INCORPORATING AN OUTSOURCING STRATEGY AND IN-HOUSE QUALITY ASSURANCE INTO THE PRODUCTION-SHIPMENT DECISION MAKING

Singa Wang Chiu1, Yi-Ying Li2, Victoria Chiu2, Hong-Dar Lin2,*
1Chaoyang University of Technology, Department of Business Administration, Taichung, Taiwan
2Chaoyang University of Technology, Department of Industrial Engineering & Management, Taichung, Taiwan
3State University of New York at Oswego, Department of Accounting, Finance and Law, Oswego, USA

To stay competitive in turbulent business environments, manufacturing firms' managers today constantly seek ways to reduce order response time, smooth production schedules, ensure the quality of their products, and lower overall making and shipping costs. This study incorporates an outsourcing strategy and in-house quality assurance into a production-shipment problem to address the aforementioned operational goals. The objectives are to simultaneously find the optimal fabrication batch size and frequency of delivery that minimize the system's relevant costs and reveal in-depth information regarding the impact of diverse system parameters on the optimal policy and system cost. This study develops a model and uses the optimization method to resolve the problem. The research results facilitate managerial decisions in such a real-life situation.

Key words: production and operation management, replenishment lot-size, multiple shipments, outsourcing, product quality assurance

INTRODUCTION

This study incorporates outsourcing and in-house quality assurance matters into production-shipment decision-making. Outsourcing for meeting product demand is a helpful strategy often used by the management of manufacturing firms to resolve occasional capacity short supply, shorten replenishment cycle (response) time, or smooth production schedules, or reduce overall system cost. Leavy [1] reviewed the increasing influences of business strategies in present-day's companies, including outsourcing, with a focus on the perception of learning in strategic analysis. Chalos and Sung [2] presented a model wherein outsourcing takes over in-house fabrication, and they argued that outsourcing could increase managerial incentives. They set up situations for a producer who favors applying the outsourcing, and their study included having outsourcers for publicly and privately held corporations. As a result, the researchers offered diverse comments on outsourcing practices. Levi-na and Ross [3] studied the potential values gained from outsourcing information technology (IT). They carefully examined IT vendor strategy and practices from a successful outsourcing contract. They found that a vendor can derive economic profits from its capability of developing a fine set of core competencies through a variety of its IT projects. Based on these findings and existing knowledge of the client-vendor relationship, they suggested ways to assess IT outsourcing's values. Serrato et al. [4] used a Markov decision to investigate outsourcing in reverse logistics functions (RLF), especially if returns are unsteady. Using capacity and operation cost as reward function, they constructed an analytical Markov model for outsourcing decision-making on either carrying out RLF in-house or outsourcing them. Accordingly, the researchers identified some sufficient conditions on system parameters to ensure an optimal outsourcing policy. Proff [5] explored the competencies shift from automobile manufacturers to module suppliers in components outsourcing policy, especially for those manufacturers who implemented differentiation strategy. Based on core competency and transaction cost theories, the researcher recommended a few possible tactical actions. Balachandran et al. [6] studied the influence of in-house fabrication capability on supply chain decisions. Different scenarios regarding the producer's in-house ability and its outsourcer's incentive to invest in the fabrication process were carefully examined and discussed. The authors revealed the effect of in-house capability on supply chain interdependent decisions and efficiencies. Additional articles [7-11] also studied various features of outsourcing options on manufacturing and supply-chain systems.

Maintaining high and steady product quality is one of the essential operation goals in most manufacturing firms. Production of faulty items is inevitable owing to different factors in the fabrication process. The capabilities of identifying and removing the scrap items, and reworking the repairable products, are significant tasks in quality management. Rosenblatt and Lee [12] explored the lot sizing for an economic production quantity (EPQ) model with stochastic nonconforming produced in the manufacturing process’s out-of-control state. The researchers...
presented fairly accurate solutions for optimal lot-size. Rahim and Ben-Daya [13] investigated the combined effects of arbitrary deteriorating products and fabrication processes on the EPQ policy, optimal schedule for inspection, and quality control procedure. They used numerical examples to express and explain their models and results. Ojha et al. [14] studied a quality assured integrated fabrication-inventory problem including supplier, producer, and customer. Their model assumed a constant defective rate and a rework process. The entire lot has to be quality assured before distribution to the customer. Various scenarios were presented and examined to decide the optimal operating policies. Additional studies that explored diverse features of quality assurance matters in fabrication systems can also be referred to [15-19].

As assumed in this study, multiple shipments policy is practically employed in most supply chain systems for transporting stocks, unlike the continuous inventory issuing policy described in the classic EPQ model [20]. Goyal [21] presented an approach to solving a single-vendor, single-buyer supply chain system. Through illustrative examples, the author confirmed his proposed solution procedure. Banerjee [22] explored a customer-vendor integrated EPQ model intending to minimize the combined system cost. The author commented that a price adjustment consideration in ordering decision-making could benefit both parties. Viswanathan [23] studied vendor-buyer integrated inventory systems using two distinct strategies extracted from past literature. The first one assumes fixed quantity delivery, and the other considers the delivery of the vendor’s all available stocks. Through in-depth numerical illustrations that provide various performance indicators of these strategies, the study concluded that no one approach gave the best solution to the problem’s potential variables. Sarker and Diponegoro [24] explored a multi-supplier single-producer multi-customer integrated supply chain system. Their system purchased raw materials from multiple non-competing suppliers and shipped the finished goods to various buyers at a constant time interval. Their objectives were to decide the optimal procurement, production, and shipment strategies that minimize the combined system costs. Additional works that investigated diverse features of multi-shipment policies in different aspects of supply chain systems can be found elsewhere [25-29]. Since few studies focused on exploring the combined effects of outsourcing, in-house quality assurance matters, and multiple deliveries on the optimal fabrication-delivery policy, this paper aims to fill the gap.

ASSUMPTIONS, MODELLING, AND FORMULATIONS

The proposed EPQ-based system incorporates outsourcing and in-house quality assurance into production-shipment decision making to meet the annual demand \( \lambda \). A \( \pi \) proportion of the lot size \( Q \) (where \( 0 < \pi < 1 \)) is outsourced in each cycle, i.e., \((1-\pi)Q\) amount is made in-house. The outside provider guarantees the quality of outsourcing products, and the receiving time of outsourced items is set at the end of in-house rework time (refer to Figure 1). Relevant costs associated with the outsourcing portion include fixed setup cost \( K_\pi \) and variable outsourcing cost \( C_\pi (\pi Q) \), where \( K_\pi = [(1+\beta_1) K]_\pi \), \( C_\pi = [(1+\beta_2) C]_\pi \), and \( K \) and \( C \) denote setup and unit cost of the in-house process; \( \beta_1 \) stands for the relating factor of \( K_\pi \) and \( K \); \( \beta_2 \) represents the relating factor of \( C_\pi \) and \( C \) (where \( -1 < \beta_1 < 0 \) and \( \beta_2 > 0 \)).

The annual in-house fabrication rate is \( P \) units, and a random nonconforming proportion \( x \) of the lot may be fabricated in uptime, with a rate \( d = P x \). A \( \theta \) portion of nonconforming items is categorized as scraps, and the other \((1-\theta)\) portion is identified as the re-workable items. In each cycle, a rework process begins when the regular fabrication ends (see Figure 1), and the annual rework rate is \( P_1 \) units. Furthermore, a \( \theta_1 \) portion of the reworked items fails and must be scrapped. The scrap items’ production rate \( d_1 \) during the rework time is \( P_1 \theta_1 \) (see the status of scraps in Fig. 2). No stock-out situations are permitted, so \( P - d - \lambda > 0 \) must hold. The outsourcing products are received before the beginning of the entire batch’s delivery time (Figure 1).

Figure 1: Status of perfect quality inventories in the proposed hybrid replenishment system (in blue) compared to that in a system with continuous distributing policy and pure in-house fabrication (in black)

Figure 2: Status of scrap inventories in the proposed system
Then, \( n \) fixed quantity installments of the batch are supplied to the buyer at a specified time interval in delivery time \( t_{mn} \). Figures 3 and 4 illustrate the status of on-hand inventories during \( t_{mn} \) at the producer side and in each cycle at the customer side, respectively.

**Formulations of the proposed supply-chain system**

From Figures 1 to 2, the following formulations are obtained from the in-house fabrication of \((1-\pi)Q\):

\[
t_{w} = \frac{(1-\pi)Q}{P} = \frac{H_{1}}{P-d} 
\]

(1)

\[
H_{2} = (P-d) t_{w} 
\]

(2)

\[
t_{2n} = \frac{x(1-\pi)Q}{P} (1-\theta) 
\]

(3)

\[
H_{2} = H_{1} + (P-d) t_{2n} 
\]

(4)

\[
T_{E} = t_{w} + t_{2n} + t_{3n} = \frac{Q(1-x)[(1-\pi)]}{\lambda} 
\]

(5)

\[
t_{3n} = T_{E} - t_{w} - t_{2n} 
\]

(6)

The outsourcing products are scheduled to be received before the beginning of \( t_{3n} \), so the maximal on-hand perfect quality inventory level \( H \) is as follows:

\[
H = H_{2} + \pi Q 
\]

(7)

The maximal number of nonconforming items \( dt_{1\pi} \) and maximal scraps \( \varphi[x(1-\pi)Q] \) in a cycle are given below:

\[
d_{t_{1\pi}} = xP_{t_{1\pi}} = xP_{t_{1\pi}} - \pi Q 
\]

(8)

\[
\varphi[x(1-\pi)Q] = \theta[x(1-\pi)Q] + \theta(1-\theta)[x(1-\pi)Q] = \theta + (1-\theta) \theta x(1-\pi)Q 
\]

(9)

From Figure 3, total inventories in \( t_{3n} \) are [7] as follows:

\[
H_{t} = H_{t} + H_{t} + H_{t} = \frac{1}{n} \left[ \sum_{i=1}^{n} H_{t} \right] 
\]

(10)

From Figures 3 and 4, the following formulations and total stocks at the customer side can be obtained [7] as follows:

\[
t_{mn} = t_{mn} - t_{n} 
\]

(11)

\[
D = \frac{H}{n} 
\]

(12)

\[
I = D - \left( A_{t_{mn}} \right) 
\]

(13)

\[
\text{Total inventories} = \left[ \frac{D+t_{w}}{2} \right] t_{mn} + \left( \frac{n!}{2} \right) t_{t_{w}+t_{2n}} + \left( D+2l \right) + A_{t_{mn}} t_{mn} 
\]

(14)

Substitute Eqs. (11) to (13) in Eq. (14), and with additional derivations, the total inventories at the customer side become
Total inventories $= \left( D - \frac{M_{tn}}{2} \right) t_{mn} + \left( D + 2 \frac{M_{tn}}{2} \right) t_{mn} + \cdots + \left( D + (n-1) \frac{M_{tn}}{2} \right) t_{mn} + \frac{n(n-1)}{2} h_{mn} + \frac{n(n+1)}{2} t_{mn} + 2 t_{2m}$

\begin{equation}
= n \left( D - \frac{M_{tn}}{2} \right) t_{mn} + \frac{n(n-1)}{2} h_{mn} + \frac{n(n+1)}{2} t_{mn} + 2 t_{2m}
\end{equation}

Cost analysis

The fixed and variable outsourcing costs are as follows:

\begin{equation}
K_{m} + C_{m} (\pi Q) = (1 + \beta) K + (1 + \beta) C (\pi Q)
\end{equation}

The in-house fabrication costs include setup and variable manufacturing costs, variable rework and disposal costs, fixed and variable shipping costs, holding cost during $t_{2m}$, and total holding costs in $T_{n}$.

\begin{equation}
K + C \left[ (1 - \pi) Q + C_{m} (1 - \theta) \right] x (1 - \pi) Q + h_{P} \frac{P_{t}}{2} (t_{m}) + C_{m} \phi [x (1 - \pi) Q] + n K + C \left[ Q (1 - \phi x (1 - \pi)) \right] + h_{P} + H_{L} (t_{m}) + n t_{m} \left( H - M_{tn} \right)
\end{equation}

The customer’s side has the following holding cost:

\begin{equation}
h_{2} \left( \frac{H}{n} t_{2m} + T_{n} (H - M_{tn}) \right)
\end{equation}

From Eqs. (16) to (18), TC(Q, n) becomes as follows:

\begin{equation}
TC(Q,n) = (1 + \beta) K + (1 + \beta) C (\pi Q) + K + C \left[ (1 - \pi) Q + C_{m} (1 - \theta) \right] x (1 - \pi) Q + n K + C \left[ Q (1 - \phi x (1 - \pi)) \right] + h_{P} + H_{L} (t_{m}) + n t_{m} \left( H - M_{tn} \right)
\end{equation}

The expected value of $x$ is used to consider the non-conforming rate’s randomness by substituting all relevant variables from Eqs. (1) to (9) into Eq. (19), $E[TCU(Q,n)]$ can be found as follows:

\begin{equation}
E[TCU(Q,n)] = E[TC(Q,n)] - \frac{1}{Q} \left[ (1 + \beta) K + (1 + \beta) C (\pi Q) \right]
\end{equation}

\begin{equation}
+ A \left[ (1 + \beta) K + (1 + \beta) C (\pi Q) \right] E_{0} s \left( \frac{Q}{2} \right) \left( E_{1} \right)
\end{equation}

\begin{equation}
\frac{\phi \left[ x (1 - \pi) Q \right]}{1 - \phi} \left[ E_{0} \right] \left( 1 - \phi \right) \left[ E_{1} \right] \left( 1 - \phi \right) \left[ E_{2} \right]
\end{equation}

where

\begin{equation}
E_{0} = \frac{1}{\phi E_{x} \left[ x (1 - \pi) \right]} ; \quad E_{1} = \frac{E_{x} \left[ x (1 - \pi) \right]}{\phi E_{x} \left[ x (1 - \pi) \right]}
\end{equation}

Determining the optimal replenishment batch size and deliveries

Apply the Hessian matrix equations [30] to Eq. (20) to first prove that $E[TCU(Q,n)]$ is convexity, i.e., to show the following equation holds:

\begin{equation}
\frac{\partial^{2} E[TCU(Q,n)]}{\partial Q^{2}} > 0
\end{equation}

\begin{equation}
\frac{\partial^{2} E[TCU(Q,n)]}{\partial Q^{2}} > 0
\end{equation}

Detailed derivation of the Hessian matrix equation [30] is exhibited in Appendix A. Substitute Eqs. (A-2), (A-4), and (A-5) in Eq. (21), one obtains Eq. (22):

\begin{equation}
E^{2}[TCU(Q,n)] - \frac{2 A (2 + 3 \pi) K + (\pi Q)}{\phi E_{x} \left[ x (1 - \pi) \right]}
\end{equation}

\begin{equation}
+ 2 \left( \frac{1}{E_{0}} \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right) \left( 1 - \phi \right)
\end{equation}

It is worth noting that the result of the number of shipments per cycle obtained in Eq. (24) is a real number; however, in real-life application, it should only be an integer. The following process helps find the optimal integer value $n^*$: First, find two adjacent integers of $n$ (as obtained from Eq. (24)), let $n^*$ be the smallest integer greater than $n$ and $n^*$ denote the largest integer less than $n$. Then, substitute $n^*$ and $n^*$ in Eq. (23) to find their corresponding values of $Q$, and apply the resulting $(Q, n^*)$ and $(Q, n^*)$ in Eq. (20) to obtain their respective system costs. Lastly, select the one that has a minimum value of $E[TCU(Q,n)]$ as our optimal operating policy of $(Q^*, n^*)$.

Numerical illustration, sensitivity analyses, and discussion

Applying Eqs. (23) and (24), one finds the optimal $Q^* = 1141$ and $n^* = 3$. Computation of Eq. (20) with $Q^*$ and $n^*$, one obtains $E[TCU(Q^*, n^*)] = 516,142$. The combined impacts of $Q$ and $n$ on the $E[TCU(Q,n)]$ for $\pi = 0.4$ are exposed in Figure 5.
Effects of different outsourcing portion $\pi$ on various system parameters are analyzed and displayed in Table 1. It is noted that for the case of outsourcing all products (i.e., $\pi=1$), $E[TCU(Q^*, n^*)]=519,926$, which enables us to locate the critical ratio of $\pi=0.503$ for the make-or-buy decision (see Figure 6). It indicates that as $\pi$ increases, in-house machine utilization decreases accordingly (see Figure 7). In our example, at $\pi=0.40$, the utilization declines slightly over 40% (i.e., 40.5%, refer to Table 1 for details).

Table 1: Effects of different outsourcing portion $\pi$ on various system parameters

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>$n^*$</th>
<th>$Q^*$</th>
<th>Outsourcing Cost</th>
<th>In-house Production Cost</th>
<th>Total Delivery Cost</th>
<th>Customer’s Stock Holding Cost</th>
<th>$E[TCU(Q^<em>, n^</em>)]$</th>
<th>Cost Increase %</th>
<th>$T^*$ (in year)</th>
<th>Machine Utilization($t_1+ t_2)/T^*$</th>
<th>Utilization Decrease %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>869</td>
<td>$0$</td>
<td>$463,969$</td>
<td>$9,508$</td>
<td>$21,777$</td>
<td>$495,253$</td>
<td>0.00%</td>
<td>0.2131</td>
<td>0.20387</td>
<td>0%</td>
</tr>
<tr>
<td>0.05</td>
<td>2</td>
<td>971</td>
<td>$30,732$</td>
<td>$440,201$</td>
<td>$8,710$</td>
<td>$24,094$</td>
<td>$503,738$</td>
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<td>0.2384</td>
<td>0.19349</td>
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<td>0.10</td>
<td>2</td>
<td>980</td>
<td>$55,066$</td>
<td>$417,793$</td>
<td>$8,646$</td>
<td>$24,056$</td>
<td>$505,561$</td>
<td>2.08%</td>
<td>0.2407</td>
<td>0.18313</td>
<td>-10.2%</td>
</tr>
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<td>0.15</td>
<td>3</td>
<td>1087</td>
<td>$75,556$</td>
<td>$396,556$</td>
<td>$10,973$</td>
<td>$20,969$</td>
<td>$507,288$</td>
<td>2.43%</td>
<td>0.2675</td>
<td>0.17279</td>
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<td>$10,874$</td>
<td>$20,798$</td>
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<td>$20,615$</td>
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</tr>
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<td>$10,512$</td>
<td>$20,003$</td>
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<td>$19,537$</td>
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<td>$19,286$</td>
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<td>$19,023$</td>
<td>$523,596$</td>
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<td>$18,748$</td>
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<td>$18,459$</td>
<td>$527,459$</td>
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<td>$18,159$</td>
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<td>6.90%</td>
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</table>
es a little, but the system cost $E[TCU(Q, n)]$ increases significantly. And at $\beta_2=0.2$ as our example assumes, $Q^*=1141$.

Moreover, the effects of $x$ and different $\varphi$ on the system cost $E[TCU(Q, n)]$ are demonstrated in Figure 10. It specifies that as $x$ increases, $E[TCU(Q, n)]$ knowingly increases, simply due to quality assurance cost rises. As the overall scrap rate $\varphi$ rises, the system cost $E[TCU(Q, n)]$ increases as expected.

Finally, the proposed system enables us to explore the critical ratio of $\varphi$ for the make-or-buy decision (see Figure 11). In our numerical example, the further analytical result indicates that for a pure in-house fabrication system (i.e., $\pi=0$), if the overall scrap rate $\varphi$ exceeds 0.626, then switch to a ‘buy’ system (i.e., $\pi=1$) becomes a better decision in saving system cost.

**CONCLUSIONS**

This paper explores an intra-supply chain type of production-shipment problem, featuring outsourcing, in-house product quality assurance, and discontinuous multi-shipment products issuing policy. A mathematical model is constructed to describe the proposed problem precisely.
By the use of optimization techniques, the optimal production-shipment policy is derived. This study demonstrates the applicability of research results through a numerical example and reveals diverse unseen important information of this specific problem. The latter, main findings of the present study, includes (1) effects of different outsourcing portion π on various system parameters (Table 1); (2) exploring the critical ratio of π for make-or-buy decision making (Fig. 6); (3) effects of variations in π on in-house machine utilization (Fig. 7); (4) joint effects of variations in β and π on system cost (Fig. 8); (5) joint effects of variations in x and different φ on system cost (Fig. 9); (6) effects of variations in x and different φ on system cost (Fig. 10); and (7) investigating the critical ratio of φ for make-or-buy decision making (Fig. 11), etc. Without an in-depth investigation of such a specific problem, the crucial managerial information mentioned above is inaccessible. For future work, one may explore the effect of probabilistic demand on the same problem.

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**NUMERICAL ILLUSTRATION**

To show the applicability of research results, this section provides a numerical example with the following system parameters:

\[ \lambda = 4,000 \text{ units}, \]
\[ P = 20,000 \text{ units}, \]
\[ K = 5,000 \text{ per run}, \]
\[ C = 100 \text{ per product fabricated in-house}, \]
\[ h = 30 \text{ per item}, \]
\[ \pi = 0.4 \text{, the outsourcing proportion of replenishment batch}, \]
\[ K_p = 1,500 \text{, the fixed cost per outsourcing items}, \]
\[ \beta_1 = -0.7, \]
\[ C_n = 120 \text{, unit outsourcing cost}, \]
\[ \beta_2 = 0.2, \]
\[ x \text{, over the interval of } [0, 0.2], \]
\[ \theta = 0.1, \]
\[ P_s = 5000 \text{ units per year}, \]
\[ C_n = 60 \text{ per reworked item}, \]
\[ h_1 = 40 \text{ per reworked item per year}, \]
\[ \theta_1 = 0.1, \]
\[ C_0 = 20 \text{, unit disposal cost}, \]
\[ K_s = 800 \text{, the fixed cost per delivery}, \]
\[ C_r = 0.5 \text{, unit delivery cost}, \]
\[ h_2 = 80 \text{ per item per year}, \]

**APPENDIX A**

The detailed derivations of the Hessian Matrix Equations. The following partial derivatives are obtained from Eq. (20):
\[ \frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} = \frac{2\lambda}{Q} \left[ E_0 \left\{ 1 - \beta_1 + \frac{1}{P} \right\} + \left( h_2 - h \right) \left( \frac{1}{n} - \frac{1}{P} \right) \lambda E_2 \right] \]  
\[ \text{(A-2)} \]

\[ \frac{\partial E[TCU(Q,n)]}{\partial n} = \frac{\lambda}{Q} E_0 \left\{ 1 - \beta_1 + \frac{1}{P} \right\} - \frac{1}{2} \left( h_2 - h \right) \left( \frac{1}{n^2} - \frac{1}{P} \right) \lambda E_2 \]  
\[ \text{(A-3)} \]

\[ \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} = \frac{Q}{2} \left( h_2 - h \right) \left( \frac{1}{n^2} - \frac{1}{P} \right) \lambda E_2 \]  
\[ \text{(A-4)} \]

\[ \frac{\partial E[TCU(Q,n)]}{\partial Q n} = \frac{\lambda}{Q} E_0 \left\{ 1 - \beta_1 + \frac{1}{P} \right\} + \left( h_2 - h \right) \left( \frac{1}{n^2} - \frac{1}{P} \right) \lambda E_2 \]  
\[ \text{(A-5)} \]