Istraživanja i projektovanja za privredu
ISSN 1451-4117
DOI:10.5937/jaes0-41482
www.engineeringscience.rs

publishing

Journal of Applied Engineering Science
Vol. 21, No. 1, 2023
Original Scientific Paper
Paper number: 21(2023)1, 1079, 361-373

COMBINATION OF DOE AND PIV METHODS FOR MULTI-CRITERIA DECISION MAKING

Do Duc Trung*, Tran Ngoc Tan

Faculty of Mechanical Engineering, Hanoi University of Industry, Hanoi, Vietnam

** doductrung@haui.edu.vn*

When performing the multi-criteria decision making to choose the best solution, if some solutions are removed from the list of solutions or some solutions are added to the list of solutions, the decision making must be re-performed from the begining. This study proposes a new method to remove this limitation. The combination of the DOE (Design Of Experimental) method and PIV (Proximity Indexed Value) method is proposed in this paper. This combination is used to build the relationship between the scores of the solutions and the criteria. When the list of solution to be ranked has been removed or have been added some solutions, the ranking of some solutions only needs to use this relationship without having to recalculate from the beginning. Four different examples were applied to evaluate the effectiveness of the proposed method. The obtained results show that the proposed method ensures the required accuracy as well as its outstanding advantages. The limitations of the proposed method that need to be overcome are also pointed out at the end of this paper.

Keywords: MCDM, PIV method, DEO method, Combination of DOE and PIV

1 INTRODUCTION

MCDM (Multi-Criteria Decision-Making) is the ranking of available solutions to choose out the best solution [1, 2]. There are hundreds of different MCDM methods have been proposed by scientists [3, 4]. Although each method has different features, they all have a limitation which is the phenomenon of rank reversal [5]. Rank reversal is a phenomenon that the rankings of alternatives are reversed when adding one or more alternatives to the list of solutions that needs to be ranked.

PIV method is a MCDM method found in 2018 [6]. This method has the advantage of minimizing the rank reversal phenomenon [7]. With this advantage, it is strongly exploited to rank the solutions in many different cases such as ranking the online learning websites [8], choosing the materials to manufacture some parts in F1 racing cars [9], choosing natural fibers for car roofs [10], evaluating the logistics performance index of EU countries [11], selecting the additives for a material production process [12], selecting the turning processes [13, 14], selection of selection of grinding options [15], choosing renewable energy source [16], evaluating the financial performance of companies [17], evaluating the information and communication technology developments of G7 Countries [18], selecting the location for construction of the storehouse in logistics [19], selecting the location for construction of textile facilities in Sivas province in Turkey [20], selecting the suppliers [21], choosing a house design model to minimize the consumption of materials when building [22], selecting offshore location for construction of Offshore wind farm (OWF) in the energy industry [23], ect. Thus, it can be seen that PIV has been successfully used in many cases in many different fields.

However, like all other MCDM methods, using PIV method, when several solutions are removed or added to the list of solutions, the ranking of the solutions must be done from the beginning [24, 25]. Some scientists have proposed a new approach to solve this problem. The way that is proposed is a combination of the DOE method with a certain MCDM method such as the combination of the DOE method and the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method [24], the combination of the DOE method with the MABAC (Multi-Attributive Border Approximation Area Comparison) method [25]. That combination aims to create the relationship between the scores of the solutions and the criteria. Then the ranking of the solutions in the list of solutions (after one solution is added or removed from the the list) simply uses this relationship without having to be done from the beginning. These combinations have confirmed the advantages in comparing to the case using only a certain MCDM method [24, 25]. Thus, it seems that the combination of DOE with a certain MCDM method has promoted its advantages when a certain solution is removed or added to the list of the solutions. However, the number of studies of this type is still very few. If it is verified to be successful, the combination of the DOE method with any MCDM method would also be recognized as the contributions to this research direction. With the advantages of the PIV method as mentioned above, in this study, it will be chosen to combine with the DOE method. The steps to rank the solutions according to the PIV method are presented in section 2 of this paper. The combination of DOE method and PIV method is presented in section 3. Section 4 presents four different examples to evaluate the proposed method. The drawn conclusions and the further works are the final contents of this study.

2 PIV METHOD

The ranking steps of the solutions to select the best solution according to the PIV method is presented as follows [6]:

Step 1: Building the decision matrix in the form of Table 1.

Table 1. Decision matrix

No.	C_1	C_2	C_j	C_{n-1}	C_n
A_1	x_{11}	x_{12}	x_{1j}	x_{1n-1}	x_{1n}
A_2	x_{21}	x_{22}	x_{2j}	x_{2n-1}	x_{2n}
A_i	x_{i1}	x_{i2}	x_{ij}	x_{in-1}	x_{in}
A_{m-1}	x_{m-11}	x_{m-12}	x_{m-1j}	x_{m-1n-1}	x_{m-1n}
A_m	x_{m1}	x_{m2}	x_{mj}	x_{mn-1}	x_{mn}

where: n is the number of criteria, m is the solution, xij is the value of criterion j at solution i.

Step 2: Calculating the normalized values by Eq. (1).

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad (1)$$

Step 3: Calculating the normalized values with considering the weights of the criteria by Eq. (2).

$$v_{ij} = w_j \times n_{ij} \quad (2)$$

where w_j is the weight of the creterion *j*.

Step 4: Evaluate the weighted asymptote index (u_i) according to the following Equations.

- For the larger the better criteria.

$$u_i = v_{\max} - v_i \quad (3)$$

- For the smaller the better criteria.

$$u_i = v_i - v_{\min} \quad (4)$$

Step 5: Calculate the scores for the solutions by Eq. (5).

$$S_i = \sum_{j=1}^{n} u_i \quad (5)$$

Step 6: Rank the solutions according to the principle that the best solution is the one with the smallest value of Si.

3 PROPOSED METHOD: COMBINATION OF DOE AND PIV METHOD

For the proposed method, the ranking of solutions is performed in the following orders:

Step 1: Similar to the step 1 of PIV method.

Step 2: Determine the minimum and maximum values of each criterion as in the Table 2.

Table 2. Minimum and maximum values of each criterion

No.	Min	Max
C_1	$Min(C_{i1})$, i = 1÷m	$Max(C_{i1})$, i = 1÷m
C_2	$Min(C_{i2})$, i = 1÷m	$Max(C_{i2})$, i = 1÷m
C_j	$Min(C_{ij})$, i = 1÷m	$Max(C_{ij})$, i = 1÷m
C_n	$Min(C_{in})$, i = 1÷m	$Max(C_{in})$, i = 1÷m

Step 3: Build an experimental matrix with the input parameters as the criteria to evaluate the solutions. In which, the minimum value and the maximum value of each criterion were determined in Table 2.

Step 4: Calculate the scores for the experiments by PIV method. In essence, this step is a synthesis from step 2 to step 5 of the PIV method.

Step 6: Build the relationship between the experiment scores and the criteria. This relationship is called the regression equation.

Step 7: Use the regression equation to calculate the scores for the solution that needs to be ranked.

Step 8: Rank the solutions according to the principle that the best solution is the one with the smallest score, and vice versa.

Note that, when having once the regression equation, ranking the solutions is simply using the regression equation to calculate the scores for the solutions, i.e. just using step 7 and step 8. When one solution is added to the list, the ranking of the solutions only needs to use the regression equation instead of having to recalculate from the beginning. Some examples are presented in the next sections of this study for the purpose of evaluating the effectiveness of the proposed method.

4 APLICATION EXAMPLES

In this section, four examples will be peformed to evaluate the effectiveness of the proposed method. Four examples were applied in four different fields. The difference in the four examples is also reflected in the number of solutions and the number of criteria. The first three examples are carried out to evaluate the effectiveness of the proposed method by comparing the identified best solution by the proposed method with the identified best one by other MCDM methods. In the first three examples, sensitivities will also analyze in different scenarios. The form of creating the scenarios in each example is also not the same. Many differences were performed in these three examples so that the most general conclusions can be drawn. In the last example, the ranking of the solutions was conducted with the created hypothetical cases when one solution was added to the list of solutions.

Example 1

This example refered the data on nine solutions of a milling process [26]. Surface Roughness (Ra) and material removal rate (MRR) that are the two criteria were used to evaluate each solution. The first criterion is the smaller the better criterion, the other is the bigger the better criterion. The data for example 1 is presented in Table 3. Determining one solution of nine that simultaneously ensuring the minimum Ra and the maximum MRR is the task of the multi-criteria decision making. Five methods including PIV, TOPSIS, EDAS (Evaluation Based on Distance from Average Solution), MARCOS (Measurement Alternatives and Ranking according to COmpromise Solution), and MOORA (Multiobjective Optimization on the basis of Ratio Analysis), were also used to rank the solutions. When using these five methods, the weights of the two criteria Ra and MRR were determined to be 0.6641 and 0.3359, respectively [26].

Table 3. The data of example 1 [26]

No.	Ra	MRR
A1	0.571	1120
A2	0.818	2400
A3	0.493	4160
A4	0.426	1680
A5	0.470	3200
A6	0.851	2080
A7	0.437	2240
A8	0.694	1600
A9	0.717	3120

Performing the multi-criteria decision making according to the proposed method as follows:

- Build the decision matrix, as presented in table 3.
- The minimum and maximum values of Ra are 0.426 and 0.851, respectively; The minimum and maximum values of the MRR are 1120 and 4160, respectively.
- Build an experimental matrix with two main input parameters including Ra and MRR. The experimental matrix form used is the full two-level form (2k), where k is the number of input parameters [24, 25]. Here, the two value levels of each criterion are understood as the maximum and the minimum values of that criterion. In this example, with two input parameters, the experimental matrix consists of four experiments as presented in Table 4.

Table 4. The experimental matrix and the score of each experiment

Exp.	Ra	MRR	Si
1	0.851	1120	0.37731
2	0.426	1120	0.16760
3	0.426	4160	0.00000
4	0.851	4160	0.20971

- Calculate the scores for the four experiments in Table 4 using the steps from step 2 to step 5 of the PIV method, where the weights of the criteria were chosen as in reference [26]. The scores of each experiment were also summarized in Table 4.
- From the data in Table 4, the relationship between the scores of the experiments and the criteria was built as a mathematical function called the regression equation as Eq. (6). All three coefficients including R-Sq, R-Sq(pred), and R-Sq(adj) of this regression equation are equal to 1. It shows that the regression equation has very high accuracy [27, 28].

$$S = 0.01914 + 0.49343 \cdot R_a - 5.51323 \cdot 10^{-5} \cdot MRR \quad (6)$$

- The regression equation (6) was used to calculate the scores for the nine milling solutions in Table 3, the calculated results are presented in Table 5.
- Ranking the solutions according to their score values, the ranked results are also summarized in Table 5.

Table 5. Scores and ranking of the solutions

No.	Ra	MRR	Si	Rank
A1	0.571	1120	0.23914	6
A2	0.818	2400	0.29045	8
A3	0.493	4160	0.03305	1
A4	0.426	1680	0.13672	4
A5	0.470	3200	0.07463	2
A6	0.851	2080	0.32437	9
A7	0.437	2240	0.11127	3
A8	0.694	1600	0.27337	7
A9	0.717	3120	0.20092	5

Thus, based on the data in Table 5, the ranking orders of the solutions according to the proposed method are: A3 > A5 > A7 > A4 > A9 > A1 > A8 > A2 > A6.

The ranked results of the solutions by different MCDM methods are presented in Table 6.

Table 6. Ranking of the solutions using different methods

No.	*Rank [26]*					Proposed method
	EDAS	*MARCOS*	*PIV*	*MOORA*	*TOPSIS*	
A1	9	6	6	6	6	6
A2	6	7	8	8	8	8
A3	1	1	1	1	1	1
A4	4	4	4	4	4	4
A5	2	2	2	2	2	2
A6	8	9	9	9	9	9
A7	3	3	3	3	3	3
A8	7	8	7	7	7	7
A9	5	5	5	5	5	5

The results in Table 6 show that the solutions from rank 1 to rank 5 are completely coincidental when using all six different MCDM methods; The solutions from rank 6 to rank 9 also have a great degree of similarity when using the proposed method in comparing to the other five methods. In other words, determining the best solution according to the proposed method were confirmed to be successful. However, to evaluate the effectiveness of the proposed method, the final step is sensitivity analysis [29]. For sensitivity analysis, the scenarios can be generated by changing the weights of the criteria, changing the criterion type, or removing one solution from the list of solutions [30, 31]. The first approach (changing the weight of the criteria) has been used in several studies [30, 31]. In addition to the used weight set (scenario S4), four randomly selected weight sets were generated as presented in table 7.

Table 7. Weight of criteria in different scenarios

Criteria	S1	S2	S3	S4	S5
Ra	0.2	0.4	0.6	0.6641	0.8

Criteria	S1	S2	S3	S4	S5
MRR	0.8	0.6	0.4	0.3359	0.2

Using the weights of the criteria in the scenarios S1, S2, S3, S5, the regression equations were built as the Eq. (7), Eq. (8), Eq. (9), and Eq. (10).

$$S = 0.48293 + 0.14860 \cdot R_a - 1.31306 \cdot 10^{-4} \cdot MRR \quad (7)$$

$$S = 0.28306 + 0.29720 \cdot R_a - 9.84798 \cdot 10^{-5} \cdot MRR \quad (8)$$

$$S = 0.08320 + 0.44581 \cdot R_a 6.56532 -\cdot 10^{-5} \cdot MRR \quad (9)$$

$$S = -0.11666 + 0.59441 \cdot R_a - 3.28266 \cdot 10^{-5} \cdot MRR \quad (10)$$

Figure 1 presents the ranked results of the solutions in five different scenarios. It can be seen that the best solution (A3) is consistently determined for all five different scenarios. In other words, in this example, the best solution that was determined by the proposed method was not affected by the weights of the criteria.

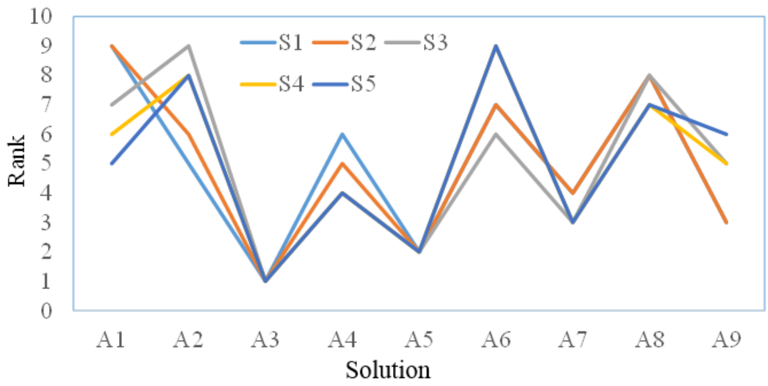

Fig. 1. Ranking of solutions in different scenarios

Finally, we can conclude that the proposed method was confirmed to be successful when applying in this example.

Example 2

This example refered data on three suppliers in the Indian steel industry [32]. Six criteria were used to evaluate each supplier including Technological Innovation (TI), Competitive Advantage (CA), ProcessS Innovation (PSi), Managerial Innovation (MI), Productive Innovation (PTI), and Greening the Supplier (GS). All six of the above criteria are the bigger the better criterion. The data for the three suppliers is shown in Table 8. Determining the solution that simultaneously ensures all six criteria is considered the greatest is the task of multi-criteria decision making. Six *MCDM* methods including *MABAC*, *MARCOS*, *CODAS* (COmbinative Distance-based ASsessment), *EDAS* (Evaluation Based on Distance from Average Solution), *SAW* (Simple Additive Weighting), and *WASPAS* (Weighted Aggregates Sum Product ASsessmen) were used to perform this task with the weights of the criteria TI, CA, PSI, MIN, PTI, and GS to be 0.395, 0.197, 0.132, 0.099, 0.099, and 0.079, respectively [32].

Table 8. Data of example 2 [32]

No.	TI	CA	PSI	MI	PTI	GS
A1	57	55	45	52	62	50
A2	52	58	47	55	57	51
A3	60	57	53	51	62	51

The proposed method is used for multi-criteria decision making in the following order:

- Building a decision matrix, as presented in table 8.
- The minimum and maximum values of the six criteria are determined as shown in Table 9.
- Build an experimental matrix with six main input parameters, which are six criteria, two levels (minimum and maximum values) of each criterion are presented in Table 9. The experimental matrix includes sixty-four experiments (because *k = 6*), and a part of this matrix is shown in Table 10.

Table 9. Minimum and Maximum values of each creterion in example 2

Criteria	TI	CA	PSI	MI	PTI	GS
Min	52	55	45	51	57	50
Max	60	58	53	55	62	51

Table 10. A part of experimental matrix and score of each experiment

Exp.	TI	CA	PSI	MI	PTI	GS	Si
1	60	58	45	55	62	51	0.26312
2	60	55	45	55	62	51	0.41082
3	60	58	53	51	57	51	0.22257
4	52	55	53	51	57	50	1.17801
5	52	55	53	55	62	50	0.95544
…	…	…	…	…	…	…	…
63	52	58	45	51	62	51	1.15005
64	52	55	45	51	62	50	1.31749

- Five steps (from step 2 to step 5) of the *PIV* method were again used to calculate the scores for the experiments in Table 10, the results were also summarized in this table.
- The regression equation (11) that presents the relationship between scores of experiments and criteria was built based on the data of table 10. All three coefficients of this equation that include R-*Sq, R-Sq(pred),* and *R-Sq(adj)* are also equal to 1, which means that this equation also has very high accuracy.

$$S = 14.4093 - 0.09849 \cdot TI - 0.04923 \cdot CA - 0.03289 \cdot PSI$$
$$- 0.02473 \cdot MI - 0.02472 \cdot PTI - 0.01974 \cdot GS \quad (11)$$

- Eq. (11) is used to calculate the scores for the three solutions in Table 8, the results are presented in Table 11.
- Ranking the solutions according to their score value, the ranked results were also summarized in Table 11.

Table 11. Score and ranking of the solutions in example 2

No.	TI	CA	PSI	MI	PTI	GS	Si	Rank
A1	57	55	45	52	62	50	0.80196	2
A2	52	58	47	55	57	51	1.11060	3
A3	60	57	53	51	62	51	0.14990	1

The ranked results of the solutions by different methods are summarized in Table 12.

Table 12. Ranking of the solutions using different methods for example 2

	Rank [32]						Proposed method
No.	MABAC	CODAS	EDAS	MARCOS	SAW	WASPAS	
A1	3	2	2	2	2	2	2
A2	2	3	3	3	3	3	3
A3	1	1	1	1	1	1	1

Thus, the ranked results of the solutions according to the proposed method are completely consistent with the ranked results when using five other methods (including *CODAS*, *EDAS*, *MARCOS*, *SAW*, and *WAPPAS*). The special thing is that all seven used methods identified A3 as the best solution. In other words, the proposed method is confirmed to have succeeded in determining the best solution in this example.

The sensitivity analysis was again performed with different scenarios. Twelve different scenarios were used for sensitivity analysis using different sets of weights, of which one set (S5) was just used above. In the scenarios S1, S2, S3, S4, and S6, the weights of the criteria are chosen at random within a relatively narrow range. In contrast, in the S7 to S12 scenario, the weights of the criteria are intentionally assigned. In each of those scenarios, one criterion will be assigned a weight by five times of the remaining criteria weights. For example, in scenario S7, the weight of TI is 0.5, and the weights of the remaining criteria are equal to 0.1. Similarly, in each of the remaining scenarios (S8, S9, S10, S11, and S12), there is a criterion whose weight is five times the weight of the other five criteria. Intentionally creating different values of the weights (both random and intentional) makes the problem to enrich more. The weights of the criteria for the twelve different scenarios are summarized in Table 13.

Table 13. Weight of criteria in different scenarios for example 2

Criteria	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
TI	0.167	0.211	0.234	0.300	0.395	0.420	0.5	0.1	0.1	0.1	0.1	0.1
CA	0.167	0.152	0.173	0.109	0.197	0.101	0.1	0.5	0.1	0.1	0.1	0.1
PSI	0.167	0.251	0.256	0.112	0.132	0.112	0.1	0.1	0.5	0.1	0.1	0.1
MI	0.167	0.114	0.069	0.233	0.099	0.146	0.1	0.1	0.1	0.5	0.1	0.1
PTI	0.167	0.126	0.088	0.111	0.099	0.152	0.1	0.1	0.1	0.1	0.5	0.1
GS	0.167	0.146	0.180	0.135	0.079	0.069	0.1	0.1	0.1	0.1	0.1	0.5

In addition to the S5 scenario as performed above, using the remaining eleven regression equations were also built as follows:

- *Using S1*

$$S = 14.1065 - 0.04156 \cdot TI - 0.04165 \cdot CA - 0.04152 \cdot PSI - 0.04163 \cdot MI - 0.04162 \cdot PTI - 0.04166 \cdot GS \quad (12)$$

- *Using S2*

$$S = 14.0540 - 0.05261 \cdot TI - 0.03798 \cdot CA - 0.06254 \cdot PSI - 0.02847 \cdot MI - 0.03147 \cdot PTI - 0.03649 \cdot GS \quad (13)$$

- *Using S3*

$$S = 13.9952 - 0.05835 \cdot TI - 0.04323 \cdot CA - 0.06378 \cdot PSI - 0.01723 \cdot MI - 0.02198 \cdot PTI - 0.04499 \cdot GS \quad (14)$$

- *Using S4*

$$S = 14.1892 - 0.07480 \cdot TI - 0.02724 \cdot CA - 0.02790 \cdot PSI - 0.05820 \cdot MI - 0.02772 \cdot PTI - 0.03374 \cdot GS \quad (15)$$

- *Using S6*

$$S = 14.4668 - 0.10473 \cdot TI - 0.02524 \cdot CA - 0.02790 \cdot PSI - 0.03647 \cdot MI - 0.03796 \cdot PTI - 0.01724 \cdot GS \quad (16)$$

- *Using S7*

$$S = 14.4486 - 0.12468 \cdot TI - 0.02499 \cdot CA - 0.02491 \cdot PSI - 0.02498 \cdot MI - 0.02497 \cdot PTI - 0.02499 \cdot GS \quad (17)$$

- *Using S8*

$$S = 14.2618 - 0.02493 \cdot TI - 0.12495 \cdot CA - 0.02491 \cdot PSI - 0.02498 \cdot MI - 0.02497 \cdot PTI - 0.02499 \cdot GS \quad (18)$$

- *Using S9*

$$S = 13.7463 - 0.02493 \cdot TI - 0.02499 \cdot CA - 0.12458 \cdot PSI - 0.02498 \cdot MI - 0.02497 \cdot PTI - 0.02499 \cdot GS \quad (19)$$

– *Using S10*

$$S = 13.9600 - 0.02493 \cdot TI - 0.02499 \cdot CA - 0.02491 \cdot PSI - 0.12491 \cdot MI - 0.02497 \cdot PTI - 0.02499 \cdot GS \quad (20)$$

– *Using S11*

$$S = 14.6584 - 0.02493 \cdot TI - 0.02499 \cdot CA - 0.02491 \cdot PSI - 0.02498 \cdot MI - 0.12489 \cdot PTI - 0.02499 \cdot GS \quad (21)$$

– *Using S12*

$$S = 13.5636 - 0.02493 \cdot TI - 0.02499 \cdot CA - 0.02491 \cdot PSI - 0.02498 \cdot MI - 0.02497 \cdot PTI - 0.12499 \cdot GS \quad (22)$$

Figure 2 presents the ranked results of the solutions in twelve different scenarios. We find that solution A3 is determined to be the best for eleven different scenarios (except S10). It can be determined that A3 is the best solution indeed, it is similar to those ones when using the other six methods (including *MABAC*, *CODAS*, *EDAS*, *MARCOS*, *SAW*, and *WASPAS*) [32]. In other words, the best solution that was determined by the proposed method changed little when using different sets of weights.

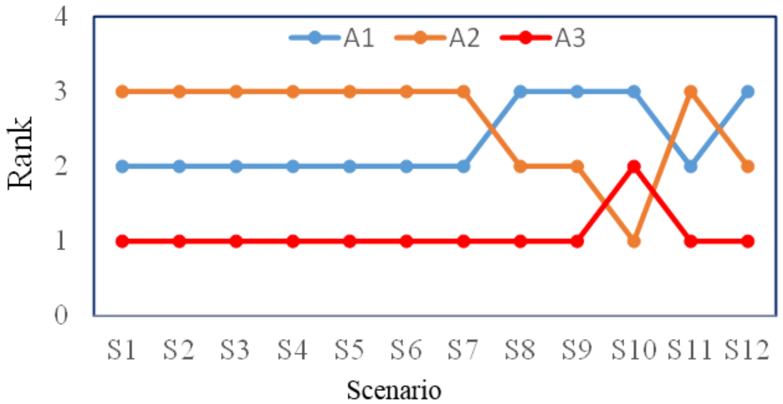

Fig. 2. Ranking of solutions in different scenarios for example 2

Finally, we can conclude that the proposed method was also confirmed to be successful when applying in this example.

Example 3

Table 14 presents the data on eight logistics service providers [33]. In which, seventeen criteria are used to evaluate each solution including Quality (Q), Lead time (L), Cost (C), Delivery and services (D), RElationship (RE), Innovativeness (I), Pollution controls (P), Resource Consumption (RC), Remanufacturing and Reuse (RR), Green technology capability (G), Environmental management system (E), Health and safety (H), Employment Stability (ES), Customer Satisfaction (CS)), Reputation (R), Respect for the Policy (RP), and Contractual Stakeholders Influence (CSI). Where L, C, and RC are the three smaller the better criteria, the remaining fourteen criteria are the larger the better criteria. Six methods including *TOPSIS*, *VIKOR*, *WASPAS, MULTIMOORA*, *ARAS (*Additive Ratio Assessment), and *MACONT (*Mixed Aggregation by COmprehensive Normalization Technique) were used for multi-criteria decision making. The weights of the criteria are presented in the last row of table 14 [33]

Table 14. Data of example 3 [33]

No.	Q	L	C	D	RE	I	P	RC	RR	G	E	H	ES	CS	R	RP	CSI
A1	22	22	850	34	3.5	13	17	11039	46	6	7	27	3.8	78	5	57	3.4
A2	34	38	1450	67	7.9	6	4	14326	37	3	2	63	5.9	89	6	66	6.8
A3	27	30	1068	29	5	21	11	12765	41	5	4	64	7.3	80	4	74	4.3
A4	19	41	729	37	4.3	26	9	10343	16	7	5	82	4.1	67	3	85	3.7
A5	15	76	697	45	2.8	8	13	6390	32	4	3	45	6.3	56	4	90	3.2
A6	32	25	1371	74	6.7	5	8	15789	24	2	4	38	5.2	92	7	69	7.5
A7	28	68	1190	63	5.4	23	14	13270	62	8	2	50	6.4	82	5	73	4.6
A8	17	64	798	42	3.1	19	16	8356	58	6	3	57	4.7	34	8	92	3.9
Weight	0.048	0.067	0.085	0.026	0.017	0.034	0.098	0.087	0.065	0.113	0.046	0.079	0.047	0.025	0.072	0.08	0.011

Performing by the same way as in the two above examples, the solutions were ranked according to the proposed method. Table 15 presents the ranked results of the solutions when using the proposed method and when using other MCDM methods.

Table 15. Ranking of solutions using different methods for example 3

No.	Rank [33]						Proposed method
	MACONT	TOPSIS	VIKOR	WASPAS	ARAS	MULTIMOORA	
A1	4	2	5	3	3	4	3
A2	8	8	7	8	8	7	7
A3	6	5	3	4	5	3	5
A4	3	4	4	5	4	5	4
A5	5	6	6	6	6	6	6
A6	7	7	8	7	7	8	8
A7	2	3	2	2	2	2	2
A8	1	1	1	1	1	1	1

The results in Table 15 show that when using the proposed method, the rank 1 and rank 2 of the solutions completely match the ranked results when using the other six methods. It means that the determination of the best solution was successfully performed using the proposed method.

Three different scenarios were used for the sensitivity analysis including:

- The first scenario (S1) that is the one is just done above.
- In the S1 scenario, the weight of G is the largest, otherwise the weight of the CSI is the smallest. In the second scenario (S2), the weights of the G and CSI criteria were intentionally changed. Specifically, in the S2 scenario, the weights of these two criteria were swapped to those ones in comparing with the S1 scenario.
- In scenario S1, in three the smaller the better criteria (L, C, and RC), L is the criterion with the smallest weight; in the remaining criteria (the bigger the better the criteria), G is the criterion with the largest weight. In the S3 scenario, weights of these two criteria were swapped to each other. Specifically, the weight of criterion L is chosen to be 0.113, and the weight of CSI is chosen to be 0.067.

Changing the weights of the criteria from the smallest to the largest and vice versa as above aims to produce the largest difference in the weights of the criteria. By this way, the obtained results in the sensitivity analysis will be as general as possible. The weights of the criteria in the three scenarios are presented in Table 16.

Table 16. Weight of the criteria in different scenarios for example 3

	Q	L	C	D	RE	I	P	RC	RR	G	E	H	ES	CS	R	RP	CSI
S1	0.048	0.067	0.085	0.026	0.017	0.034	0.098	0.087	0.065	0.113	0.046	0.079	0.047	0.025	0.072	0.08	0.011
S2	0.048	0.067	0.085	0.026	0.017	0.034	0.098	0.087	0.065	0.011	0.046	0.079	0.047	0.025	0.072	0.08	0.113
S3	0.048	0.113	0.085	0.026	0.017	0.034	0.098	0.087	0.065	0.067	0.046	0.079	0.047	0.025	0.072	0.08	0.011

Figure 3 presents a graph showing the ranking results of the solutions corresponding to three different scenarios. We find that, although the weights of the criteria in the scenarios were intentionally changed as analyzed above (from smallest to largest, vice versa), in all three cases, A8 is still determined to be the best solution. In other words, the proposed method always identifies the best solution in different scenarios.

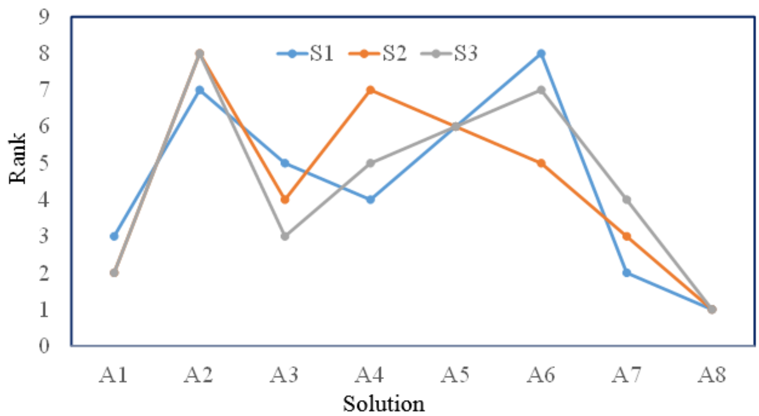

Fig. 3. Ranking of solutions in different scenarios for example 3

Finally, we also reach the conclusion that the proposed method was also confirmed to be successful when applying in this example.

There are many differences between three examples such as: belonging to different fields, different number of solutions, very different number of criteria (there are two criteria in example 1, there are six criteria in example 2, and there are seventeen criteria in example 3), the number of scenarios is different, the creation of scenarios is also conducted according to different intentions. However, in all those three examples we find that:

- The determined best solution by the proposed method is always the same as the best solution when using other MCDM methods.
- The weights of the criteria have little influence on the ranking of the best solution. Specifically, in example 1 and example 3, the best solution is determined to be exactly the same that one when using different scenarios. In example 2, the best solution was determined to be consistent when using 11/12 different scenarios.

All of these confirmed the accuracy of the proposed method. In addition, a special advantage of the proposed method is that it is possible to rank the solutions when one/several solutions are added to the list of solutions. An example that is performed shortly below will present lighter on this issue.

Example 4

The data on the seven types of robots in Table 17 were used in this example [6]. The five criteria for evaluating each type of robot include Load capacity (L), Repeatability (R), Maximum tip Speed (MS), Memory Capacity in (MC), and Manipulator Reach (MR). In which, R is the smaller the better criterion, the remaining four criteria are the form of the larger the better criterions. *TOPSIS* and *PIV* are two methods that were also used for multi-criteria decision making with weights of criteria as shown in the last row of table 17 [6].

Table 17. Data of example 4 [6]

No.	L	R	MS	MC	MR
A1	60	2.5	2540	500	990
A2	6.35	6.667	1016	3000	1041
A3	6.8	10	1727	1500	1676
A4	10	5	1000	2000	965
A5	2.5	9.8	560	500	915
A6	4.5	12.5	1016	350	508
A7	3	10	1778	1000	920
Weight	0.1761	0.2042	0.2668	0.1243	0.2286

To demonstrate the power of the proposed method, it is supposed that initially only the first six solutions need to be ranked (from A1 to A6). Performing the steps as in the 3 above examples, the regression equation was built as in Eq. (23). Note that this equation is built when only six solutions are considered.

$$S = 0.18466 - 7.33114 \cdot 10^{-4} \cdot L + 0.004 \cdot R - 2.56440 \cdot 10^{-5} \cdot MS$$
$$-1.02886 \cdot 10^{-5} \cdot MC - 3.26330 \cdot 10^{-5} \cdot MR \quad (23)$$

Using Eq. (23), the scores for the solutions from A1 to A6 are calculated and these solutions are ranked. However, this is only a hypothetical case, and in fact, we need to rank for seven solutions (from A1 to A7). It means that, A7 must be returned to the list of solutions. Now, Eq. (23) will be used to calculate the scores for all seven solutions and the rank them. The ranked results of seven solutions by the proposed method and some other *MCDM* methods are presented in Table 18.

Table 18. Ranking of solutions using different methods for example 4

No.	Rank [6]		Proposed method
	TOPSIS	PIV	
A1	1	1	1
A2	4	4	3
A3	2	2	2
A4	6	5	4
A5	7	7	6
A6	5	6	7
A7	3	3	5

The data in Table 18 show that when using the proposed method, the best and the rank 2 solutions are completely consistent to those ones when using the other two methods (*TOPSIS* and *PIV*). It means that the decision-making task was successfully performed using the proposed method. From this point, it is clear that the equation (23) is constructed with considering only six solutions, but it can be used to rank all seven solutions. It means that when one solution is added to the list of solutions, it is only necessary to use the equation (23) to calculate the scores for the solutions, not to perform the calculation from the begining like the current *MCDM* methods. This is considered an outstanding advantage of the combination of *DOE* and *PIV* methods.

After the proposed method was used in four above examples, we find that:

1. This method always determines the best solution equivalent to other *MCDM* methods.
2. The identified best solution by this method is minimally influenced by the weights of the criteria.
3. In particular, this method can quickly rank the added solution to the list of the solutions. This is much faster than when using current MCDM methods.

5 CONCLUSION

A new *MCDM* method has been proposed in this study. The proposed method is a combination of *DOE* and *PIV* methods. The effectiveness of the proposed method was tested through its application to multi-criteria decision making in four different cases. The results show that: this method always determines the best solution like other *MCDM* methods; the weights of the criteria have little effect on the best solution that was detemined by proposed method; In particular, this method can be used to quickly rank when one solution is added to the list of solutions.

When one solution is added to the list of solutions that needs to be ranked, if in a certain criterion, the minimum/maximum value of that solution is less than/greater than the values of all previous solutions, the regression equation cannot be used to calculate the score for the added solution. This is considered as the biggest limitation of the proposed method in this study, and it needs to be resolved as soon as possible to complete this method.

The combination of the *DOE* method with other *MCDM* methods (*VIKOR*, *WASPAS*, *MACONT*, ect.) in the same way as in this study are also the works that should be developed and tested in the next studies.

6 REFERENCE

[1] Zizovic, M., Pamucar, D., Albijanic, M., Chatterjee, P., Pribicevic, I. (2020). Eliminating Rank Reversal Problem Using a New Multi-Attribute Model—The RAFSI Method. Methematics, vol. 8, no. 1015, 1-16, DOI: 10.3390/math8061015

[2] Tien, D. H., Trung, D. D., Thien, N. V. (2022). Comparison of multi-criteria decision making methods using the same data standardization method. Strojnícky časopis - Journal of Mechanical Engineering, vol. 72, no. 2, 2022, 57-72, DOI: 10.2478/scjme-2022-0016

[3] Dua, T. V. (2022). Application of the Collaborative unbiased rank list integration method to select the materials. Applied Engineering Letters, vol.7, no.4, 133-142, DOI: 10.18485/aeletters.2022.7.4.1

[4] Ardil, C. (2020). Aircraft Selection Process Using Preference Analysis for Reference Ideal Solution (PARIS). International Journal of Aerospace and Mechanical Engineering, vol. 14, no. 3, 89-90.

[5] Mufazzal, S., Muzakkir, S. M. (2018). A new multi-criterion decision making (MCDM) method based on proximity indexed value for minimizing rank reversals. Computers & Industrial Engineering, vol. 119, 427–438, DOI: 10.1016/j.cie.2018.03.045

[6] Mufazzal, S., Muzakkir, S. (2018). A New Multi-Criterion Decision Making (MCDM) Method Based on Proximity Indexed Value for Minimizing Rank Reversals. Computers & Industrial Engineering, vol. 2018, 1-39, 2018, DOI: 10.1016/j.cie.2018.03.045

[7] Yu, Y., Wu, S., Yu, J., Chen, H., Zeng, Q., Xu, Y., Ding, H. (2022). An integrated MCDM framework based on interval 2-tuple linguistic: A case of offshore wind farm site selection in China. Process Safety and Environmental Protection, vol. 164, 613-628, DOI: 10.1016/j.psep.2022.06.041

[8] Khan, N. Z., Ansari, T. S. A., Siddiquee, A. N., Khan, Z. A. (2019). Selection of E-learning websites using a novel Proximity Indexed Value (PIV) MCDM method. Journal of Computers in Education, 6, 241-256, DOI: 10.1007/s40692-019-00135-7

[9] Wakeel, S., Bingol, S., Bashir, M. N., Ahmad, S. (2020). Selection of sustainable material for the manufacturing of complex automotive products using a new hybrid Goal Programming Model for Best Worst Method–Proximity Indexed Value method. Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications, vol. 235, no. 2, 1-15, DOI: 10.1177/1464420720966347

[10] Wakeel, S., Bingol, S., Ahmad, S., Bashir, M. N., Emamat, M. S. M. M., Ding, Z., Fayaz, H. (2021). A New Hybrid LGPMBWM-PIV Method for Automotive Material Selection. Informatica, vol. 45, 105–115, DOI: 10.31449/inf.v45i1.3246

[11] Ulutas, A., Karakoy, C. (2019). An analysis of the logistics performance index of EU countries with an integrated MCDM model. Economics and Business Review, vol. 5, no. 4, 49-69, DOI: 10.18559/ebr.2019.4.3

[12] Raigar, J., Sharma, V. S., Srivastava, S., Chand, R., Singh, J. (2020). A decision support system for the selection of an additive manufacturing process using a new hybrid MCDM technique. Sadhana, vol. 45, no. 101, 1-14, DOI: 10.1007/s12046-020-01338-w

[13] Trung, D. D. (2021). A combination method for multi-criteria decision making problem in turning process. Manufacturing review, vol. 8, no. 26, 1-17, DOI: 10.1051/mfreview/2021024

[14] Trung, D. D. (2021). Application of TOPSIS an PIV Methods for Multi - Criteria Decision Making in Hard Turning Process. Journal of Machine Engineering, vol. 21, no. 4, 57–71, DOI: 10.36897/jme/142599

[15] Trung, D. D. (2021). The Combination of Taguchi – Entropy – WASPAS - PIV Methods for Multi-Criteria Decision Making when External Cylindrical Grinding of 65G Steel. Journal of Machine Engineering, vol. 21, no. 4, 90–105, DOI: 10.36897/jme/144260

[16] Goswam, S. S., Mohanty, S. K., Behera, D. K. (2022). Selection of a green renewable energy source in India with the help of MEREC integrated PIV MCDM tool, Materialstoday: Proceeding, vol. 52, 1153-1160, DOI: 10.1016/j.matpr.2021.11.019

[17] Ersoy, N. (2021). Application of the PIV method in the presence of negative data: an empirical example from a real-world case. Hitit Journal of Social Sciences, vol. 14, no. 2, 318-337, DOI: 10.17218/hititsbd.974522

[18] Ersoy, N. (2022). Comparative analysis of MCDM methods for the assessment of ICT development in G7 countries. Kafkas Universitesi Iktisadi ve Idari Bilimler Fakültesi Dergisi, vol. 25, 55-73, DOI: 10.36543/kauiibfd.2022.003

[19] Ulutas, A., Balo, F., Sua, L., Demir, E., Topal, A., Jakovljevic, V. (2021). A new integrated grey MCDM model: Case of warehouse location selection. Facta Universitatis, Series: Mechanical Engineering, vol. 19, no. 3, 515-535, DOI: 10.22190/FUME210424060U

[20] Ulutas, A., Karakus, C. B. (2021). Location selection for a textile manufacturing facility with GIS based on hybrid MCDM approach. Industria textilas, vol. 72, no. 2, 126-132, DOI: 10.35530/IT.072.02.1736

[21] Soni, P. K. (2021). Integrated Multi-Criteria Decision-Making Methods for Selection of Supply Chain Partner for Supply Chain Management. International Journal for Research in Applied Science & Engineering Technology, vol. 9, no. 12, 952-957.

[22] Shahid, M., Karimi, M. N. (2019). Multi-criteria decision-making approach for finding optimal energy efficient bulding model. Journal of Emerging Technologies and Innovative Research, vol. 6, no. 4, 175-183.

[23] Yu, Y., Wu, S., Yu, J., Chen, H., Zeng, Q., Xu, Y., Ding, H. (2022). An integrated MCDM framework based on interval 2-tuple linguistic: A case of offshore wind farm site selection in China. Process Safety and Environmental Protection, vol. 164, 613-628, DOI: 10.1016/j.psep.2022.06.041

[24] Wang, P., Zhu, Z., Huang, S. (2017). The use of improved TOPSIS method based on experimental design and Chebyshev regression in solving MCDM problems. Journal of Intelligent Manufacturing, vol. 28, 229–243, DOI: 10.1007/s10845-014-0973-9

[25] Chattopadhyay, R., Das, P. P., Chakraborty, S. (2022). Development of a rough-MABAC-DoE-Based metamodel for supplier selection in an iron and steel industry. Operational Research in Engineering Sciences: Theory and Applications, vol. 5, no. 1, 20-40, DOI: 10.31181/oresta190222046c

[26] Trung, D. D. (2021). Application of EDAS, MARCOS, TOPSIS, MOORA and PIV Methods for Multi-Criteria Decision Making in milling process. Strojnícky časopis - Journal of Mechanical Engineering, vol. 71, no. 2, 69-48, DOI: 10.2478/scjme-2021-0019

[27] Trung, D. D. (2021). Influence of Cutting Parameters on Surface Roughness in Grinding of 65G Steel. Tribology in Industry, vol. 43, no. 1, 167-176, DOI: 10.24874/ti.1009.11.20.01

[28] Dean, A., Voss, D., Draguljic, D. (2007). Design and Analysis of Experiments - Second Edition, Springer.

[29] Trung, D. D. (2022). Application of FUCA method for multi-criteria decision making in mechanical machining process. Operational Research in Engineering Sciences: Theory and Applications, vol. 5, no. 3, 131-152, DOI: 10.31181/oresta051022061d

[30] Bobar, Z., Bozanic, D., Djuric, K., Pamucar, D. (2020). Ranking and Assessment of the Efficiency of Social Media using the Fuzzy AHP-Z Number Model - Fuzzy MABAC. Acta Polytechnica Hungarica, vol. 17, no. 3, 43-70

[31] Le, H.A., Hoang, X.T., Trieu, Q.H., Pham, D.L., & Le, X. H. (2020). Determining the Best Dressing Parameters for External Cylindrical Grinding Using MABAC Method. Applied scicences, vol. 12, no. 16, 8287, DOI: 10.3390/app12168287

[32] Badi, I., Muhammad, L. J., Abubakar, M., Bakır, M. (2022). Measuring Sustainability Performance Indicators Using FUCOM-MARCOS Methods. Operational Research in Engineering Sciences: Theory and Applications, vol. 5, no. 2, 99-116, DOI: 10.31181/oresta040722060b

[33] Wen, Z., Liao, H., Zavadskas, E. K. (2020). MACONT: Mixed Aggregation by Comprehensive Normalization Technique for Multi-Criteria Analysis. Informatica, vol. 31, no. 4, 857–880, DOI: 10.15388/20-INFOR417

Paper submitted: 30.11.2022.
Paper accepted: 26.12.2022.