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ANALYTICAL METHOD FOR ELASTIC RECOVERY PREDICTION OF AIR BENDING SHEET

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The estimation of the elastic recovery is of great importance during the planning of plastic deformation processes, because this estimation could reduce the number of required steps to reach the target geometry. The sheet bending process is one of the most widely used industrial processes, and that is why this paper seeks to provide an analytical model based on an elastic behavior with potential hardening of the material that could be combined with the geometric information of the process to estimate the degree of recovery of the components. The potential hardening model was selected due to its simplicity and good fit with the experimental data observed in steel sheets in this application. The effectiveness of the model was compared with the results obtained by several authors. The effectiveness of the model is significantly influenced by the parameters of the bending process and the method used to estimate the radius of curvature.

Keywords: springback, analytical model, plastic deformation, elastic recovery, potential hardening

1 INTRODUCTION

The manufacturing processes of components and products by plastic deformation are significantly affected by the elastic recovery when the loads are removed. Therefore, during the planning of the processes, this phenomenon must be contemplated to compensate for the dimensional and geometric variations and ensure that the final product matches the specifications.

The manufacture of different types of components involves the bending of sheets, plates, and/or platens through the action of a punch on the material, which could be supported on rollers or resting on a die. The application of loads on the material during the forming process induces stresses greater than the yield stress, so that the material is permanently deformed. In the bending process, it is often necessary to determine, with a certain level of precision, the final geometrical configuration that the component will reach once the load has been removed, since this could deviate due to the phenomenon of elastic recovery (springback) [1].

Authors such as Wagoner, Lim, and Lee [2], Marciniak [3], and Gardiner [4], among others, have established that the level of elastic recovery in components manufactured through bending depends on several factors, including the induced moment, material thickness, and elastoplastic properties of the material, such as the modulus of elasticity and yield stress [3].

To estimate this phenomenon, it is crucial to determine the relationship between operating conditions (applied force, point of application, type and number of supports, process speed, among others), the initial and final geometry of the component, its mechanical properties, and the level of stress achieved.

The springback phenomenon has been extensively studied due to its implications in plastic deformation processes. The number of stages and operational characteristics of each process depend on the accuracy of recovery estimation. Several researchers have developed models based on experimental data, results from simulations using finite elements, or by analyzing the elastoplastic behavior of the material.

In this context, Wagoner's work [2] compiles advancements in elastic recovery analysis, particularly through the use of finite element analysis, and the consideration of nonlinear hardening, and the variation of the elastic modulus during material recovery. Other authors, such as Y. Hou and collaborators [5], have incorporated functions proposed by Hill and Barlat to estimate yield surface variation and the kinematic hardening model proposed by Yoshida-Uemori in finite element simulations. Simultaneously using these functions, along with the kinematic hardening model, has improved the prediction of elastic recovery.

Li, Carden, and Wagoner [6] focused on the effect of the number of integration points on the accuracy of simulating elastic recovery in sheets using finite elements.

They found that the difference between using 9 and 51 integration points is not significant, which is highly beneficial in reducing computational costs. Notably, for relationships between the radius of curvature and thickness greater than 6, the accuracy of the response increases.

In the work of Hino, Hamasaki, and Yoshida [7], the effect of considering an isotropic hardening model was compared with the kinematic hardening models of Yoshida-Uemori. Although both cases yield good predictions, they conclude that the isotropic model tends to underestimate elastic recovery. This aligns with the findings of Sitar, Kosel, and

Brojan [8], who confirmed that elastic recovery is strongly influenced by the maximum applied load (bending moment) and the radius of curvature. They found that the effect on prediction when considering kinematic or isotropic hardening is not negligible.

Lepadapu and collaborators [9] employed a different approach, optimizing the estimation of elastic recovery using the statistical response surface technique. This methodology requires a higher degree of experimentation under real process conditions.

Sumikawa, Ishiwatari, and Hiramoto [10] propose a new model that combines the use of kinematic and isotropic hardening to predict the evolution of the yield surface and model the relationship between stress and deformation. This modification allowed the authors to achieve a better prediction of elastic recovery.

It is important to note that there is no universal model, and the accuracy of the model depends on the assumptions made and the data it is based on. Variability in mechanical properties, the hardening models used to describe their behavior, the precision and accuracy of geometric measurements, and the parameters used (such as the types of elements and the number of integration points) in the simulations, among others, can affect the response obtained from the application of different models [11], [12].

The design of experiments for data collection in industrial processes, as well as the optimization of measurement processes [9], [13], and the use of automated control systems, would be ideal conditions. However, these often require sophisticated instrumentation [13], [14], the implementation of data analysis and process control systems [14], consumption of process time, and may even lead to a loss of raw materials, making the process more complex and expensive.

Although analytical models are also limited, they are of great interest since their implementation, in general, requires only basic information about the process and the mechanical properties of the materials and does not involve the use of sophisticated technologies, software, or specialized personnel.

The present work proposes a model of elastic recovery based on an elastic behavior with potential hardening, which could be combined with the geometric information of the bending process to establish the level of recovery of the material. The parameters of this model (resistance coefficient and hardening exponent), along with the modulus of elasticity and yield stress, can be easily obtained through a tensile test and, for metals such as steel, offer a level of accuracy comparable to that of other elastoplastic models. The simplicity of this model has allowed for the development of a mathematical expression that enables the estimation of material's elastic recovery based on specific processes, and material parameters.

2 DEVELOPMENT OF THE ANALYTICAL MODEL

2.1 Analytical models of elastic recovery

Elastic recovery, or springback, is the phenomenon by which the material partially returns to its original shape when the forming tool is removed from the part. In the case of materials that are deformed plastically by bending, this recovery is measured as the variation of the angle between the bent part in relation to the angle produced by the tool [3]. That is, the angle reached during the bending process and the final angle acquired by the part when the load is removed. These variations in bending angle (□) are also reflected in the modification of the radius of curvature (R) of the workpiece (Fig. 1).

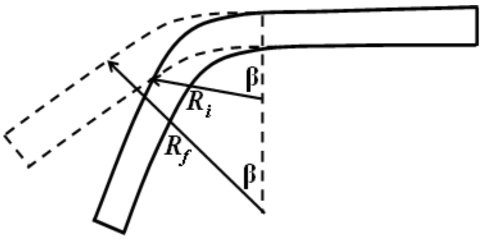


Figure 1. Springback

Ri=Curvature radius during load application, Rf=Curvature radius after load removal

It is known that there is a close relationship between the radius of curvature (Ri and Rf) of the bent piece, the bending moment (M) it experiences, its modulus of elasticity (E), and its area moment of inertia (I). The variation in the radius of curvature could be estimated as a function of these parameters, as shown in the equation (1)[12]

$$\frac{1}{R_f} - \frac{1}{R_i} = \frac{M}{EI} \tag{1}$$

In the case of a narrow beam, the conventional modulus of elasticity could be used because it could be assumed that the material is only subjected to normal stresses in the longitudinal direction. If the width and thickness of the beam are important, an effective or modified modulus of elasticity must be used. The elastic recovery is usually expressed in terms of the variation in bending angle (β), as shown in equation (2)[16].

$$\Delta\beta = \beta_i\left(\frac{R_i}{R_f} - 1\right) \quad (2)$$

The analytical models used are based on characteristic premises of classical materials mechanics: uniform thickness, isotropic material, no residual stress in the material before the process, sections remain flat during bending and thickness changes are negligible [1],[16]-[18].

Marciniack [3] established in his book "Mechanical of sheet metal forming" that for a perfectly plastic elastic model the elastic recovery could be estimated by the equation:

$$\Delta\beta = -3\beta_i\left(\frac{R_i\sigma_y}{Et}\right) \quad (3)$$

Where: σ_y: yield stress; E: modulus of elasticity; t: thickness; R_i: initial radius of curvature.

One of the most used models is the one formulated by Gardiner and quoted by several authors [4],[19]. This model is also based on an elastic material - perfectly plastic.

$$\frac{R_i}{R_f} = 4\left(\frac{R_i\sigma_y}{Et}\right)^3 - 3\left(\frac{R_i\sigma_y}{Et}\right) + 1 \quad (4)$$

The modulus of elasticity could be corrected when the width of the material is very large in relation to the thickness. In this case the behavior is not that of a beam but of a sheet or plate.

$$E' = \frac{E}{(1-\nu^2)} \quad (5)$$

Where, v: Poisson´s ratio. So, the elastic recovery could be estimated as follows:

$$\Delta\beta = \beta_i\left[4\left(\frac{R_i\sigma_y}{E't}\right)^3 - 3\left(\frac{R_i\sigma_y}{E't}\right)\right] \quad (6)$$

This model is often used as a first approximation to predict recovery, but since it does not consider the plastic strain hardening of the materials, it is not accurate enough. It could be seen that the second term of the Gardiner model corresponds to the perfectly plastic behavior established in the Marciniak model.

Other authors have established recovery models based on hardened materials. Querneer-Angelis [16] and Hosford-Cadell [20] have based their models on the law of potency. The simplified elastic recovery model obtained by Querneer and Angelis is the following:

$$\Delta\beta \approx -\left(\frac{2R_i}{t}\right)^{1-n}\left[\frac{3K(1-\nu^2)}{\left(\frac{3}{4}\right)^{\frac{(n+1)}{2}}(n+2)E}\right]\beta_i \quad (7)$$

Where K: Strength coefficient and n: hardening exponent. This latter term indicates how marked the level of hardening achieved by the material is.

The Hosford-Cadell Model [20] estimates the elastic recovery using the following expression:

$$\Delta\beta = \beta_i\left(\frac{6}{2-n}\right)\left[K\left(\frac{4}{3}\right)^{\frac{n+1}{2}}\frac{(1-\nu^2)}{Et}\right]R_i\left(\frac{t}{2R_i}\right)^n \quad (8)$$

The values of the hardening exponent and the strength coefficient of metals depend on the type of material and the processes and treatments to which it has been subjected. In the case of the hardening exponent, it is related primarily on the type of unit cell in which it is found upon solidification (atomic organization) [21].

According to the text by Askeland and Puhlé [21], the characteristic value of n for stainless steels is 0.52 and the value of the strength coefficient is around 1517 MPa, while for a medium carbon steel that has been tempered and annealed (thermal treatments), n = 0.10 and K = 1572 MPa. In the work on sheet bending by M.L. Garcia-Romeu [16], the values obtained for stainless steel AISI 304 were: n=[0.419; 0.435] and K=[1409.51; 1551.67] Mpa.

Other authors, such as Appiah and M. Jain [22] propose a kinematic hardening model based on Armstrong's model and compare it with 6 other hardening models (Ziegler-Prandtl, Armstrong-Lemaitre, Caboched, Geng-Wagon, Chun-lee and Yoshida). The implementation of these models for estimating the elastic recovery involves the experimental determination of other material properties or assuming more complex hardening models.

In the work of H. Yi et al [23] six possible levels of deformation were established that influence the springback phenomenon and related them to the modulus of elasticity, the yield field, the applied stress, and the geometry generated in the process.

To study the elastic recovery of beam-type elements subjected to bending, the 3-point bending model with a load applied at the center was used, as shown in Fig. 2.

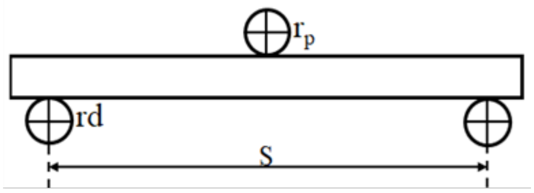


Figure 2. Initial set-up of the 3-point bending process

r_p=radius of curvature of the punch, r_d=radius of support rollers

The different models to estimate the variation in the bending angle or the elastic recovery, require the initial radius of curvature value. That radius [14] could be estimated in a simplified way using the expression:

$$R_i = \frac{s\tan\theta + \frac{t}{2} - p}{sec\theta - 1} \tag{9}$$

Where: p: displacement of the punch, t: thickness and s: distance between centers (supporting rollers).

2.2 Development of the recovery model

To determine the angles θ and β produced during the bending process, could be used the arc model of circumference and two straight sides to represent the curvature of the sheet. Fig. 3 and Fig. 4 shows a schematic view of this model and the bending process in general.

The point P2 in Fig. 4 corresponds to the point of tangency of the arc of circumference with the straight section of the beam that is in contact with the support roll.

The center-to-center spacing of the support rollers is constant, but the position of the beam contact point with these rollers is not constant and changes according to the punch displacement and consequently the bending angle.

The model does not consider the crushing in the contact zone, or the fact that the lateral sections of the beam that are not in contact with the punch are not perfectly straight.

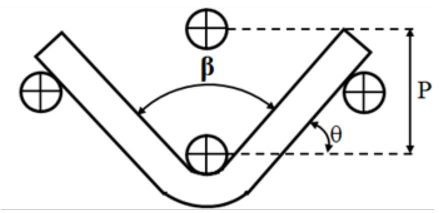


Figure 3. Flexed beam

In Fig. 4 the inner circle represents the punch, and the outer circle represents the curvature of the upper part of the beam in contact with the piece mentioned above.

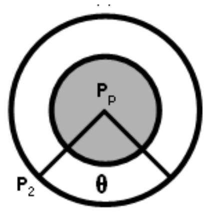
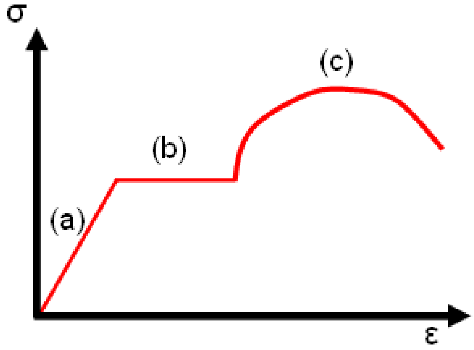


Figure 4. Contact model

The angle (θ) produced during bending depends on the process parameters: punch penetration or displacement, punch radius, support roller radius, support spacing, and sheet thickness. To determine this angle, it could be used a drawing software such as AutoCad®, or geometric analysis tools such as Geogebra®, or use the approximated analytical expression (see Fig. 3 and Fig. 4).

$$\theta = 2tan\left(\frac{x_p - x_2}{y_p - y_2}\right) \tag{10}$$

For the determination of the springback or recovery model, it was assumed that the material could be represented as having a linear elastic zone (a), a perfectly plastic transition zone (b) and finally a zone with potential hardening (c) (Fig. 5).



y

Figure 5. Elastoplastic model

(a) σ=E.ϵ if $\sigma<\sigma_y$ (11)

(b) $\sigma=\sigma_y$ if σ=σy (12)

(c) $\sigma = K.\epsilon^n$ if $\sigma \geq \sigma_y$ (13)

From classical mechanics, it is known that the bending moment (M) of a rectangular section beam could be determined by the following relationship [24]:

$$M = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma y dA = 2b\left\{\int_0^{y_c}(E\epsilon)ydy + \int_{y_c}^{ay_c}\sigma_y ydy + \int_{ay_c}^{\frac{t}{2}} K\epsilon^n ydy\right\} \quad (14)$$

Where t and b correspond to the thickness and width of the sheet.

The strain by bending of the internal fibers of the sheet could be estimated by the following expression ϵ=y/R and that the critical height of the beam for which the yield point is reached is:

$$y_c = \frac{R\sigma_y}{E}$$

“a” is a factor which indicates the distance from the centroid to the fiber where the plastic transition zone ends. Its value varies between 1 and t/(2yc). Where 1 represents that there is no transition zone and t/(2yc) would indicate that the rest of the fibers experience plastic behavior without hardening.

Substituting ϵ=y/R and yc in the equation we obtain the following expression:

$$M = 2b\left\{\int_0^{y_c}\frac{E}{R}y^2dy + \int_{y_c}^{a.y_c}\sigma_y ydy + \int_{ay_c}^{\frac{t}{2}} K\frac{y^n}{R^n}ydy\right\} \quad (15)$$

$$M = 2b\left(\frac{E}{R}\right)\left(\frac{R\sigma_y}{E}\right)^3\left[\frac{n+2-3a^2}{3(n+2)}\right] + b(a^2 -)\sigma_y\left(\frac{R\sigma_y}{E}\right)^2 + \frac{2b}{(n+2)a^n}\left(\frac{R}{E}\right)^{-1}\left(\frac{t}{2}\right)^{n+2}\left(\frac{R\sigma_y}{E}\right)^{1-n} \quad (16)$$

Remembering that the moment due to the elastic recovery could be estimated from the relationship:

$$M = EI\left(\frac{1}{R} - \frac{1}{R_f}\right) \quad (1)$$

Which for a rectangular section beam is equivalent to:

$$M = E\frac{bt^3}{12}\left(\frac{1}{R} - \frac{1}{R_f}\right) \quad (17)$$

By equalizing the moments expressed in the equations (16) and (17), multiplying both sides by the initial radius of curvature and rearranging terms, is obtained:

$$\left(1 - \frac{R}{R_f}\right) = \left\{8\left(\frac{R\sigma_y}{tE}\right)^3\left[\frac{n+2-3a^2}{(n+2)}\right] + \frac{6}{2^n(n+2)a^n}\left(\frac{R\sigma_y}{tE}\right)^{1-n}\right\} \quad (18)$$

The recovery angle (Δβ) of the element is determined by the general expression (Eq.2):

$$|\Delta\beta| = \beta_i\left|1 - \frac{R}{R_f}\right|$$

The initial bending angle could be measured directly during the process or estimated from the process geometric information. By replacing equation (18), an expression is obtained that allows to determine the angle of elastic recovery of the beam:

$$|\Delta\beta| = \left|\beta_i\left[8\left(\frac{R\sigma_y}{tE}\right)^3\left(\frac{n+2-3a^2}{(n+2)}\right)+\frac{6}{2^n(n+2)a^n}\left(\frac{R\sigma_y}{tE}\right)^{1-n}\right]\right| \quad (19)$$

The bending radius is not a constant parameter and depends on the value of the bending moment. For materials that do not exhibit a transition zone between the elastic zone and the hardened plastic zone, the value of "a" is equal to 1, therefore the expression to evaluate the moment is reduced to:

$$M = 2b\left(\frac{E}{R}\right)\left(\frac{R.\sigma_y}{E}\right)^3\left[\frac{(n-1)}{3(n+2)}\right]+\frac{2.b}{(n+2)}\left(\frac{E}{R}\right)\left(\frac{t}{2}\right)^{n+2}\left(\frac{R\sigma_y}{E}\right)^{1-n} \quad (20)$$

And the recovery angle for this case is estimated by the formula:

$$|\Delta\beta| = \left|\beta_i\left(1-\frac{R}{R_f}\right)\right| = \left|\beta_i\left\{-8\frac{(1-n)}{(n+2)}\left(\frac{R\sigma_y}{tE}\right)^3+3\frac{2^{1-n}}{(n+2)}\left(\frac{R\sigma_y}{tE}\right)^{1-n}\right\}\right| \quad (21)$$

3 RESULTS AND DISCUSSIONS

To validate the model performance, the data recorded by Garcia-Romeu was used [16], [25]. This data corresponds to stainless Steel AISI 304. The Fig. 6 shows the engineering stress-deformation diagram.

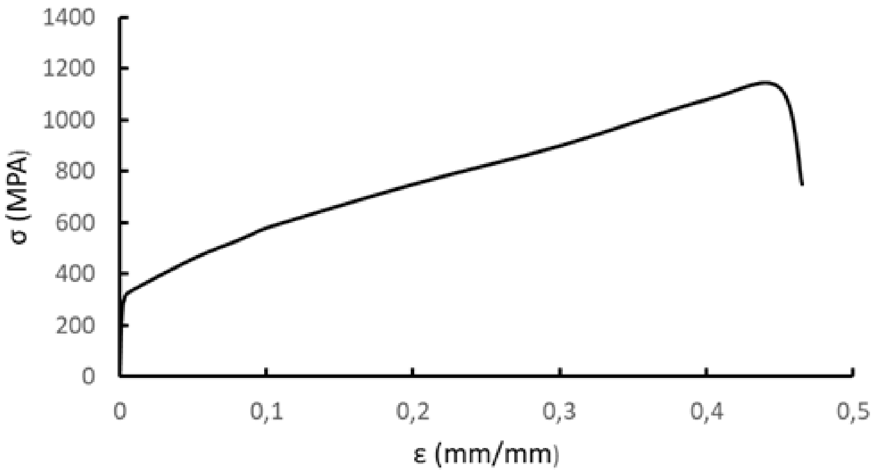


Figure 6. Trust Stress-Strain diagram AISI 304 (thickness=1mm)

In Table 1 geometric parameters of process are shown, and Table 2 shows the elastoplastic properties of material, initial and final bending angles and results obtained with the model application. The experimental results obtained by the author include initial and final bending angles. The initial bending radius was calculated using the equation:

$$R_i = \frac{s\tan\theta + \frac{t}{2} - P}{sec\theta - 1} \quad (9)$$

Where s is the distance between supports, t is the sheet thickness and p is the displacement of the die.

Table 1. Geometric parameters from A-304 sheet bending

rp (mm)	s (mm)	rd (mm)
0.8	54	2

The change in bending angle and supplementary angle is equal in magnitude.

$$\Delta\theta = -\Delta\beta \quad (22)$$

Elastic recovery was evaluated with the proposed model:

$$|\Delta\beta| = \left|\beta_i\left[8\left(\frac{R\sigma_y}{tE}\right)^3\left(\frac{n+2-3a^2}{(n+2)}\right)+\frac{6}{2^n(n+2)a^n}\left(\frac{R\sigma_y}{tE}\right)^{1-n}\right]\right| \quad (23)$$

Table 2. Elastoplastic properties and results from elastic recovery model for stainless Steel AISI 304

t (mm)	E (GPa)	σy (MPa)	K (MPa)	n	Ai (°)	Af (°)	SB (°)	Ri (mm)	SB-NM (°)	% Error	% Error
1	215.4	311.6	1491	0.419	33	20	13	54	14	4.5	4.0
					38	24	14	46	15	1.9	
					44	28	16	41	16	3.6	
					50	33	17	37	17	0.3	
					55	38	17	34	18	3.3	
					62	42	20	32	19	2.9	
					67	47	21	30	20	2.3	
					80	58	22	28	23	5.3	
					88	66	22	27	25	13.1	
1.5	224.4	292.5	1423	0.428	46	32	14	40	12	9.2	5.0
					53	38	15	36	13	9.1	
					64	48	16	31	15	7.8	
					68	51	17	30	15	7.1	
					74	57	17	29	17	5.3	
					79	61	18	28	17	3.7	
					83	65	18	27	18	1.9	
					88	69	19	27	19	0.3	
					91	72	19	27	20	2.1	
2	244.2	292.7	1552	0.435	43	32	11	50	10	12.1	15.0
					51	38	13	43	11	14.4	
					63	48	15	37	12	16.2	
					66	51	15	32	13	16.3	
					73	57	16	31	14	16.1	
					78	61	17	29	14	15.7	
					82	65	17	28	15	14.9	
					87	69	18	28	15	13.9	
					90	72	18	27	16	12.9	

Ai: initial bending angle; Af: final bending angle; SB: real springback; SB-NM: springback predicted.

It could be observed that with increasing sheet thickness, the error in the estimation of recovery increases. A lower thickness is consistent with the model used since it is not affected by the different hardening zones inside the material, and there are fewer discrepancies with the straight segment and arc-circle models used.

For small displacements and with the radius of the punch less than the thickness of the sheet, the process is more like 3-point bending. If the sheet thickness increases, the effect is expected to be like that of increasing the punch radius, however the equation used to estimate the radius of curvature does not directly include this effect.

The model with the data proposed by Vorkok et al. was also used in their work on elastic recovery of high strength steels with high punch radius [26].

The material used was Weldox 1100. Fig. 7 and tables 3 and 4 specify the properties and characteristics of the material and the process.

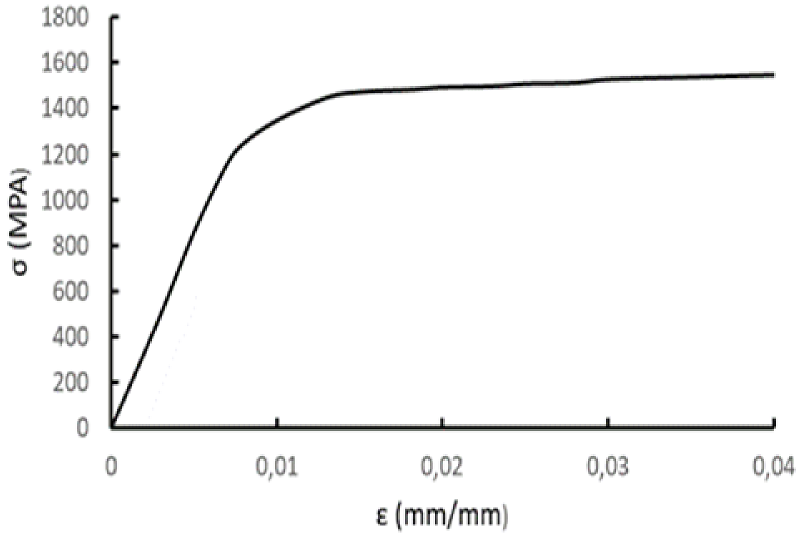


Figure 7. Trust Stress-Strain Diagram – Weldox 1100

Table 3. Geometric parameters of Weldox 1100 sheet bending

rp(mm)	s(mm)	rd(mm)
40	54	2

Table 4. Elastoplastic properties and results of the Weldox 1100 Steel recovery model

T (mm)	E (GPa)	σy (MPa)	K (MPa)	n	Ai (°)	Af (°)	SB (°)	Ri (mm)	SB-NM (°)	% Error
4.15	175	1078.7	2627.5	0.1496	91	68	23	41	21	4.0

It could be seen that the calceulated radius of curvature coincides quite well with the radius of the punch. The bending angle is high, so the punch tends eto embrace the sheet. The model estimates the elastic recovery with a suitable error. The authors simulated the springback process by means of the finite element and obtained errors between 2.1% and 4.7% with respect to the experimental data.

4 CONCLUSIONS

The proposed method of estimating elastic recovery shows adequate behavior for low sheet thicknesses, (1.0 mm and 1.5 mm) and bending angles between 44° and 90°, using a tool radius of 0.8 mm. For larger thicknesses the error is not acceptable.

Comparison against a larger data set is required, but the results obtained for high forming angles and large punch radius have an error like the obtained by finite element simulations.

The obtained model could be implemented easily because it only requires basic mechanic properties and basic process parameters (sheet thickness, clearance, punch radius, support roller radius, and punch displacement).

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