Istraživanja i projektovanja za privredu
ISSN 1451-4117
DOI:10.5937/jaes0-50407
www.engineeringscience.rs

publishing

Journal of Applied Engineering Science
Vol. 22, No. 2, 2024
Original Scientific Paper
Paper number: 22(2024)2, 1184, 245-252

ANALYTICAL AND NUMERICAL SOLUTION FOR FREE VIBRATIONS OF LAMINATED COMPOSITE PLATES

Vasilije Bojović*, Marina Rakočević

Faculty of Civil Engineering, University of Montenegro, Podgorica, Montenegro

** vasilije.b@ucg.ac.me*

In this paper, a comprehensive analysis of the free vibration characteristics of laminated composite plates (LCP) with different thicknesses, is conducted using both analytical and numerical methods. The study encompasses various boundary conditions and considers laminates with cross-ply and angle-ply orientations. The ply orientation, defined as the angle between the x-axis and the fibre direction within each ply, is a key focus of the investigation. Cross-ply laminates are characterized by ply orientations θ of 0° or 90°, while angle-ply laminates exhibit ply orientations of θ and -θ, where 0° ≤ θ ≤ 90°. The analytical and numerical calculation of natural frequencies of vibrations for laminates with symmetric and antisymmetric arrangement of layers is performed. The analytical solution is developed using Single-Layer Theories, including the CLPT (Classical Laminated Plate Theory) and the FSDT (First-order Shear Deformation Theory). The numerical calculation is made using the finite element-based software ANSYS. The influence of dimensions, boundary conditions, and ply orientation, on the values of natural frequencies, is analysed. The conducted analysis detected the significant impact of dimensions, boundary conditions, and ply orientation, on the free vibrations of the considered plates. Increasing the thickness of the plate leads to an increase in shear deformation. Therefore, in the analysis of thicker plates, it is noted that the FSDT yields more accurate results compared to the CLPT. It has been established that the application of Equivalent Single-Layer Theories (CLPT and FSDT) is justified in the analysis of moderately thick plates. In contrast, for the analysis of thick plates, the use of Layerwise theories is recommended.

Keywords: natural frequencies, free vibrations, analytical solution, numerical solution, laminated composite plates

1 INTRODUCTION

In recent decades, the interest of researchers in the application of new materials, such as polymers, as well as combinations of polymers with glass, carbon, etc. has increased. Laminated composite materials are the most commonly used “modern” materials in the industry. These materials are obtained by combining the layers of composite materials. Layers consist of continuous unidirectional fibres, carrying in one direction and embedded in matrix mass. Laminated composite materials possess significantly better characteristics than conventional materials. These materials have disadvantages as well. The main disadvantage is their high cost of production. As well, during the production process, because of faults in the lamination of layers, interlayer cracks can occur. These cracks are called delaminations and they can cause the degradation of the material.

In civil engineering, laminated composite materials are frequently utilized for the rehabilitation and reconstruction of existing structures.

In the previous three decades, a large number of scientists and researchers, following the trend of progress in the application of mathematical methods, gave suggestions for solving the problem of natural vibrations of laminated composites. In the development of models, the authors outline the contribution of Reddy and Reddy et al. [1] to [3], Sharma et al. [4] to [5], Vuksanović [6], Vuksanović and Marjanović [7] to [8].

Available theories for the calculation of LCP are Equivalent single-layer 2D theories (ESLT) and Layerwise theories (LWT). In ESL theories [9] to [11], a heterogeneous laminated plate is regarded as a statically equivalent single layer, and a 3D continuum problem is reduced to a 2D problem. This makes these theories simple to use. In Layerwise theories [1], [12] to [14], considerations are conducted at the layer level, and by applying these theories it is possible to take into account geometric imperfections in the connections of the layers of laminated plates. However, due to a large number of unknowns, the calculation is significantly more complicated. Therefore, it is necessary to define in which situations it is justified to use Layerwise theories, and in which ESL theories. ESL 2D theories are CLPT, FSDT, and Higher-order shear deformation theories (HSDT).

For determining the natural frequencies of vibrations, an analytical solution is developed using the CLPT and the FSDT. In addition to the analytical solution, a numerical solution is obtained using the ANSYS software package.

The thick and moderately thick plates, with the symmetric and antisymmetric configuration of layers through the plate thickness, are analysed in this paper. Considered plates have different boundary conditions and various ply (layer) orientations (cross-ply and angle-ply). Cross-ply laminates have ply orientation θ of 0° or 90°. Angle-ply layers have ply orientation θ and -θ, where 0° ≤ θ ≤ 90°. Ply orientation is determined by the angle of orientation of the fibres in the layer.

This analysis aims to explore the effects of dimensions, boundary conditions, and ply orientation on the dynamic characteristics of LCP. The further objective of the paper is to determine the influence of applied theory on the values of frequencies.

It is assumed that geometric and material properties, as well as the applied theory have a significant impact on free vibrations of LCP.

2 METHODOLOGY

2.1 Characteristics of the plate layers

In order to perform the calculation of LCP it is necessary to define the stiffness Q of each layer in the local coordinate system. The stiffness of the k-th layer is given by [1]:

$$Q_{11}^{(k)}=\frac{E_1^k}{1-\nu_{12}^k\nu_{21}^k},\quad Q_{12}^{(k)}=\frac{\nu_{21}^k E_1^k}{1-\nu_{12}^k\nu_{21}^k},\quad Q_{22}^{(k)}=\frac{E_2^k}{1-\nu_{12}^k\nu_{21}^k},\quad Q_{66}^{(k)}=G_{12}^k,\quad Q_{44}^{(k)}=G_{23}^k,\quad Q_{55}^{(k)}=G_{13}^k \tag{1}$$

The following expressions present the transformation of stiffness from a local to a global coordinate system [1]:

$$\begin{aligned}
\bar{Q}_{11}&=Q_{11}\cos^4\theta+2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta+Q_{22}\sin^4\theta\\
\bar{Q}_{12}&=(Q_{11}+Q_{22}-4Q_{66})\sin^2\theta\cos^2\theta+Q_{12}(\sin^4\theta+\cos^4\theta)\\
\bar{Q}_{22}&=Q_{11}\sin^4\theta+2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta+Q_{22}\cos^4\theta\\
\bar{Q}_{16}&=(Q_{11}-Q_{12}-2Q_{66})\sin\theta\cos^3\theta+(Q_{12}-Q_{22}+2Q_{66})\sin^3\theta\cos\theta\\
\bar{Q}_{26}&=(Q_{11}-Q_{12}-2Q_{66})\sin^3\theta\cos\theta+(Q_{12}-Q_{22}+2Q_{66})\sin\theta\cos^3\theta\\
\bar{Q}_{66}&=(Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta\cos^2\theta+Q_{66}(\sin^4\theta+\cos^4\theta)\\
\bar{Q}_{44}&=Q_{44}\cos^2\theta+Q_{55}\sin^2\theta\quad \bar{Q}_{45}=(Q_{55}-Q_{44})\cos\theta\sin\theta\quad \bar{Q}_{55}=Q_{55}\cos^2\theta+Q_{44}\sin^2\theta
\end{aligned} \tag{2}$$

Axial stiffness A_{ij}, flexural-axial coupling stiffness B_{ij}, and flexural stiffness D_{ij} are given by the equation:

$$\begin{aligned}
A_{ij}&=\int_{z_k}^{z_{k+1}}\bar{Q}_{ij}^{(k)}dz=\sum_{k=1}^{N}\bar{Q}_{ij}^{(k)}(z_{k+1}-z_k)\\
B_{ij}&=\int_{z_k}^{z_{k+1}}\bar{Q}_{ij}^{(k)}z\,dz=\frac{1}{2}\sum_{k=1}^{N}\bar{Q}_{ij}^{(k)}(z_{k+1}^2-z_k^2)\\
D_{ij}&=\int_{z_k}^{z_{k+1}}\bar{Q}_{ij}^{(k)}z^2dz=\frac{1}{3}\sum_{k=1}^{N}\bar{Q}_{ij}^{(k)}(z_{k+1}^3-z_k^3)
\end{aligned} \tag{3}$$

For developing the analytical solution, ESL theories are used. The analytical solution is obtained using the CLPT and the FSDT.

2.2 Classical Laminated Plate Theory – CLPT

The CLPT is the simplest ESL theory. It posits that the cross-sections of a deformed plate remain straight and perpendicular to the deformed midplane. The CLPT is widely used in the analysis of plates with smaller thickness. However, its application to LCP with higher thickness, which have high anisotropy, often yields unsatisfactory results due to its neglect of shear deformation.

In this paper, the CLPT is used for the calculation of moderately thick and thick plates, to compare the results with the FSDT, in order to determine the influence of shear deformation on plates of different thicknesses.

According to the CLPT, the displacement field is given in the form [1]:

$$\begin{aligned}
u(x,y,z,t)&=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}\\
v(x,y,z,t)&=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}\\
w(x,y,z,t)&=w_0(x,y,t)
\end{aligned} \tag{4}$$

where u_o, v_o and w_o are the displacement components in the midplane of the plate (z = 0).

Using the CLPT, the procedure of analytical calculation for symmetric laminates is different compared to antisymmetric laminates.

2.2.1 Free vibrations of symmetric laminates

According to [1], the conditional equation, for symmetric laminates, is given by the relation:

$$D_{11}\frac{\partial^4 w_0}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_0}{\partial y^4} + I_0 \ddot{w}_0 - I_2\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) = 0 \tag{5}$$

where I_0 and I_2 are moments of inertia which are equal to:

$$\begin{aligned} I_0 &= \sum_{k=1}^{N} \rho_0^{(k)}(z_{k+1} - z_k) \\ I_2 &= \frac{1}{3}\sum_{k=1}^{N} \rho_0^{(k)}(z_{k+1}^3 - z_k^3) \end{aligned} \tag{6}$$

where ρ_o is layer density and N represents the total number of layers (Figure 1).

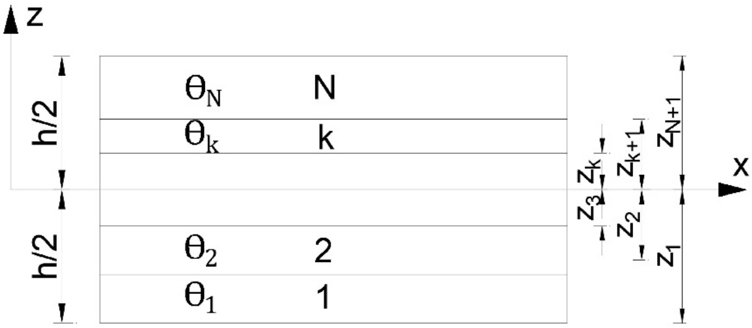

Fig. 1. Laminated composite with different layer thicknesses [1]

A periodic solution is assumed [1]:

$$W_{mn}(t) = W_{mn}^0 e^{i\omega\, t} \tag{7}$$

where $i = \sqrt{-1}$ and ω is the frequency of natural vibration. Substituting (7) in (5), it is obtained:

$$\{D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 - \omega^2[I_0 + (\alpha^2 + \beta^2)I_2]\}W_{mn}\, sin\, \alpha\, x\, sin\, \beta\, y = 0 \tag{8}$$

Since the equation must hold for each value of x and y ($0 < x < a$ and $0 < y < b$) the expression inside the braces has to be equal to zero, for each value of m and n. It follows that [1]:

$$\omega_{mn}^2 = \frac{\pi^4}{\tilde{I}_0 b^4}[D_{11}m^4 s^4 + 2(D_{12} + 2D_{66})m^2 n^2 s^2 + D_{22}n^4] \tag{9}$$

where

$$\tilde{I}_0 = I_0 + I_2\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] \tag{10}$$

$$s = \frac{a}{b} \tag{11}$$

In order to simplify the calculation, the rotary inertia I_2 is neglected.

2.2.2 Free vibrations of antisymmetric laminates

According to the CLPT, for free vibrations of antisymmetric laminates, a periodic solution is assumed [1]:

$$\begin{aligned} U_{mn}(t) &= U_{mn}^0 e^{i\omega t} \\ V_{mn}(t) &= V_{mn}^0 e^{i\omega t} \\ W_{mn}(t) &= W_{mn}^0 e^{i\omega t} \end{aligned} \tag{12}$$

Since all applied loads are equal to zero, the equation of motion takes the form [1]:

$$\left(\begin{bmatrix} \hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13} \\ \hat{c}_{12} & \hat{c}_{22} & \hat{c}_{23} \\ \hat{c}_{13} & \hat{c}_{23} & \hat{c}_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} \widehat{m}_{11} & 0 & 0 \\ 0 & \widehat{m}_{22} & 0 \\ 0 & 0 & \widehat{m}_{33} \end{bmatrix}\right)\begin{Bmatrix} U_{mn}^0 \\ V_{mn}^0 \\ W_{mn}^0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{13}$$

where

$$\begin{aligned}&\hat{c}_{11}=A_{11}\alpha^2+A_{66}\beta^2, \quad \hat{c}_{12}=(A_{12}+A_{66})\alpha\beta, \quad \hat{c}_{13}=-B_{11}\alpha^3-(B_{12}+2B_{66})\alpha\beta^2,\\&\hat{c}_{22}=A_{66}\alpha^2+A_{22}\beta^2, \quad \hat{c}_{23}=-(B_{12}+2B_{66})\alpha^2\beta-B_{22}\beta^3,\\&\hat{c}_{33}=D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4\end{aligned} \tag{14}$$

For a nontrivial solution, $U_{mn}^0 \neq 0$, $V_{mn}^0 \neq 0$, $W_{mn}^0 \neq 0$. Thus, the determinant of the matrix coefficients must be equal to zero. If the inertias in-plane are disregarded ($\hat{m}_{11}=\hat{m}_{22}=0$), the expression for the natural frequency takes the form [1]:

$$\omega_{mn}^2=\frac{1}{\hat{m}_{33}}\left(\hat{c}_{33}-\frac{\hat{c}_{13}\hat{c}_{22}-\hat{c}_{23}\hat{c}_{12}}{\hat{c}_{11}\hat{c}_{22}-\hat{c}_{12}\hat{c}_{12}}\hat{c}_{13}-\frac{\hat{c}_{11}\hat{c}_{23}-\hat{c}_{12}\hat{c}_{13}}{\hat{c}_{11}\hat{c}_{22}-\hat{c}_{12}\hat{c}_{12}}\hat{c}_{23}\right) \tag{15}$$

where $\hat{m}_{33}=[I_0+I_2(\alpha^2+\beta^2)]$.

2.3 First-order Shear Deformation Theory – FSDT

The FSDT arose due to the limitations of the CLPT. The theory includes constant shear along the plate thickness. As per the FSDT, the cross-section remains straight after deformation, but not perpendicular to the deformed midplane.

According to the FSDT, the displacement field is established through the following relations [1]:

$$\begin{aligned}&u(x,y,z,t)=u_0(x,y,t)+z\phi_x(x,y,t),\\&v(x,y,z,t)=v_0(x,y,t)+z\phi_y(x,y,t)\\&w(x,y,z,t)=w_0(x,y,t)\end{aligned} \tag{16}$$

where ϕ_x and ϕ_y are rotations around the x and y axes.

When employing the FSDT, it is observed that the analytical solutions for symmetric and antisymmetric laminates are equivalent.

According to [1], a periodic solution is given by the expression:

$$\begin{aligned}&U_{mn}(t)=U_{mn}^0e^{i\omega t}\\&V_{mn}(t)=V_{mn}^0e^{i\omega t}\\&W_{mn}(t)=W_{mn}^0e^{i\omega t}\\&X_{mn}(t)=X_{mn}^0e^{i\omega t}\\&Y_{mn}(t)=Y_{mn}^0e^{i\omega t}\end{aligned} \tag{17}$$

The conditional equation is given in the form [1]:

$$\left(\begin{bmatrix}\hat{s}_{11}&\hat{s}_{12}&0&\hat{s}_{14}&\hat{s}_{15}\\\hat{s}_{12}&\hat{s}_{22}&0&\hat{s}_{24}&\hat{s}_{25}\\0&0&\hat{s}_{33}&\hat{s}_{34}&\hat{s}_{35}\\\hat{s}_{14}&\hat{s}_{24}&\hat{s}_{34}&\hat{s}_{44}&\hat{s}_{45}\\\hat{s}_{15}&\hat{s}_{25}&\hat{s}_{35}&\hat{s}_{45}&\hat{s}_{55}\end{bmatrix}-\omega^2\begin{bmatrix}\hat{m}_{11}&0&0&0&0\\0&\hat{m}_{22}&0&0&0\\0&0&\hat{m}_{33}&0&0\\0&0&0&\hat{m}_{44}&0\\0&0&0&0&\hat{m}_{55}\end{bmatrix}\right)\begin{Bmatrix}U_{mn}^0\\V_{mn}^0\\W_{mn}^0\\X_{mn}^0\\Y_{mn}^0\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\\0\end{Bmatrix} \tag{18}$$

where

$$\begin{aligned}&\hat{s}_{11}=(A_{11}\alpha^2+A_{66}\beta^2), \quad \hat{s}_{12}=(A_{12}+A_{66})\alpha\beta, \quad \hat{s}_{14}=(B_{11}\alpha^2+B_{66}\beta^2),\\&\hat{s}_{15}=\hat{s}_{24}=(B_{12}+B_{66})\alpha\beta, \quad \hat{s}_{22}=(A_{66}\alpha^2+A_{22}\beta^2), \quad \hat{s}_{25}=(B_{66}\alpha^2+B_{22}\beta^2),\\&\hat{s}_{33}=K(A_{55}\alpha^2+A_{44}\beta^2), \quad \hat{s}_{34}=KA_{55}\alpha, \quad \hat{s}_{35}=KA_{44}\beta, \quad \hat{s}_{44}=(D_{11}\alpha^2+D_{66}\beta^2+KA_{55}),\\&\hat{s}_{45}=(D_{12}+D_{66})\alpha\beta, \quad \hat{s}_{55}=(D_{66}\alpha^2+D_{22}\beta^2+KA_{44})\end{aligned} \tag{19}$$

When the rotary inertia I_2 is neglected, equation (18) is simplified. Then the coefficients X_{mn} and Y_{mn} are equal to zero and (18) takes the form [1]:

$$\left(\begin{bmatrix}\bar{s}_{11}&\bar{s}_{12}&\bar{s}_{13}\\\bar{s}_{12}&\bar{s}_{22}&\bar{s}_{23}\\\bar{s}_{13}&\bar{s}_{23}&\bar{s}_{33}\end{bmatrix}-\omega^2\begin{bmatrix}\hat{m}_{11}&0&0\\0&\hat{m}_{22}&0\\0&0&\hat{m}_{33}\end{bmatrix}\right)\begin{Bmatrix}U_{mn}^0\\V_{mn}^0\\W_{mn}^0\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \tag{20}$$

where

$$
\begin{aligned}
&\bar{s}_{11} = \hat{s}_{11} - (\hat{s}_{14}\hat{s}_{55} - \hat{s}_{15}\hat{s}_{45})\hat{s}_{14} / \hat{s}_{00} - (\hat{s}_{15}\hat{s}_{44} - \hat{s}_{14}\hat{s}_{45})\hat{s}_{15} / \hat{s}_{00} \\
&\bar{s}_{12} = \hat{s}_{12} - (\hat{s}_{24}\hat{s}_{55} - \hat{s}_{25}\hat{s}_{45})\hat{s}_{14} / \hat{s}_{00} - (\hat{s}_{25}\hat{s}_{44} - \hat{s}_{24}\hat{s}_{45})\hat{s}_{15} / \hat{s}_{00} \\
&\bar{s}_{13} = -(\hat{s}_{34}\hat{s}_{55} - \hat{s}_{35}\hat{s}_{45})\hat{s}_{14} / \hat{s}_{00} - (\hat{s}_{35}\hat{s}_{44} - \hat{s}_{34}\hat{s}_{45})\hat{s}_{15} / \hat{s}_{00} \\
&\bar{s}_{22} = \hat{s}_{22} - (\hat{s}_{24}\hat{s}_{55} - \hat{s}_{25}\hat{s}_{45})\hat{s}_{24} / \hat{s}_{00} - (\hat{s}_{25}\hat{s}_{44} - \hat{s}_{24}\hat{s}_{45})\hat{s}_{25} / \hat{s}_{00} \\
&\bar{s}_{23} = \hat{s}_{23} - (\hat{s}_{34}\hat{s}_{55} - \hat{s}_{35}\hat{s}_{45})\hat{s}_{24} / \hat{s}_{00} - (\hat{s}_{35}\hat{s}_{44} - \hat{s}_{34}\hat{s}_{45})\hat{s}_{25} / \hat{s}_{00} \\
&\bar{s}_{33} = \hat{s}_{33} - (\hat{s}_{34}\hat{s}_{55} - \hat{s}_{35}\hat{s}_{45})\hat{s}_{34} / \hat{s}_{00} - (\hat{s}_{35}\hat{s}_{44} - \hat{s}_{34}\hat{s}_{45})\hat{s}_{35} / \hat{s}_{00}, \quad \hat{s}_{00} = \hat{s}_{44}\hat{s}_{55} - \hat{s}_{45}\hat{s}_{45}
\end{aligned}
\tag{21}
$$

When the in-plane and rotary inertias are disregarded, the coefficients are set to zero. Therefore, (20) is simplified and frequencies of free vibrations are obtained by the expression [1]:

$$
\omega^2 = \frac{1}{\hat{m}_{33}}\left(\bar{s}_{33} - \frac{\bar{s}_{13}\bar{s}_{22} - \bar{s}_{23}\bar{s}_{12}}{\bar{s}_{11}\bar{s}_{22} - \bar{s}_{12}\bar{s}_{12}}\bar{s}_{13} - \frac{\bar{s}_{11}\bar{s}_{23} - \bar{s}_{12}\bar{s}_{13}}{\bar{s}_{11}\bar{s}_{22} - \bar{s}_{12}\bar{s}_{12}}\bar{s}_{23}\right) \tag{22}
$$

2.4 Numerical solution

The numerical solution is commonly used in the analysis of structural problems due to its applicability, in contrast to the limitations of the analytical solution. In this paper, numerical analysis is performed using the ANSYS software (no. 1.058.610). The software is used in the analysis of structural statics and fluid dynamics, as well as in mechanical, thermal, and electromagnetic analyses. The ANSYS is specifically tailored for the analysis of composite structures. The program contains a large database of different finite element (FE) types. According to the programme instructions [15] to [16], for the analysis of free vibrations, SHELL181, and SOLID185 (layered) finite elements are adopted.

According to the user guide [15] to [16], the finite element designated as SHELL181 is well-suited for modelling plates and shells of smaller thickness (thin to moderately thick). This particular element comprises four nodes, each endowed with six degrees of freedom (disp. and rot. in three directions). It is important to note that the accuracy of the finite element model is constrained by the FSDT.

The SOLID185 (layered) finite element is used for the analysis of LCP with higher thickness. This element comprises eight nodes, each featuring three degrees of freedom (disp. in the three directions). Using this finite element, the layers can be modelled independently [15] to [16]. This study utilizes the SOLID185 (layered) element for system discretization, combined with additional, contact finite element at layers contact CONTA174, which has the same geometric characteristics as the contact surface. The contact material is designated as an elastic material whose friction coefficient is equal to zero while other material characteristics are not defined. The contact between the layers is specified as "Always bonded", utilizing the MPC algorithm [9], [11].

In this analysis, the SHELL181 FE is used for modelling moderately thick plates, while the calculation of thick plates is performed with SOLID185 (layered) element.

3 RESULTS AND DISCUSSION

The results of the analysis of natural vibrations for symmetric and antisymmetric laminates are presented below. Simply supported and clamped plates with four layers are analysed [10]. The calculation of fundamental frequencies is made. The thicknesses of the plates are h = 0.1 *m* (b/h = 10) and h = 0.25 *m* (b/h = 4). The adopted material of the analysed plates is Epoxy Carbon whose material characteristics are given in Table 1:

Table 1. Material characteristics of the plate layers

E_1 [GPa]	E_2 [GPa]	E_3 [GPa]	ν_{12}	ν_{13}	ν_{23}	G_{23} [GPa]	G_{13} [GPa]	G_{12} [GPa]	ρ [kg/m³]
123.34	7.78	7.78	0.27	0.27	0.42	3.08	5.0	5.0	1518.0

3.1 Symmetric arrangement of layers

Symmetric laminates with ply orientation 0°/90°/90°/0° are analysed. Rectangular plates of dimensions *a* x *b*, are considered. The dimension *a* varied in the range of 1 *m* – 6 *m*, and dimension *b* = 1 *m*.

In the graphs given in Figure 2, the values of fundamental frequencies of simply supported plates calculated analytically and numerically are presented. For both plate thicknesses, by the ratio a/b = 2, the frequencies decrease, and then gradually converge to a final value. Therefore, the frequencies of plates with smaller thickness (Figure 2-right), are approximately 60% lower than for the thicker plates (Figure 2-left). The difference in the values of frequencies obtained by analytical and numerical solutions is higher in the thick plates than in the moderately thick plates. This occurs due to the limitations of ESL theories (CLPT and FSDT) in the calculation of thick plates.

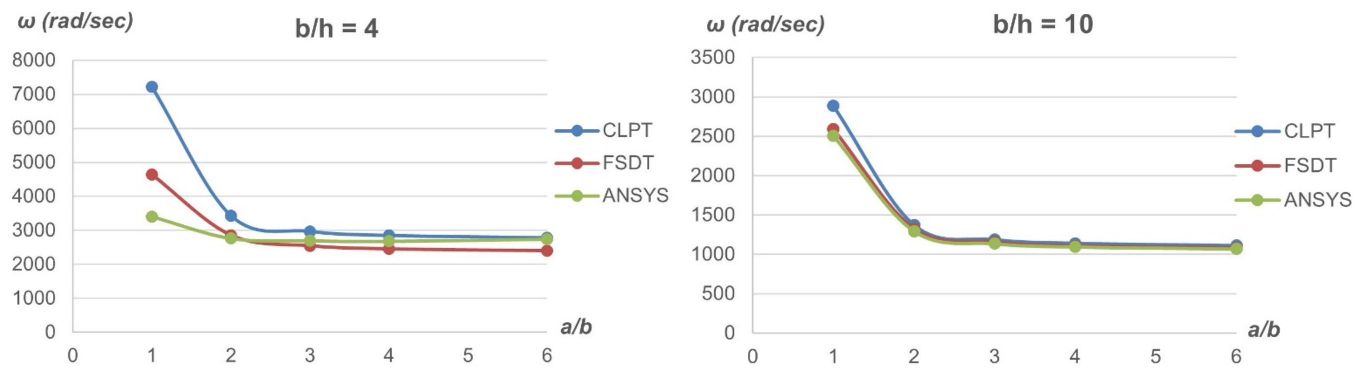

Fig. 2. Fundamental frequencies of simply supported plates

In Figure 3, frequencies of clamped composite plates are shown. The analytical solution for clamped plates cannot be obtained, so only a numerical calculation is made. As with simply supported plates, a similar decrease-converge trend appears with clamped plates. For plates with ratio *b/h* = 10, frequencies are approximately 50% lower when compared to the thick plates (*b/h* = 4). As well, it is found that frequencies of clamped plates are higher than frequencies of simply supported plates.

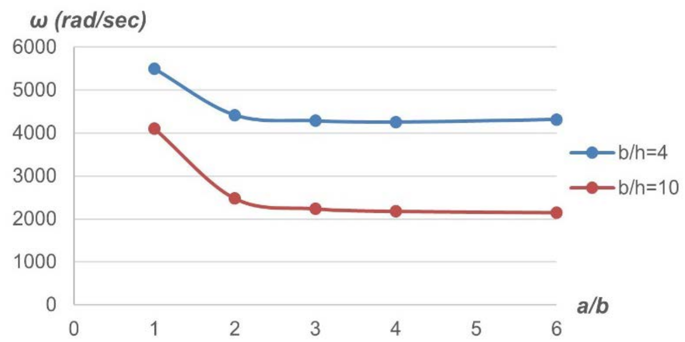

Fig. 3. Fundamental frequencies of clamped plates

The above indicates the significant impact of dimensions and boundary conditions on the free vibrations of LCP.

3.2 Antisymmetric arrangement of layers

Antisymmetric, square plates with the orientation of layers *θ/-θ/θ/-θ*, whereby *θ* = 0°, 15°, 30°, 45°, 60°, 75°, and 90 are considered. Adopted dimensions are *a* = *b* = 1 *m*.

The results of free vibrations are graphically presented in Figure 4. Fundamental frequencies of simply supported plates increase to the ply orientation of *θ* = 45° by approximately 30%. By further increasing the angle, frequencies decrease to the previous value. Frequencies obtained by numerical calculation are lower than the frequencies obtained analytically. Difference in the values is a consequence of the limitation of ESL theories (CLPT and FSDT). ESL theories treat the plate as a statically equivalent single layer, which in the analysis of thick plates does not always give satisfactory accurate results. Therefore, in this case, numerical calculation provides more accurate results, than analytical calculation.

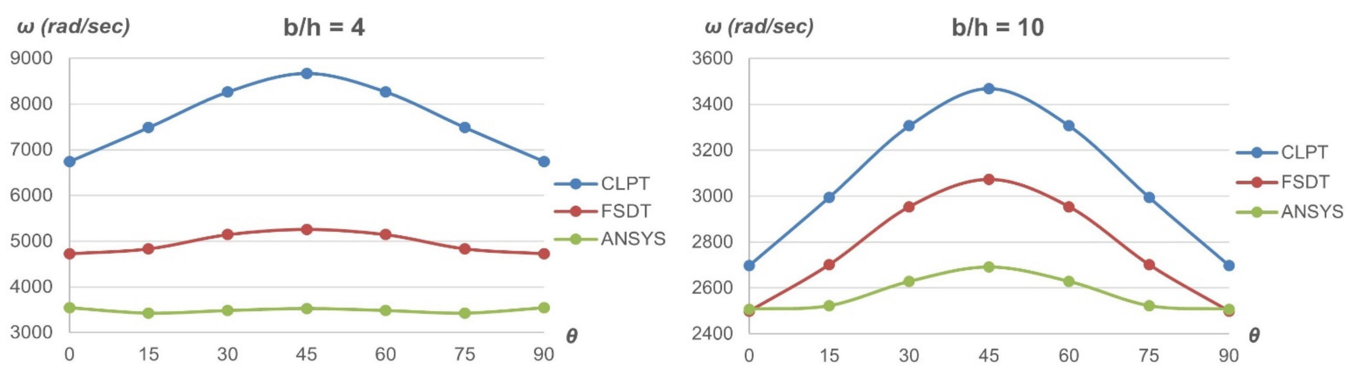

Fig. 4. Fundamental frequencies of simply supported plates

In Figure 5, frequencies of clamped plates are presented. Unlike the simply supported plates, increasing the orientation angle of clamped composite plates does not lead to a noticeable change of frequencies. The influence of prevented rotation in supports is significant, and therefore the increase of the angle *θ* is not sufficient to cause frequency changes.

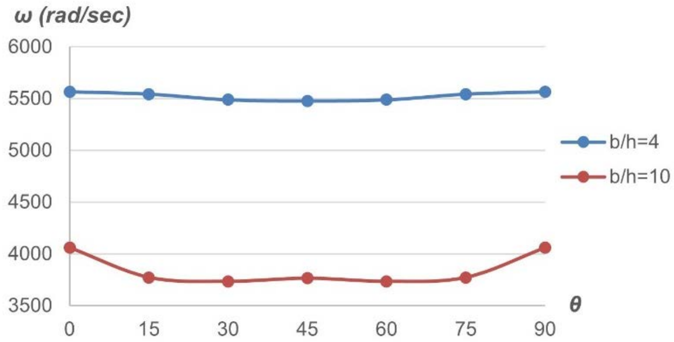

Fig. 5. Fundamental frequencies of clamped plates

4 CONCLUSIONS

After the conducted analysis, it can be concluded that dimensions, boundary conditions, and ply orientation greatly impact the dynamic properties of laminated composite plates. By analysing the influence of the aforementioned variables, the goal of the analysis has been achieved. The hypothesis of the research has been confirmed as well.

The analytical and numerical analysis is performed in this paper. It is found that in the analysis of moderately thick plates, the values of frequencies, calculated according to the CLPT and the FSDT, are approximately equal. In the analysis of thick plates, the FSDT gives more accurate results than the CLPT, due to a significant influence of shear on the deformation. Therefore, it is determined that the CLPT provides reliable results in the analysis of plates with smaller thickness. Increasing the thickness of the plate reduces the accuracy of the CLPT solution.

The numerical solution gives better matches with the analytical solution for the moderately thick plates, compared to the thick plates. In the analysis of thick plates, layers should be analysed independently, which is included in the numerical solution. However, the analytical calculation, using the ESL theories (CLPT and FSDT), cannot consider layers independently. Therefore, in the analysis of thick plates, the application of the aforementioned theories may not provide satisfactory accurate results. Thus, the second objective of the research is achieved.

Further research should be focused on analysing plates with geometric imperfections in the connections of the layers, specifically delaminations. When considering delaminations, Layerwise theories should be applied, in order to consider each layer independently.

5 REFERENCES

[1] Reddy, J.N. (2003). Mechanics of laminated composite plates and shells – Theory and Analysis, II edition. CRC Press, Boca Raton, DOI: 10.1201/b12409.

[2] Reddy, J.N., Khdeir, A.A. (1989). Buckling and Vibration of Laminated Composite Plates Using Various Plate Theories. *AIAA Journal*, vol. 27, no. 12, 1808-1817, DOI: 10.2514/3.10338.

[3] Liew, K.M., Huang, Y.Q., Reddy, J.N. (2003). Vibration analysis of symmetrically laminated plates based on FSDT using the moving least squares differential quadrature method. *Computer Methods in Applied Mechanics and Engineering*, vol. 192, no. 19, 2203-2222, DOI: 10.1016/S0045-7825(03)00238-X.

[4] Sharma, A.K., Mittal, N.D. (2010). Review on stress and vibration analysis of composite plates. *Journal of Applied Sciences*, vol. 10, no. 23, 3156-3166, DOI: 10.3923/jas.2010.3156.3166.

[5] Sharma, A.K., Mittal, N.D., Sharma, A. (2011). Free vibration analysis of moderately thick anti-symmetric cross-ply laminated rectangular plates with elastic edge constraints. *International Journal of Mechanical Sciences*, vol. 53, no. 9, 688–695, DOI: 10.1016/j.ijmecsci.2011.05.012.

[6] Vuksanović, Đ. (2000). Linear analysis of laminated composite plates using single layer higher-order discrete models. *Composite Structures*, vol. 28, 205-211, DOI: 10.1016/S0263-8223(99)00096-3.

[7] Marjanović, M., Vuksanović, Đ. (2014). Layerwise solution of free vibrations and buckling of laminated composite and sandwich plates with embedded delaminations. *Composite Structures*, vol. 108, 9-20, DOI: 10.1016/j.compstruct.2013.09.006.

[8] Vuksanović, Đ., Marjanović, M. (2014). Free vibrations of delaminated composite and sandwich plates. The 5th international conference Civil engineering – science and practice, 363-370.

[9] Bojović, V., Rakočević, M. (2022). Free vibration analysis of symmetric cross-ply laminated composite plates. The 8th international conference Civil engineering – science and practice, 101-109.

[10] Bojović, V. (2021). Stress strain analysis of laminated composite plates. Master thesis, Faculty of Civil engineering, University of Montenegro, Podgorica.

[11] Rakočević, M., Bojović, V. (2021). Analysis of symmetric angle-ply laminated composite plates. *Researches 2020* - Special issue of the Journal Istraživanja/Researches, on the occasion of the 40th anniversary of the Faculty of Civil Engineering in Podgorica, 1980-2020, 147-156.

[12] Rakočević, M. (2016). Analytical solution for simply supported laminated composite plate based on Partial layerwise theory. *Journal of Applied Engineering Science*, vol. 14 no. 1, 102-108, DOI: 10.5937/jaes14-10470.

[13] Rakočević, M., Popović, S., Ivanišević, N. (2017). A computational method for laminated composite plates based on layerwise theory. Composites Part B, vol. 122, 202-218, DOI: 10.1016/j.compositesb.2017.03.044.

[14] Rakočević, M., Popović, S. (2018). Bending analysis of simply supported rectangular laminated composite plates using a new computation method based on analytical solution of layerwise theory. Archive of Applied Mechanics, vol. 88, 671-689, DOI: 10.1007/s00419-017-1334-x.

[15] ANSYS Inc. Ansys Mechanical User's Guide.

[16] ANSYS Inc. (2009). Verification Manual for the Mechanical APDL Application – Release 12.1.

Paper submitted: 10.03.2024.

Paper accepted: 10.06.2024.