy load effect number, and the fuzzy reliability is defined as the ratio of the membership function of $\tilde{M}$ which is greater than 0 to the total area of the membership funtion of $\tilde{M}$ (Fig. 2)

$$FR=\frac{\int_{M>0} \mu_M(x)dx}{\int \mu_M(x)dx} \tag{5}$$

where μM(x) is the membership function of fuzzy number M˜

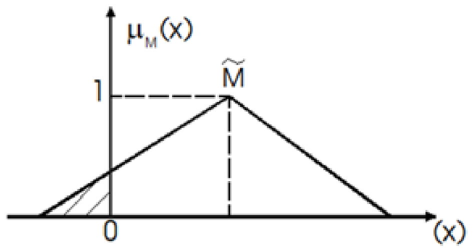

Figure 2: The method [6, 7]

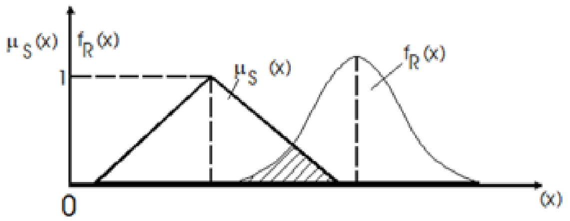

Figure 3: The interference model [10, 11]

The formula (5) is approriate for the geometrical definition of probabiliy in [16]. Nevertheless, it should be realized that the membership functions in the fuzzy sets theory and the density distribution functions in the probability theory are not equivalent representations of uncertainty. Hence, the fuzzy reliability in the formula (5) is only similiar to the probabilistic reliability in the classical reliability theory. Based on the formula (5), Park et al. [7] determined the fuzzy reliability when the membership funtion $\mu_M(x)$ is triangular and trapezoidal. Rezaei et al. [8] extended formula (5) in case fuzzy variables formed the pyramid to asess fuzzy reliability of the wing flutter speed.

In [9], Li et al. proposed a formula for fuzzy reliability analysis of mechanical structures when the stress S was modeled as a fuzzy number with given membership function $\mu_S(x)$ and the strength R was modeled as a random variable with given distribution function $f_R(x)$ (Fig. 3). The authors idealized that fuzzy reliability in the fuzzy stress - random strength interference model was a real value, and the Zadeh's notion for the probability of a fuzzy event [17] is used to calculate. The fuzzy reliability is given as follows

$$FR=1-FF=1-\int \mu_S(x) f_R(x)dx \tag{6}$$

The drawback of this method is that two functions are under integral in the formula (6) have different measurements, the area between the curve $f_R(x)$ and the abscissa is unit but for the fuzzy number μS(x) isn't. In order to overcome this drawback, Jiang and Chen [10] supposed that the α-cuts of a fuzzy number were the linear distributions, then computed the conventional probability at various α-cuts and the fuzzy reliability was the average of the reliabilities at the α-cuts. Tang et al. [11] also utilized this assumption and computed the fuzzy failure probability by applying the Gauss-Legendre quadrature formula. However, it should be seen that different distribution assumptions of the α-cuts lead to totally different fuzzy reliability results.

From the above analysis, one realizes that it had diversity approaches for evaluating the structural fuzzy reliability. Nevertheless, the meaning of the existing formulas is less evident than the reliability in the classical reliability theory. In order to overcome this drawback, using the traditional reliability analysis theory as a basis for assessing fuzzy reliability is a reasonable approach, due to the fact that the traditional probabilistic methods remain dominant in the field of measurements [18] and are well established in the decision making problems [19]. Hence, the aim of this study is to establish a novel approach for calculating structural fuzzy reliability by using the classical reliability theory. Firstly, the transformation from fuzzy number to equivalent normal random variables is presented. As a consequence, a fuzzy reliability problem is replaced by the traditional reliability problems, and the classical methods could be utilized to solve. Additionally, two notions including the central fuzzy reliability and the standard deviation of fuzzy reliability are proposed. While the central fuzzy reliability is the mean value of fuzzy reliability, the remain notion represents a measure of the spread of fuzzy reliability around its own central. Lastly, the ultimate fuzzy reliability is constituted based on three sigma rule, and is easily compared with the allowable reliability in the structural design codes. Numerical results are thoroughly explored to demonstrate the accuracy of the proposed method.

METHODS

The formula for computing the deviation of the equivalent normal random variable

The transformation from fuzzy numbers into random quantities and conversely should be taken into acount in any problem where heterogeneous uncertain and imprecise data appear together (e.g. information deficit, lingustic variables, statistical data). The representative transformation principles, such as insufficient reason, maximum specify, uncertainty invariance, are proposed by Dubois et al. [18, 20], Dubois [21] and Klir [15]. Based on combining the principles of insufficient reason and maximum specify, Tuan and Huynh [22] proposed an innovation transformation to calculate the deviation of the equivalent normal random variable. For the transformation from standardized symmetric triangular fuzzy number (Fig. 4) into random quantity, the error of probability measure between equivalent probability density function p(x) of standardized symmetric triangular fuzzy number got by the principle of insufficient reason and normal random variable $p_1(x)$ is expressed by the following formula

$$(P(A)-P_1(A))^2\to min\quad \forall x\in[-1,0) \tag{7}$$

where p(x), $p_1(x)$, P(A), $P_1(A)$ are determined as follows

$$p(x)=\begin{cases}-\frac{1}{2}ln(-x)\ ;x\in[-1,0)\\ -\frac{1}{2}ln(x)\ ;x\in(0,1]\end{cases} \tag{8a}$$

$$p_1(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{\left(-\frac{x^2}{2\sigma^2}\right)} \tag{8b}$$

$$P(A)=\frac{1}{2}\left[x-xln(-x)+1\right] \tag{8c}$$

$$P_1(A)=\int_{-1}^{x}\frac{1}{\sqrt{2\pi}\sigma}e^{\left(-\frac{x^2}{2\sigma^2}\right)}dx \tag{8d}$$

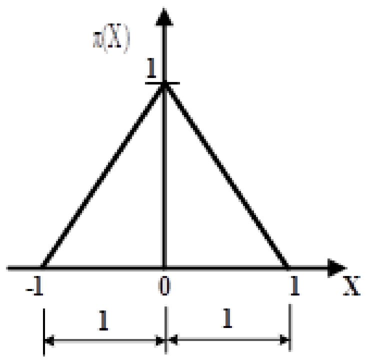

Figure 4: Standardized fuzzy variable

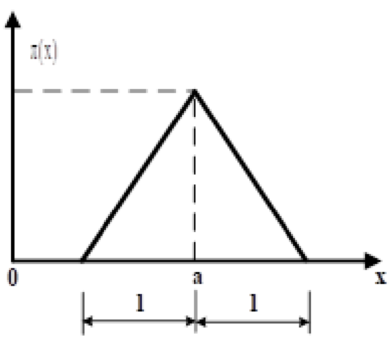

Figure 5: Symmetric triangular fuzzy variable

From (7) we get

$$F_1(\sigma)=\int_{-1}^{0}(P(A)-P_1(A))^2dx\to min \tag{9}$$

Due to the different domain between probability density function p(x) and normal random variable $p_1(x)$, in order that probability of density function $p_1(x)$ in $(-\infty,1)$ be insignificant, one needs

$$F_2(\sigma)=P_1\left[A^*:x_o\in(-\infty,-1)\right]=\int_{-\infty}^{-1}\frac{1}{\sqrt{2\pi}\sigma}e^{\left(-\frac{x^2}{2\sigma^2}\right)}dx\to min \tag{10}$$

Combine (9) and (10), we have

$$F(\sigma)=F_1(\sigma)+F_2(\sigma)=\int_{-1}^{0}(P(A)-P_1(A))^2dx+\int_{-\infty}^{-1}\frac{1}{\sqrt{2\pi}\sigma}e^{\left(-\frac{x^2}{2\sigma^2}\right)}dx\to min \tag{11}$$

For the reverse transformation from normal random variable into equivalent fuzzy number, the error of possibility measure between equivalent fuzzy number of normal random variable got by the principle of maximum specificity and standardized fuzzy number is expressed by the following formula

$$G(\sigma)=\int_{-1}^{0}(\pi_1(x)-1-x)^2dx+\int_{-\infty}^{-1}\pi_1^{\,2}(x)dx\to min \tag{12}$$

where $\pi_1(x)$ is the equivalent fuzzy number of normal random variable determined as follows

$$\pi_1(x)=\pi_1(-x)=\int_{-6\sigma}^{x}p_1(y)dy+\int_{-x}^{6\sigma}p_1(y)dy \tag{13}$$

In order to solve the multiobjective optimization problem (11) and (12), transforms multiple objectives into a scalar objective function by multiplying each objective function by a weighting factor and summing up all contributors

$$H(s)=\gamma F(s)+(1-\gamma)G(s)\to min \tag{14}$$

where $\gamma\in[0,1]$.

For the mathematical meaning, formula (14) is an extension which modifies the equivalent characteristic according to two principles: the principle of insufficient reason when going from fuzzy number to random variable, and the principle of maximum specificity when going from radom variable to fuzzy number. For solving (14), Genetic algorithm (GA) [23] is applied using the built-in functions in Matlab. The relation between weighting factor γ and deviation σ is detailed depicted in Fig. 6.

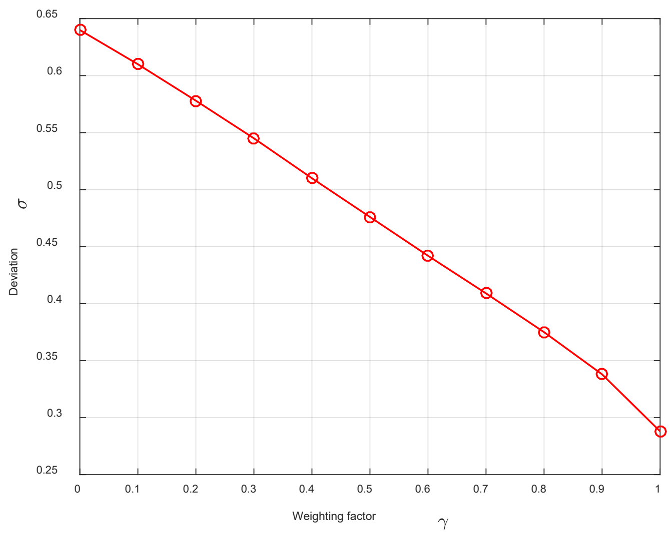

Figure 6: Representation of the relation between deviation and weighting factor

One realizes that the proposed transformation encodes a family of normal random variables including the result of Klir's method, from the standardized fuzzy number. Indeed, the deviation of normal random variable using uncertainty invariance principle is of 0.3989 [24], as the same as the result attains according to the proposed transformation when weighting factor is of 0.728. Hence, the constrains generated in the proposed transformation are more flexible than Klir's method.

For a symmetric triangular fuzzy number $\tilde{X}=(a,l)_{LR}$ (Fig. 6), the relation between it and the standardized fuzzy variable $\tilde{x}=(0,1)_{LR}$ is defined as follows [22]

$$\tilde{x}=\frac{\tilde{X}-a}{l} \tag{15}$$

The central fuzzy reliability, the standard deviation of fuzzy reliability

As a result of accomplishing a family of normal random variables, the original structural fuzzy reliablity problem is converted to the basic reliability problems using the classical reliability theory to solve. By this we mean that the calculated reliablity will vary from value to value. Therefore, the attained fuzzy reliability can be treat as a random variable with its own mean, standard deviation.

Based on the relation between the deviation of equivalent nornal random variable and weighting factor explored in the previous section, three especial values of the weighting factor γ are considered:

When $\gamma = 0.5$ we get $\sigma = 0.476$. (16.a)

When $\gamma = 1$ we get $\sigma = 0.288$. (16.b)

When $\gamma = 0$ we get $\sigma = 0.640$. (16.c)

The value $\sigma = 0.476$ with $\gamma = 0.5$ in formula (16.a) means to choice the equilibrium for the weighting factors of objective function $F(\sigma)$ and objective function $G(\sigma)$. Due to this reason, the value $\sigma = 0.476$ is utilized to calculate the central fuzzy reliablity FR_c being the mean value of fuzzy reliability. Two remain values in formula (16.b) and (16.c) are the minimum and the maximum values of the deviation of equivalen normal random variable, respectively. Therefore, we will utilize them to calculate the standard deviation of fuzzy reliability σ_{FR}.

Following that, without loss of generality, one considers the limit state function is given by

$$g(x)=R\text{-}S=g(x_F,x_r)=g(\tilde{x}_1,\tilde{x}_2,\ldots\ldots,\tilde{x}_n,x_r) \tag{17}$$

where $x_F=(\tilde{x}_1,\tilde{x}_2,\ldots\ldots,\tilde{x}_n)$ are independent fuzzy variables, as symmetric triangular fuzzy number $\tilde{x}_i=(a_i,l_i)_{LR}$; x_R is random parametter of the resistance function or the load effect.

In order to determine reliability according to the traditional reliability theory, the transformation from fuzzy variable $\tilde{x}_i=(a_i,l_i)_{LR}$ to normal random variable $x_i\sim N(\mu_i, \sigma_i)$ is accomplished based on formulas (15) and (16), it means:

When $\gamma = 0.5 : \mu_i = a_i, \sigma_i = 0.476\, l_i$ (18.a)

When $\gamma = 1.0 : \mu_i = a_i, \sigma_i = 0.288\, l_i$ (18.b)

When $\gamma = 0.0 : \mu_i = a_i, \sigma_i = 0.640\, l_i$ (18.c)

For calculating the central fuzzy reliability FRc, the mean and the deviation of the equivalent normal random variable determined by formula (18.a) are applied. Then, can utilize any techniques in the classical reliability theory, such as First-order reliability method (FORM), Second-order reliability method (SORM), Cornell reliability index, Hasofer-Lind reliability index, Monte Carlo method, so on, to estimate structural reliability.

Due to the fact that a large amount of the means and the deviations of the equivalent normal random variables are determined by solving the formulae (14) with weighting fator γ is fixed, there should be accomplished a large number of problems to determine the standard deviation σ_{FR} of fuzzy reliability. In order to reduce the number of computations, the central composite design in the response surface method [25] is applied and the standard devition of fuzzy reliability is calculated as follows

$$\sigma_{FR}=\sqrt{\frac{\sum_{k=1}^{m}\left(FR_{s_k}-FR_c\right)^2}{(m-1)}} \tag{19}$$

where FR_{sk} is the reliability value at the kth experimental of the central composite design, with its own mean, standard deviation are defined by formulas (18.b) and (18.c);

FR_c is the central fuzzy reliability;

m is the total number of test runs in the central composite design: $m = 2^n+2n$, with n is the number of fuzzy variables.

The ultimate fuzzy reliability

In order to take into account the bias of fuzzy reliability, the ultimate fuzzy reliability FRu is presented and used to compare the allowable reliability in the structural design codes. Based on three sigma rule in the probability theory, the ultimate fuzzy reliability is computed as follows

$$FR_u=FR_c\text{-}3\sigma_{FR} \tag{20}$$

When the input data has only one fuzzy variable (n=1), the ultimate fuzzy reliability is calculated as follows

$$FR_u=min(FR_1,FR_2) \tag{21}$$

where $FR_1 = P_s(x_1, x_R)$, $FR_2 = P_s(x_2, x_R)$ with x_1 and x_2 are equivalent normal random variables, which have mean μi and deviation σi is defined by formulas (18.b) and (18.c), respectively.

NUMERICAL EXAMPLES AND DISCUSSIONS

In order to verify the proposed method, the illustrative examples including mathematical examples and engineering examples are given. The limit state functions in the illustrative examples are explicit functions, in order to get to a resemblance to structural analysis methods.

Example 1

Hypothetical limit state function with three variables is indicated as follows

$$g(x)=2.0-0.32(x_1-1)^2 x_2^2-x_2+x_3^3-0.2sin(x_1x_3) \quad (22)$$

where x_1, x_2, x_3 are assumed to be symmetric triangular fuzzy numbers $(0,1)_{LR}$.

The fuzzy reliability FR of the method [5] is: FR_D=0.89053.

The fuzzy reliability FR of the method [6] is: FR_S=0.918638.

The relative difference ε between FR_D and FR_S is:

$$\varepsilon=100.\frac{(FR_D-FR_S)}{FR_S}=-3.05293\% \quad (23)$$

The Monte Carlo method [2, 3] with a number of trials N_s=106 is used to calculate in the proposed method. Tab. 1 displays the results of the central and the ultimate fuzzy reliability of the proposed method and the relative difference ε of the fuzzy reliability between the proposed method and the method [6].

Table 1: The reliability of the proposed method and the method [6]

The fuzzy reliability	The fuzzy reliability FR of the proposed method	The fuzzy reliability FR_S of the method [6]	The relative difference ε of the fuzzy reliability FR
FR_c	0.989040	0.918638	7.66371
FR_u	0.936526	0.918638	1.94726

Example 2

Consider a two-story frame structural system in Fig. 7, two girders are infinitely rigid. Elastic modulus E, ceiling height H, loads P_1 and P_2 are assumed to be symmetric triangular fuzzy numbers: $\tilde{E}=(2x10^7, 1.2x10^5)_{LR}$ (unit: kN/m²); $\tilde{H}=(3,0,2)_{LR}$ (unit: m); $\tilde{P}_1=(20,2)_{LR}$ (unit: kN). The width b and the height h of the column are certain variables with 0.2 m and 0.3m, respectively.

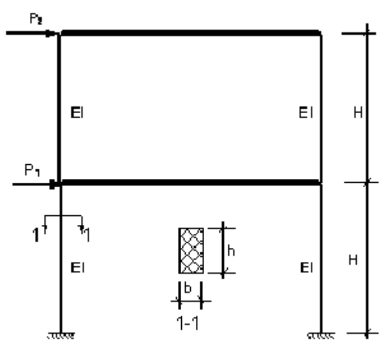

Figure 7: Two-story frame structural system

Applying the shear forces distribution method, horizontal displacement at the top of the structural system is expressed as follows

$$\Delta_t=\frac{(2P_2+P_1)H^3}{24EI} \quad (24)$$

where I = bh³/12 = 4.5x10⁻⁴ m⁴.

Because the prescribed horizontal displacement at the top of frame structural is 2H/500, the limit state function in terms of horizontal displacement at the top is given as follows

$$g(x)=\frac{H}{250}-\frac{(2P_2+P_1)H^3}{24EI} \quad (25)$$

The fuzzy reliability FR of the method [5] is: FR_D=0.892432.

The fuzzy reliability FR of the method [6] is: FR_S 0.970360.

The relative difference ε between FR_S and FR_D is:

$$\varepsilon=100.\frac{(FR_D-FR_S)}{FR_S}=-8.03085\% \quad (26)$$

The second-order reliability method SORM [27, 28] is used to calculate the central and the ultimate fuzzy reliability in the proposed method. The derived results are compared with the method [6] and represented in Tab. 2.

Table 2: The results of the proposed method and that of the method [6]

The fuzzy reliability FR	The proposed method	The fuzzy reliability FRs according to the method [6]	The relative difference ε of the fuzzy reliability FR
FR_c	0.994769	0.970360	2.51548
FR_u	0.955102	0.970360	-1.57237

Example 3

Determine the fuzzy reliability in terms of plastic collapse of the steel beam is shown in Fig. 8. Two cases are considered as follows:

Case 1: The lenght L, the width b and the height h of the steel beam are certain variables with 4m, 4 cm and 8 cm, respectively. The concentrated load $\tilde{P}$ (unit: kN) and the uniform load $\tilde{q}$ (unit: kN/m) are assummed as symmetric triangular fuzzy numbers given as following: $\tilde{P}=(10,2)_{LR}$, $\tilde{q}=(8,1)_{LR}$. The yield stress is considered as a normal distribution with the mean value of 24 kN/cm² and the standard deviation of 2 kN/cm².

Case 2: The lenght $\tilde{L}$ (unit:m), the width $\tilde{b}$ and the height $\tilde{h}$ of the steel beam (unit: cm), the concentrated load $\tilde{P}$ (unit: kN), the uniform load q (unit: kN/m) and the yield stress $\tilde{\sigma}_y$ (unit: kN/cm²) are assummed as symmetric triangular fuzzy numbers given as following: $\tilde{L}=(4,0.5)_{LR}$, $\tilde{b}=(4,1)_{LR}$, $\tilde{h}=(8,1)_{LR}$, $\tilde{P}=(15,3)_{LR}$, $\tilde{q}=(10,2)_{LR}$, $\tilde{\sigma}_y=(24,2)_{LR}$.

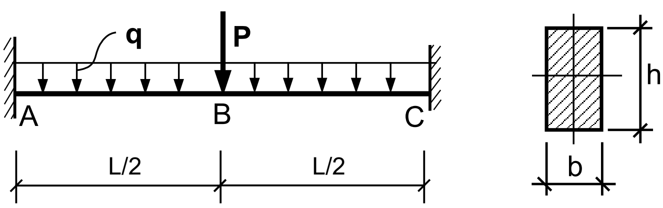

Figure 8: The built-in steel beam fixed at both ends

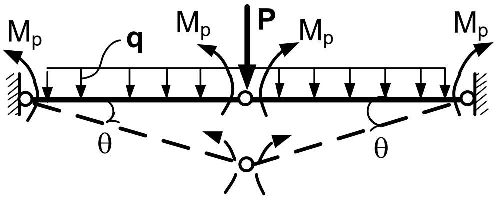

Figure 9: Plastic collapse of the steel beam fixed at both ends

The steel beam is statically indeterminate to the second degree, therefore three hinges are required to change it from a girder structure to a mechanism, as shown in Fig. 9.

Then, the principle of virtual work is applied to determine the limit state function in terms of plastic collapse.

Work done by three hinges during collapse

$$A = Mp.\theta + Mp.2\theta + Mp.\theta = 4Mp.\theta \quad (27)$$

Work done by the concentrated load P and the uniform load q

$$T=qL.\frac{L}{4}.\theta+P.\frac{L}{2}.\theta \quad (28)$$

Equating (27) and (28): $M_p=\frac{qL^2}{16}+\frac{PL}{8}$ (29)

where $M_p=\sigma_y.W_p$, with W_p - the plastic moment of resistance. Hence, the limit state function in terms of plastic collapse is given as follows

$$g(x)=\sigma_y-\frac{qL^2}{16.W_p}-\frac{PL}{8.W_p} \quad (30)$$

with the solid rectanglar section: $W_p=\frac{bh^2}{4}$

Case 1:

- The fuzzy reliability of the method [9] is: FR_L=0.840945.
- The fuzzy reliability FR of the method [10] is: FR_J=0.958025.
- The relative difference ε between FR_L and FR_J is:

$$\varepsilon=100.\frac{(FR_L-FR_J)}{FR_J}=-12.22093\% \quad (31)$$

Case 2:

- The fuzzy reliability of the method [5] is: FR_D=0.693371.
- The fuzzy reliability FR of the method [6] is: FR_S=0.792823
- The relative difference ε between FR_D and FR_S is:

$$\varepsilon=100.\frac{(FR_D-FR_S)}{FR_S}=-12.54398\% \quad (32)$$

Tab. 3 and Tab. 4 display the results of the central and the ultimate fuzzy reliability of the proposed method and the relative difference ε of the fuzzy reliability between the proposed method and the method [10] in Case 1 and the method [6] in Case 2. The Cornell reliability index is applied in terms of Case 1 and the SORM is applied in terms of Case 2 in the proposed method.

Table 3: The results of the proposed method and that of the method [10] in Case 1

The fuzzy reliability FR	The proposed method	The fuzzy reliability FR_J according to the method [10]	The relative difference ε of the fuzzy reliability FR
FR_c	0.948643	0.958025	-0.97927
FR_u	0.9221510	0.958025	-3.74460

Table 4: The results of the proposed method and that of the method [6] in Case 2

The fuzzy reliability FR	The proposed method	The fuzzy reliability FR_S according to the method [6]	The relative difference ε of the fuzzy reliability FR
FR_c	0.934335	0.792823	17.84907
FR_u	0.794347	0.792823	0.19218

Example 4

Determine reliability in terms of stability of the structural system is shown in Fig.10. Two cases are considered as follows:

Case 1: Inertia moment I = 808cm4, length l = 5m. Elastic modulus E is considered as a normal distribution with the mean value of 2.1x104 kN/cm^2 and the standard deviation of 2.5x103 kN/cm^2. Concentrated load P (units:kN) is considered as symmetric triangular fuzzy numbers given as following: $\tilde{P}=(110,10)_{LR}$.

Case 2: Inertia moment I = 808cm^4. Length l (units: m), elastic modulus E (units: kN/cm^2) and concentrated load P (units:kN) are assumed as symmetric triangular fuzzy numbers given as following: $\tilde{l}=(5,0.5)_{LR}$, $\tilde{E}=(2.1x10^4,2.5x10^3)_{LR}$, $\tilde{P}=(110,10)_{LR}$.

In order to calculate in terms of stability, the structural system is transformed to the continuous beam of two equal spans in Fig. 11. The stiffness k of the intermediate elastic support is determined by bellow formula

$$\frac{1}{k}=\frac{1}{EA}l+\frac{l^3}{48.(2EI)}=\frac{49l^3}{96EI}\Rightarrow k=\frac{96EI}{49l^3} \quad (33)$$

According to formula (33), it is evident that the value of $k<16\pi^2EI/(2l)^3$ corresponds to stability loss in the symmetrical mode.

The equation in terms of stability is represented in the following form [26]

$$\eta_1(v)=-\frac{k.(2l)^3}{48EI}=-\frac{16}{49} \quad (34)$$

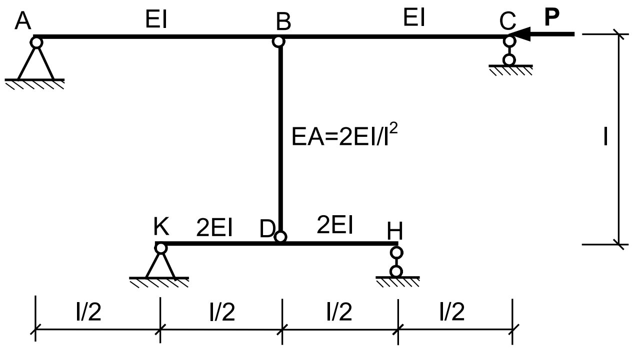

Figure 10: The structural system

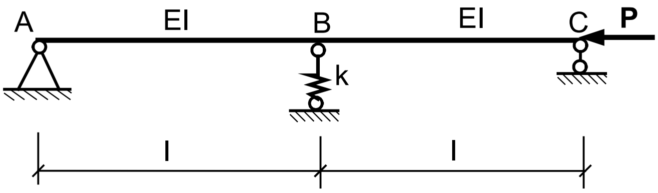

Figure 11: The diagram for calculating stability

Solve the equation (34) obtained v=1.80475.

The critical load is

$$P_{cr}=v^2\frac{EI}{l^2}=1.80475\frac{EI}{l^2} \tag{35}$$

The limit state function in terms of stability is given as follows

$$g(x)=1.80475\frac{EI}{l^2}-P \tag{36}$$

Case 1:

- The fuzzy reliability of the method [9] is: FR_L=0.812516.
- The fuzzy reliability FR of the method [10] is: FR_J=0.800996.
- The relative difference ε between FR_L and FR_J is:

$$\varepsilon=100.\frac{(FR_L\text{-}FR_J)}{FR_J}=1.43826\% \tag{37}$$

Case 2:

- The fuzzy reliability of the method [5] is: FR_D=0.698673.
- The fuzzy reliability FR of the method [6] is: FR_S=0.828759.
- The relative difference ε between FR_D and FR_S is:

$$\varepsilon=100.\frac{(FR_D\text{-}FR_S)}{FR_S}=-15.6964\% \tag{38}$$

Tab. 5 and Tab. 6 display the results of the central and the ultimate fuzzy reliability of the proposed method and the relative difference ε of the fuzzy reliability between the proposed method and the method [10] in Case 1 and the method [6] in Case 2. The Cornell reliability index is applied in terms of Case 1 and the SORM is applied in terms of Case 2 in the proposed method.

Table 5: The results of the proposed method and that of the method [10] in Case 1

The fuzzy reliability FR	The pro-posed method	The fuzzy reliability FR_J accord-ing to the method [10]	The relative difference ε of the fuzzy reliability FR
FR_c	0.792281	0.800996	-1.08795
FR_u	0.783605	0.800996	-2.17113

Table 6: The results of the proposed method and that of the method [6] in Case 2

The fuzzy reliability FR	The pro-posed method	The fuzzy reliability FR_S accord-ing to the method [6]	The relative difference ε of the fuzzy reliability FR
FR_c	0.924816	0.828759	11.58050
FR_u	0.814419	0.828759	-1.73027

Example 5

In order to verify design bearing capacity of the pre-stressed concrete pile with the cross-sectional area (40x40) cm, six testing piles under static axial compressive loads have been proceed. The allowable loads calculated from the test results based on Vietnamese national standard TCVN 9393:2012 [29] are displayed in Tab. 7. The design load P_d is of 1400 kN.

Requires: Determine the safety level in terms of the design bearing capacity of the pile.

Table 7: Summary of test pile properties

Pile/Test No.	Embedded Length L (m)	The failure load (kN)	The allow-able load Pa (kN)
1	23.2	3600	1800
2	22.8	3600	1800
3	21.5	2700	1350
4	21.2	2700	1350
5	22.3	3150	1575
6	22.5	3150	1575
Mean value	22.25	3150	1575

Based on the data obtained from Tab.7, one realizes that all numbers are equally likely to appear. Due to the fact that only a small number of observations are available, so as to determine safety level in terms of the design bearing capacity of the pile based on the classical apporach, the allowable load P_a need to be supposed as a uniform distribution within a range [1350, 1800] kN.

As a results, the reliability in terms of the design bearing capacity of the pile is given as follows

$$P_s = 1 - \frac{(1400\text{-}1350)}{(1800\text{-}1350)} = 0.888889 \qquad (39)$$

In the proposed method, triangular membership fuction can be assigned to the allowable load $\tilde{P}_a$, with the belief value of the membership function equals to the mean value, and the zero α-cut of fuzzy number is created by the extreme values [1350, 1800] kN. Therefore, the allowable load $\tilde{P}_a$ is the symmetric triangular fuzzy number: $\tilde{P}_a = [1575, 225]_{LR}$ (unit: kN).

Then, the limit state function in terms of the design bearing capacity of the pile is given as follows

$$g(x) = \tilde{P}_a - P_d \qquad (40)$$

Tab. 8 displays the results of the central and the ultimate fuzzy reliability of the proposed method and the relative difference ε of the fuzzy reliability between the proposed method and the classical reliability method based on assumption for distribution of uncertain variable.

Table 8: The results of the proposed method and that of the classical reliability method based on assumption for distribution of uncertain variable

The fuzzy reliability FR	The proposed method	The classical reliability method P_S	The relative difference ε of the fuzzy reliability FR
FRc	0.948869	0.888889	6.74779
FRu	0.887870	0.888889	-0.11463

DISCUSSIONS

Analysis the results of the above examples, the following discussions are given:

1. The results of the central and the ultimate fuzzy reliability of the proposed method has an only small relative difference in comparison with that of fuzzy reliability of the method [10]. In the all of the examples, the central fuzzy reliability FRc of the proposed method produces results closed to the results of the method [10]. It is found that the fuzzy reliability of the method [10] is the average of the reliabilities at the α-cuts, which is similar to the notion of the central fuzzy reliability of the proposed method.
2. The results of the ultimate fuzzy reliability of the proposed method has an only small relative difference in comparison with that of fuzzy reliability of the method [6]. The relative differences have been occured due to the different meaning between the fuzzy reliability in the method [6] and the reliability in the classical reliability theory.
3. The result of the ultimate fuzzy reliability of the proposed method approximates that of the reliability based on the classical approach. In addition, the proposed method need'n any assumption for distribution of uncertainty variables when information deficit appears. This pointed out that the proposed method is suitable to handle the problems of reality.

CONCLUSIONS

A novel method for calculating structural fuzzy reliability is proposed in this study. In order to apply the traditional reliability theory, a family of normal random variables equalling symmetric triangular fuzzy number are presented and explored in detail. From these equivalent random ones, two novel notions of the central fuzzy reliability and the ultimate fuzzy reliability are formulated and a procedure for calculating them is defined. In order to handle the bias of fuzzy reliability, the proposed ultimate reliability need to be compared with the allowable reliability in the structural design codes. Numerical results manifest the equivalence between the ultimate reliability in the proposed method and the “exact” reliability in the classical reliability theory.

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Paper submitted: 01.02.2021.
Paper accepted: 10.04.2021.