

## PARTICLE SIZE GROUPING METHOD AS A CONTROL SYSTEM OF EFFICIENCY FLOTATION PROCESS ON THE EXAMPLE OF COAL

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### Abstract

Agglomeration of coal particles during flotation can be analysed with the Particle Size Grouping (PSG) method. Numerous experiments were carried out to theoretically explain the effect of carbon particles agglomeration, but the result still remains incomplete. In this paper the PSG method was used to analyse agglomeration groups of carbon particles of initial size 100-400  $\mu\text{m}$ , maintaining the total particle volume. The size of particles population with definite radius and density was determined for 1 Mg coal. The influence of density and size of particles with given mixing energies and parameter  $\alpha$  on agglomeration was analysed. It was stated that the size of the particles had an effect on their agglomeration. In the analysed cases the dimensionless parameter of collision turbulence  $t^*$  needed for particles agglomeration in particular size groups was importantly shorter for particles of initial size 300 and 400  $\mu\text{m}$ . The change of the mixing energy did not have influence on the agglomeration of coal particles. The theoretical analyses based on computer calculations were supplemented by the analyses of the coal flotation process on an aqueous model. Experiments lied in introducing a foaming agent in the form of aqueous solution of hexanol which, without changing pH of the pulp, lowered surface tension value, and consequently increased the dispersion of air in the suspension. The experimental results were presented in the form of flotation kinetics curves. Fine particles 100-200 $\mu\text{m}$ . turned out to be best for flotation, unlike coarse 400-500  $\mu\text{m}$ .

**Keywords:** Coal flotation; PSG Method; Agglomeration; Modelling

### 1. Introduction

One of the toughest technical and economic problems associated with coal production its processing flotation enrichment in the primary process dedicated to small particles—below 500 $\mu\text{m}$ . Optimum conditions for flotation depend on the solid particles concentration in the feed, temperature, pH of the pulp, size of particles under the flotation, amount of introduced air, as well as on the time and intensity of mixing. During the flotation coal particles are separated from the mineral fraction while surface phenomena take place on the air/particle interface. As a result of the operation of hydrophobic forces particles are adhered to the gas bubbles [1, 2].

Flotation allows for separation of fine particles of carboniferous material having different hydrophobic character [3-8]. Hard coal is naturally hydrophobic, which means that that it can be easily enriched with the flotation method [9-14]. From the mineralogical point of view, hard coal is not a homogeneous substance [12]. For this reason its surface properties depend on the petrographic composition, coalification level, oxidation of the surface, amount and kind of

mineral admixtures, in that organic matter, etc. [9]. The changes in the hydrophobic character of coals in the coalification process are mainly explained by the structure and chemical build of the organic fraction. The particles are usually analysed for their ability of agglomerate on the surface of gas bubbles on the theoretical grounds, numerically or experimentally on water models [15-22]. One of the mathematical methods with which agglomeration can be modelled is the Particle Size Grouping method [23-27]. It allows for evaluating the effect of definite physicochemical parameters of the flotation process on agglomeration:

- effect of coal density on the agglomeration of particles with the same radius and different density,
- effect of particle size on agglomeration of particles of the same densities and different radius,
- effect of physicochemical properties (parameter  $\alpha$ ) on agglomeration process,
- effect of the energy of mixing.\*

It should be noted, that the use of computer modelling imposes the use of simplifications in relation to the actual course of phenomena in technological processes. Therefore, in this paper,

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using the PSG method, an attempt was made to verify the applicability of this research (simulation) method to control the batch flotation process, based on the analogy between agglomeration process of particles and subprocesses of flotation - collision and adhesion, on the example of coal flotation as a model raw material

Flotation of coal particles depends on the size of coal undergoing flotation [28]. The experiments revealed that the bigger particle of coal was higher the probability of collision with the air bubble and formation of a coal/bubble aggregate [9] (Fig. 1).

The durability of the connection depends on the forces trying to disrupt the agglomerate. It also depends on the size of the particle and its densities. Mechanism of the impact and connection between gas bubble and spherical particle is shown in Figure 2.

The force adhering the particle to the gas bubble can be presented as below [29-30]:

$$F_{adh} = \pi \cdot R_2^2 \cdot \Delta P_m + 2\pi \cdot R_2 \cdot \sigma_m \quad (1)$$

where:

$$\Delta P_m = \sigma_m \cdot (1/R_1 - 1/R_2) \quad (2)$$

The second term of the equation accounts for the contact angles and is a balance of surface forces. The radius of the neck is calculated from the following equations:

$$(r_1 + r_2)(R_2^2 + 2R_1R_2) + 2r_1r_2R_1(\cos\Theta_1 + \cos\Theta_2) = 0 \quad (3)$$

$$R_2' = R_2 / \sin\alpha = R_2 / \sqrt{1 - \cos^2\alpha} \quad (4)$$

$$\cos\alpha = \frac{R_1(r_1\cos\Theta_1 - r_2\cos\Theta_2)}{(r_1 + r_2)(R_1 + R_2')} \quad (5)$$

where:

$\Theta_1$  and  $\Theta_2$  - contact angles, *rad*

$R_2'$  - auxiliary variable, *m*

$R_1$  - radius of surface curvature of liquid phase in the bottleneck area, *m*

$r_1$  - particle radius, *m*

$r_2$  - bubble radius, *m*

$\sigma_m$  - surface tension, *N/m*

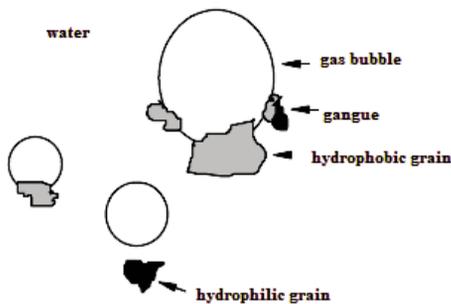


Figure 1. Flotation of coal particles. Overview drawing [9]

$\alpha$  - angle between  $R_2'$  and line connecting centers of the particles, *rad*

$F_{adh}$  - the force of adhesion, *N*

$P_m$  - pressure, *Pa*

Successive coal particles usually adhere to the previously entrapped ones. This type of connection is stable if the contact angle exceeds 90°.

It should be noted, that the use of computer modelling imposes the use of simplifications in relation to the actual course of phenomena in technological processes. Therefore, in this paper, using the PSG method, an attempt was made to verify the applicability of this research (simulation) method to control the batch flotation process, based on the analogy between agglomeration process of particles and subprocesses of flotation - collision and adhesion, on the example of coal flotation as a model raw material.

The author's analysis, can the PSG method model the process of raw material flotation with the success and can be used as a control system of this process.

## 2. Materials and Methods

### 2.1. Materials and reagents

Coal type 33 was (in polish classifications) used, and was crushed in a roll crusher dimensions below 500  $\mu\text{m}$ . The wet sieve analysis was performed, achieving the (500-400); (400-315); (315-200); (200-100)  $\mu\text{m}$  size fraction, followed by the float and sink analysis. Having regard to the ensure the appropriate amount of the raw material for planned experiments according to the analysis of separation products yields in heavy liquids, to the test batch flotation were selected the following density fractions- (-1.35),

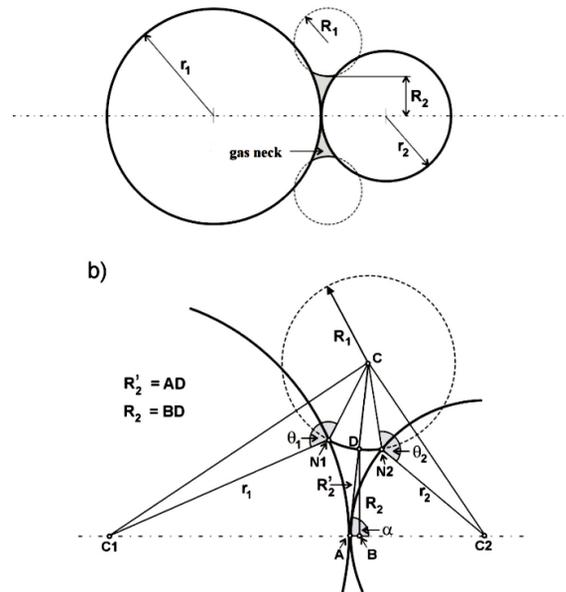


Figure 2. Mechanism of connection between gas bubble and spherical particle [29-30]

(1.35-1.5), (1.5-1.7), (1.7-1.8), (1.8-2.0), (+2.0) Mg/m<sup>3</sup>. The narrow size fraction-density fractions were stored in vacuum to limit the oxidation of coal surface and next then were used for research. Ash content was determined in every sample according to the Polish standard [12].

## 2.2. Kinetics of flotation of densimetric fractions

Flotation experiments for coal were carried out at room temperature in a Denver laboratory flotation machine with a capacity of 1 dm<sup>3</sup> and a constant rotor speed of 2020 rpm and constant air flow. The solids content was the same in all the experiments and amounted to 80 g/dm<sup>3</sup>. Such conditions ensured a constant amount and size of air bubbles in the chamber with limited turbulence of the flotation pulp caused by the rotor. The weighed portion was wetted for 15 minutes in a fixed concentration of flotation reagent, followed by stirring for 5 minutes without air. An aqueous solution of n-hexanol was used as a collecting and foaming agent, which does not change the pH of the pulp, but only reduces the surface tension at the gas-liquid interface, which increases the air dispersion in the suspension [31]. Also, the reagent adsorption on the surface of air bubbles occurs, which ensures their stabilization and prevents coalescence [32]. Appropriate reagent concentrations were selected based on the preliminary tests, ensuring the stability and appropriate structure of foam formation. It is in line with the parameters used to simulate the process by the PSG method. In the conducted series of tests, the fractionated flotation was carried out, in which the concentrates were collected in the following time intervals: 15, 30, 60, 90, 120s and every 60 s. Flotation time depended on particle size and fraction density. As a rule, the last foam product was collected after 6 minutes of flotation. Samples were weighed after drying and the ash content (*A*) was determined [14]. Experimental data were used to graph the flotation kinetics curves, which describes the progress of coal processing in time and the maximum recovery of the mineral particles in the predetermined conditions of physicochemical and hydrodynamic of this processes.

## 2.3 PSG method

The agglomeration of coal particles in the flotation process was simulated with the Particle Size Grouping method. The method lies in separating coal particles into size groups and each group is associated with a given size interval, the size of which changes with the set coefficient  $R_v$  (Eq.6) [33].

$$R_v = \frac{v_k}{v_{k-1}} = const \quad (6)$$

where:  $v$  – volume of particle (coal),  $k$  – index of a group.

When  $R_v=2$ , the critical particle  $v_k$  consists of two, one level smaller particles, i.e.  $v_{k-1}$ . In the formula,  $k$  describes the number of the smallest elements making up the particle. Hence the conclusion that the agglomerate of particles making up two size-groups, e.g.  $k-1$  and  $k-1$  may bring about a new particle, the size of which is bigger than in this group. This new agglomerate is classified as group  $k$  if its size exceeds the beginning threshold of group  $k$ ; otherwise, it will be classified as group  $k-1$ . Such a classification of particles changes the density of groups, e.g.: if one agglomerate is formed of two particles, that lowers the total number of particles.

If a collision of particles belonging to the interval  $i_{c,k-1}$  to  $k-1$  generates a particle  $k$ , then the balance of the population can be calculated with equation (7) [26-28]:

$$\begin{aligned} \frac{dn_k^*}{dt^*} = & \sum_{i=i_{c,k-1}}^{k-1} \xi_{i,k-1} (r_i^* + r_{k-1}^*)^3 n_i^* n_{k-1}^* + \\ & + \sum_{i=1}^{i_{c,k-1}} \zeta_{i,k} (r_i^* + r_k^*)^3 n_i^* n_k^* - \\ & - \sum_{i=i_{c,k}}^{M-1} (1 + \delta_{ik}) (r_i^* + r_k^*)^3 n_i^* n_k^* \end{aligned} \quad (7)$$

where dimensionless number density of size  $i$  particles are defined as:

$$n_k^* = n_k / N_0 \quad (8)$$

$$t^* = 1,3\alpha r_1^3 (\varepsilon/\nu)^{1/2} N_0 t \quad (9)$$

where:

$N_0$  – initial total number of particles,  $m^3$

$N_M$  – number of fundamental particles making up the biggest agglomeration, 1000

$r_i$  – radius of elementary monomers,  $m$

$t^*$  – dimensionless parameter of collision turbulence,

$n_i, n_{i-k}, n_k$  – number density of size  $i$ ,  $i-k$  and  $k$  particles,  $m^{-3}$

$\xi_{i,k-1}, \zeta_{i,k}$  – correction coefficient of particle density in a group,

$\nu$  – kinematic viscosity,  $m^2/s^3$

$\varepsilon$  – turbulent energy dissipation rate,  $m^2/s^3$

$\alpha$  – agglomeration coefficient

$$\xi_{i,k-1} = (v_i + v_{k-1}) / v_k \quad (10)$$

$$\zeta_{i,k} = v_i / v_k \quad (11)$$

$$r_k^* = r_k / r_1 \quad (12)$$

$i_{c,k-1}$  – size of a particle,

$v_i$  – volume of  $i$ -particle,  $m^3$

$\delta_{ij}$  – Kronecker's delta ( $\delta_{ij} = 1$  for  $i=j$ ,  $\delta_{ij} = 0$  for  $i \neq j$ ).

If  $k=3$ , the  $n_{ik}=1$ , at  $R=2$ , the sum will consist of two elements, i.e. collision of particles  $k-2=1$  and  $k-1=2$ , and the other one  $k-1$  and  $k-1$ .



Parametr  $\xi_{i,k-1}$  represents the fraction of total volume of two colliding particles, which tend to the volume of  $k$ -class:

$$\text{i.e. } \xi_{1,2} = \frac{v_1 + v_2}{v_3} = \frac{3}{4} \quad \text{for } R=2 \quad (13)$$

$$\text{and } \xi_{1,2} = \frac{2v_2}{v_3} = 1 \quad (14)$$

The simulation calculation of coal particles collisions observed in the flotation process was based on the data presented in Table 1. The process was analysed for a population of particles with the following size: 100, 200, 300, 400 $\mu\text{m}$ . The data assumed for the calculations are presented in Tables 2.

**Table 1.** Constants assumed for modelling

Agglomeration coefficient, $\alpha$	Viscosity of fluid $\nu$ , $\text{m}^2\text{s}^{-1}$	Mixing energy $\varepsilon$ , $\text{m}^2\text{s}^{-3}$
0.5, 0.8, 1	$8.937 \cdot 10^{-7}$	0.01; 0.0001

**Table 2.** Variable values adopted for modelling

Radius $R$ , m	Volume of particle $V_{k_s}$ , $\text{m}^3$	Particle Density $d$ , $\text{Mg m}^{-3}$	Mass of particle $m_k$ , kg	Mass of coal, kg	Initial number of particles $N_{\rho}$ , $\text{m}^{-3}$
0.0001	4.18E-12	1	4.18E-09	1000	2.39E+11
0.0002	3.34E-11	1	3.34E-08	1000	2.99E+10
0.0003	1.13E-10	1	1.13E-07	1000	8.86E+09
0.0004	2.68E-10	1	2.68E-07	1000	3.74E+09
0.0001	4.18E-12	1.3	5.43E-09	1000	1.84E+11
0.0002	3.34E-11	1.3	4.35E-08	1000	2.30E+10
0.0003	1.13E-10	1.3	1.47E-07	1000	6.82E+09
0.0004	2.68E-10	1.3	3.48E-07	1000	2.88E+09
0.0001	4.18E-12	1.5	6.27E-09	1000	1.59E+11
0.0002	3.34E-11	1.5	5.02E-08	1000	1.99E+10
0.0003	1.13E-10	1.5	1.69E-07	1000	5.91E+09
0.0004	2.68E-10	1.5	4.01E-07	1000	2.49E+09
0.0001	4.18E-12	1.8	7.52E-09	1000	1.33E+11
0.0002	3.34E-11	1.8	6.02E-08	1000	1.66E+10
0.0003	1.13E-10	1.8	2.03E-07	1000	4.92E+09
0.0004	2.68E-10	1.8	4.82E-07	1000	2.08E+09
0.0001	4.18E-12	2	8.36E-09	1000	1.20E+11
0.0002	3.34E-11	2	6.69E-08	1000	1.50E+10
0.0003	1.13E-10	2	2.26E-07	1000	4.43E+09
0.0004	2.68E-10	2	5.35E-07	1000	1.87E+09

The results of calculation for particular variants was presented graphically; the effect of agglomeration was analysed for a substitute time  $t^*$ .

### 3. Results and Discussion

The analysed effect of particles agglomeration on gas bubbles does not mean that the flotation process takes place. This is caused by the fact that the collision mechanism is accounted for without the outflowing effect which is associated with the flotation process. It was assumed in the applied calculation method that the collisions of the particle with the bubble or another particle adhering to the bubble were effective. However, this is a simplification because such surface phenomena on the bubble/particle interface as wettability and surface tension have not been taken into account. The agglomeration process results in the formation of bigger particles in the course of collisions between smaller particles and the gas bubble. The agglomerates which exceed the boundary values for a given size class are classified in another size

group. According to the calculation assumption in the initial phase of the process we have only particles with a given radius. e.g. 100  $\mu\text{m}$ . In the course of mixing, the particles collide and consequently new particles are formed in particular size groups. It should be noted that in the course of the process particles from the same size group and from other size groups collide. This means that the assumed initial number of particles  $N_0, \text{m}^{-3}$  does not change and remains constant during the simulation. This number undergoes grouping in a number of ways as the process proceeds in time (time  $t^*$ ).

### 3.1. Influence of particle size on agglomeration

The plot illustrating the qualitative change of particles in particular groups (Fig.3-6) shows the dynamics of coal particles removal in the process of mixing baths containing coal particles. The rate of the process is much faster in the case of bigger particles. e.g. 400 $\mu\text{m}$  (Fig. 6). In the analysed case time  $t^*$  which is needed for the removal of particles belonging to the first size group  $n_1$  is 2 orders lower as compared to particles 100 $\mu\text{m}$  – group  $n_1$  (Fig. 3).

Characteristically, in all the analysed cases the process was most dynamic in the initial phase. At the end of the process the number of particles in group 1 changes very slowly, which is caused by the

lower number of particles in this size group. In this way the probability of the collision between particles occurrence is lower.

### 3.2. Effect of particle density on agglomeration

The plots in Figures 7-10 illustrate the influence of particle density on the agglomeration in group 1  $n_{k1}/N_0$ . The calculations were performed for the parameters listed in Tables 1 and 2. Agglomeration was analysed for 1 Mg of coal of given density, hence the initial number of particles  $N_0$  with given radius. The agglomeration process is fastest for particles with radius 300 and 400  $\mu\text{m}$  and high density (Fig. 9-10).

At the first stage of the process the particles agglomeration presented for group  $n_1$  was very fast. In the case of particles with the biggest assumed radius of 400  $\mu\text{m}$ . the calculated time  $t^*$  and  $t$  (real time of agglomeration) had the lowest value. For instance when comparing agglomeration time  $t^*$  for particles with density  $\rho=2 \text{ Mg/m}^3$  and various radius agglomeration time  $t^*$  was three orders shorter for particles with radius 400 $\mu\text{m}$ .

Attention should be paid to the fact that there was low density signified higher carbon content (%C) in a sample. The mass of coal assumed in the calculations was 1 Mg. therefore the calculated number of particles  $N_0$  was different for each particle density and radius. For instance, the number of particles  $N_0$

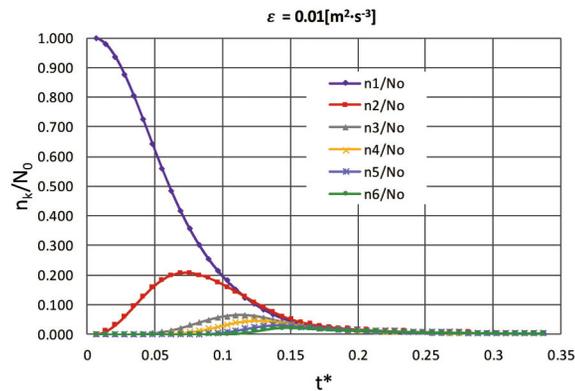


Figure 3. Results of calculations for particle  $r=100\mu\text{m}$ ,  $\varepsilon=0.01\text{m}^2\text{s}^{-3}$

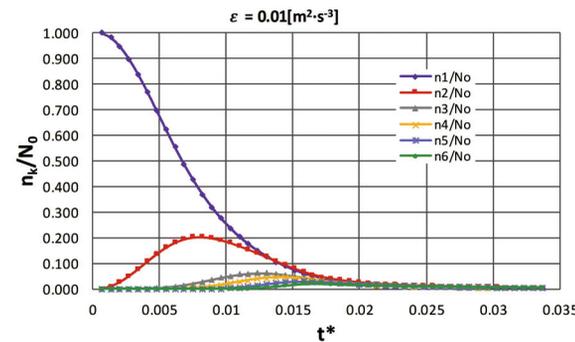


Figure 4. Results of calculations for particle  $r=200 \mu\text{m}$ ,  $\varepsilon=0.01\text{m}^2\text{s}^{-3}$

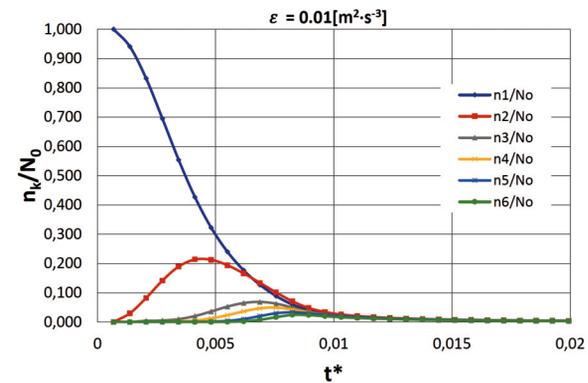


Figure 5. Results of calculations for particle  $r=300\mu\text{m}$ ,  $\varepsilon=0.01\text{m}^2\text{s}^{-3}$

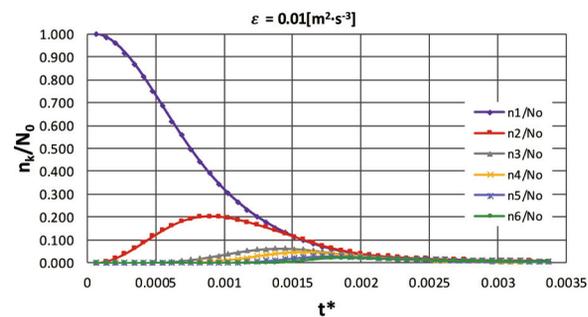


Figure 6. Results of calculations for particle  $r=400\mu\text{m}$ ,  $\varepsilon=0.01\text{m}^2\text{s}^{-3}$



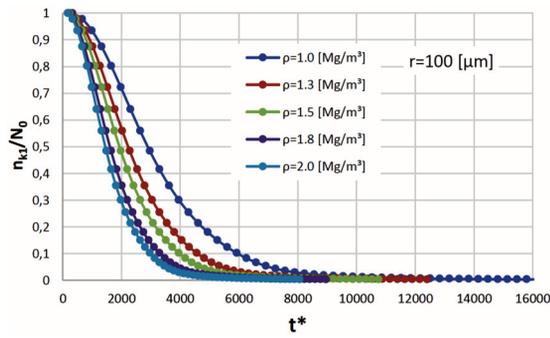


Figure 7. Influence of particle density on coal agglomeration ( $r=100 \mu\text{m}$ ) – result for size group I

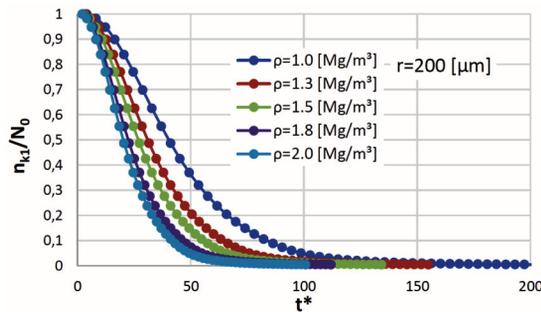


Figure 8. Influence of particle density on coal agglomeration ( $r=200 \mu\text{m}$ ) – result for size group I

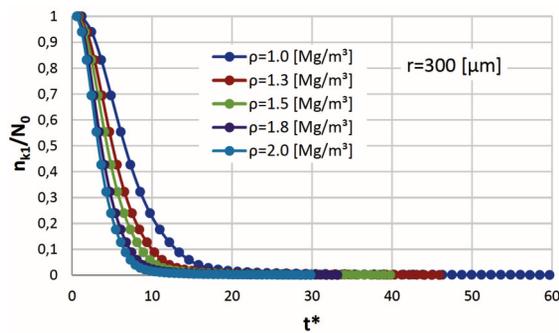


Figure 9. Influence of particle density on coal agglomeration ( $r=300 \mu\text{m}$ ) – result for size group I

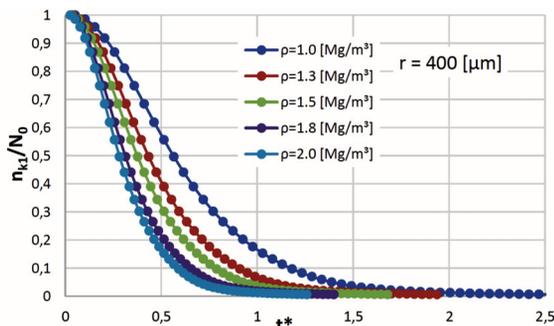


Figure 10. Influence of particle density on coal agglomeration ( $r=400 \mu\text{m}$ ) – result for size group I

for  $\rho=1\text{Mg/m}^3$  was twice smaller than  $N_0$  for  $\rho=2\text{Mg/m}^3$  (Fig. 10). The most efficient agglomeration was observed for large particles.

### 3.3. Effect of coefficient $\alpha$ on coal particles agglomeration

Agglomeration coefficient  $\alpha$  accounts for the operation of viscosity forces and London-van der Waals forces. Higashitani et al. [27] defined parameter  $\alpha$  for experimental conditions with the following equation:

$$\alpha = 0.727 \left[ \frac{\mu \cdot r_1^3 \left( \frac{\varepsilon}{\nu} \right)^{0.5}}{A_{131}} \right]^{-0.242} \quad (15)$$

where:  $A_{131}$  - effective Hamaker constant of particle 1 in medium,  $r$  - radius of particle.  $\nu$  - kinematic viscosity.

The authors of this paper assumed the following values of parameter  $\alpha$ : 1; 0.8; 0.5. The results for a size group  $n_{k,l}/N_0$  for particles with radius 100 and 400  $\mu\text{m}$  are presented in Figures 11 and 12. Parameter  $\alpha$  accounts for a few characteristic parameters, e.g., viscosity, radius and mixing energy. Based on the equation (9) it indirectly affects  $t^*$  value.

In both cases the agglomeration curves have a similar trend; the low value of the agglomeration coefficient resulting from, e.g., value of the mixing energy and viscosity has a favourable effect on this phenomenon. Accordingly, at the next stage the effect of mixing energy on the efficiency of collisions between particles and gas bubbles was calculated.

### 3.4 Effect of mixing energy on agglomeration

Figures 13-16 illustrate the results of simulations for coal particles with radius 100 and 400  $\mu\text{m}$  for mixing energy 0.01 and 0.0001  $[\text{m}^2\text{s}^{-3}]$ . Interestingly there is no significant effect of intensified mixing on agglomeration of particles in size groups.

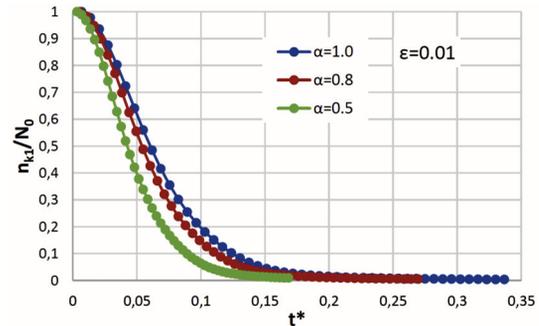


Figure 11. Effect of coefficient  $\alpha$  on agglomeration of coal particles  $r=100 \mu\text{m}$  for various  $\alpha$  and size group  $n_{k,l}$

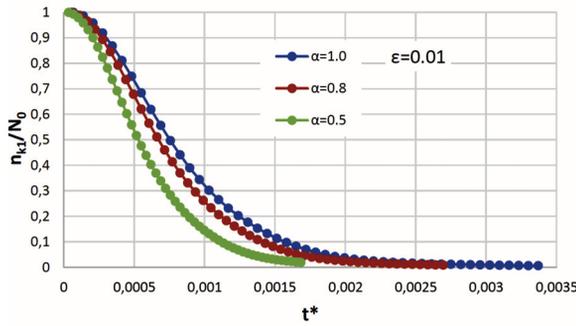


Figure 12. Effect of coefficient  $\alpha$  on agglomeration of coal particles  $r=400\mu\text{m}$  for various  $\alpha$  and size group  $n_{kl}$

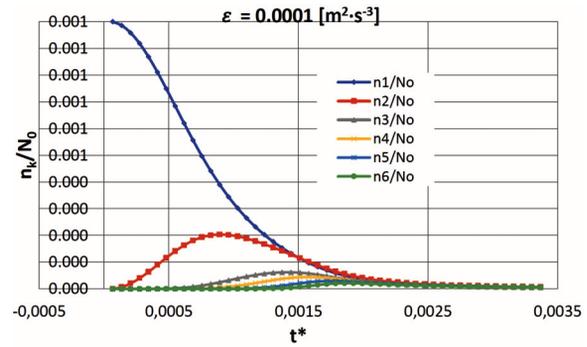


Figure 16. Effect of mixing energy on agglomeration of coal particles  $r=400\mu\text{m}$  for various  $\epsilon=0.0001\text{m}^2\text{s}^{-3}$

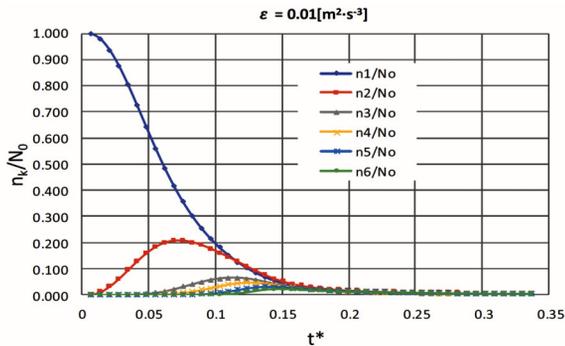


Figure 13. Effect of mixing energy on agglomeration of coal particles  $r=100\mu\text{m}$  for various  $\epsilon=0.01\text{m}^2\text{s}^{-3}$

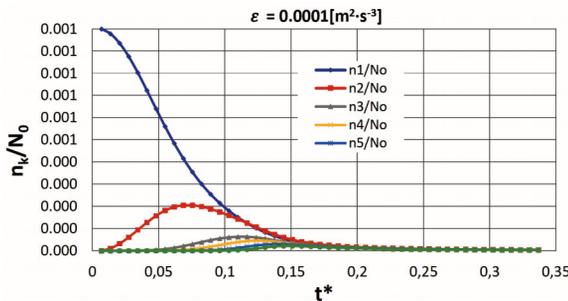


Figure 14. Effect of mixing energy on agglomeration of coal particles  $n r=100\mu\text{m}$  for various  $\epsilon=0.0001\text{m}^2\text{s}^{-3}$

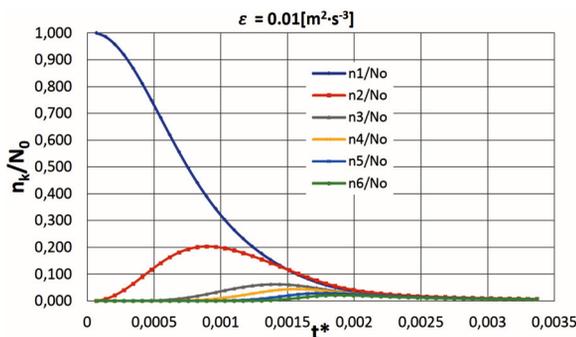


Figure 15. Effect of mixing energy on agglomeration of coal particles  $r = 400\mu\text{m}$  for various  $\epsilon=0.01\text{m}^2\text{s}^{-3}$

Characteristically the agglomeration time expressed as  $t^*$  is almost identical in Figures 13 and 14 (particle with radius  $100\mu\text{m}$ ) and figures 15 and 16 (particle with radius  $400\mu\text{m}$ ). In all four analysed cases the rate of vanishing of particles in group 1  $n_1/N_0$  was most intense at the initial stage I and continued very fast (low  $t^*$ ). These results suggest that the intensification of flotation through the agglomeration of coal particles can be realized by controlling such process parameters as viscosity and surface tension (not accounted for in calculations).

### 3.5 Results of kinetics flotation

From the analysis of the flotation kinetics equations, it follows that the adhesion of particles to gas (air) bubbles depends on the probability of collision of these particles. When the physicochemical pulp properties and hydrodynamic conditions of flotation process are constant, that the coal enrichment effectivity depends on particle size, bubbles size, density of particle and ash content. [14]. It can be assumed that for the flotation of the narrow class of the particle size, the average particle size is constant. In order to verify the results of PSG modelling method carried out, the flotation tests were based on the methodology described in section 2. The results are shown in Figures 17-21.

Density ranges of floated particle fractions included the values of these parameters used in PSG modelling. In laboratory flotation test not isolated fractions of a strictly fixed value of density but flotation tests were realised for the narrow class of particle size and density fractions. It should be noted that a real of coal samples are characterized by the distribution content of the useful component, i.e., also by the density distribution in a narrow particle size class. Empirical curves obtained from the experimental data are shown in Figures 17-21. The course of flotation kinetics curves can be described by an equation analogous to the first order chemical reaction kinetics equation. Furthermore, many

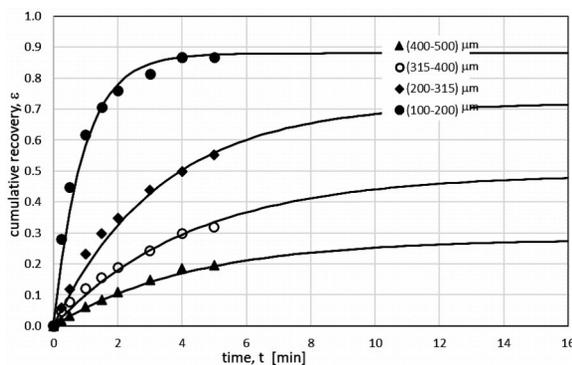


investigations of pure minerals flotation kinetics of narrow size fraction proved that under conditions of free flotation (the surface of bubbles is not a limitation for their mineralization) the flotation results can be described with the 1st order equation. [33-35]. The curves of flotation kinetics were drawn according to the results of empirical tests. The experimental data were approximated by model equations of flotation kinetics of the 1<sup>st</sup> order (equation (16)). The parameters of this equation,  $e_\infty$  and  $k$ , were calculated by transforming this formula to the linear form:

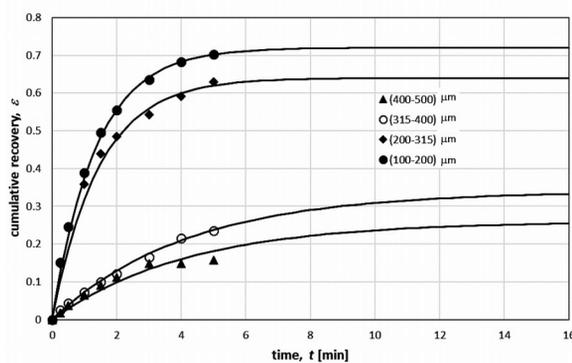
$$\varepsilon(t) = \varepsilon_\infty [1 - \exp(-k t)] \quad (16)$$

$$\ln \frac{\varepsilon_\infty - \varepsilon}{\varepsilon_\infty - \varepsilon} = k t = y \quad (17)$$

Next, the constant  $k$  was calculated by the least square method. The rate of agreement of the model dependence with empirical courses was evaluated by



**Figure 17.** Flotation kinetics for density fraction below 1.3 Mg/m<sup>3</sup> of different particle size; particle size (100-200) μm,  $e = 0.88(1 - e^{-1.09 t})$ ,  $R^2 = 0.9896$ ; particle size (200-315) μm,  $e = 0.72(1 - e^{-0.30 t})$ ,  $R^2 = 0.9956$ ; particle size (315-400) μm,  $e = 0.49(1 - e^{-0.24 t})$ ,  $R^2 = 0.9936$ ; particle size (400-500) μm,  $e = 0.28(1 - e^{-0.233 t})$ ,  $R^2 = 0.9972$



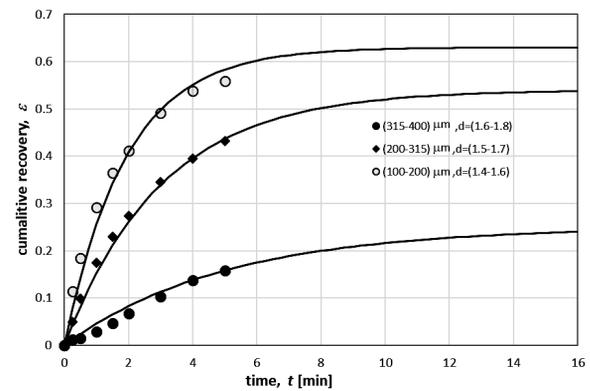
**Figure 18.** Flotation kinetics for density fraction (1.3-1.4) Mg/m<sup>3</sup> of different particle size; particle size (100-200) μm,  $e = 0.72(1 - e^{-0.74 t})$ ,  $R^2 = 0.9995$ ; particle size (200-315) μm,  $e = 0.64(1 - e^{-0.69 t})$ ,  $R^2 = 0.9927$ ; particle size (315-400) μm,  $e = 0.34(1 - e^{-0.237 t})$ ,  $R^2 = 0.9979$ ; particle size (400-500) μm,  $e = 0.26(1 - e^{-0.24 t})$ ,  $R^2 = 0.9793$

means of the curvilinear correlation coefficient  $R$ :

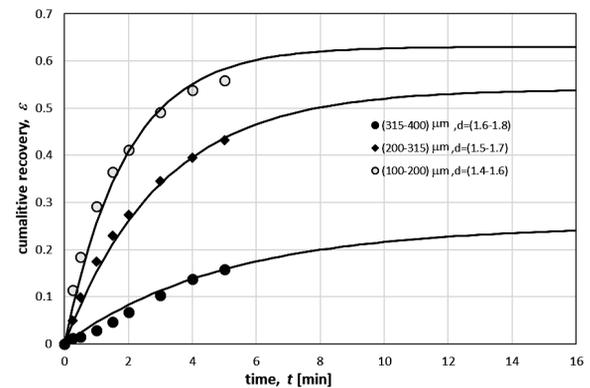
$$R = \sqrt{1 - \frac{\sum_{i=1}^n (\varepsilon_i - \check{\varepsilon}_i)^2}{\sum_{i=1}^n (\varepsilon_i - \bar{\varepsilon})^2}} \quad (18)$$

where respectively:  $\varepsilon(t)$  - recovery on time,  $\varepsilon_\infty$  - equilibrium recovery,  $\bar{\varepsilon}$  - average value of recovery;  $\check{\varepsilon}_i$  - value of recovery calculated from the model in the  $i$ -th point;  $\varepsilon_i$  - experimental value of recovery in the  $i$ -th point;  $n$  - number of experimental points. The value  $e_\infty$  was changed constant step value to reach the maximum value of the curvilinear correlation coefficient.

Figs 17 - 21 show the empirical values as points with the model curves of flotation kinetics. In all the



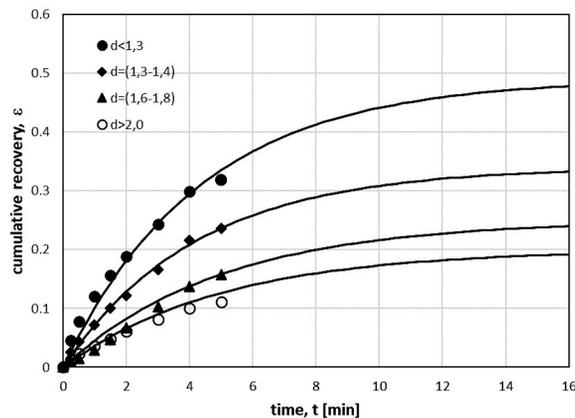
**Figure 19.** Flotation kinetics for density fraction on average 1.6 Mg/m<sup>3</sup> of different particle size; particle size (100-200) μm,  $e = 0.63(1 - e^{-0.52 t})$ ,  $R^2 = 0.9942$ ; particle size (200-315) μm,  $e = 0.54(1 - e^{-0.35 t})$ ,  $R^2 = 0.9979$ ; particle size (315-400) μm,  $e = 0.25(1 - e^{-0.20 t})$ ,  $R^2 = 0.9898$



**Figure 20.** Flotation kinetics for density fraction above 2.0 Mg/m<sup>3</sup> of different particle size; particle size (100-200) μm,  $e = 0.68(1 - e^{-0.36 t})$ ,  $R^2 = 0.9951$ ; particle size (200-315) μm,  $e = 0.28(1 - e^{-0.245 t})$ ,  $R^2 = 0.9895$ ; particle size (315-400) μm,  $e = 0.20(1 - e^{-0.20 t})$ ,  $R^2 = 0.9976$ ; particle size (400-500) μm,  $e = 0.25(1 - e^{-0.20 t})$ ,  $R^2 = 0.9898$

cases the curvilinear correlation coefficient is larger than 0.98. Model dependences  $\varepsilon(t)$  are given by each figure. Based on the experimental data it can be concluded that with the growth particles density (increase ash content) the permanent adhesion decreases and the detachment process increases. This is clearly seen in Figure 20 where maximum values of recovery are low and do not exceed 30%. In addition, the flotation progress over time is very negligible. It can be assumed that all the particles of the non-zero rate of exposition of the coal substance on the particle surface are subjected to adhesion to bubbles in the first and latest steps.

Then all the particles can flow into the foam. Yet the classical three-phase contact is formed on the surface of the particles with a large perimeter (permanent adhesion) when the particles density is low and high coal content (low ash content) or with a smaller perimeter contact for low content of coal substance (high ash content) and a low rate of expositions [13]. Impact of particle size fraction on flotation recovery is very important and explicit. This is visible for enriching each sample with a fixed density of fractions- Figures 17-20. Fine particles of size fraction (100-200) $\mu\text{m}$  are best for flotation and the least worst coarse particles of size fraction (400-500) $\mu\text{m}$ . The highest recovery values were recorded for the flotation of particles with the lowest density below 1.3  $\text{Mg}/\text{m}^3$  and for the density fraction (1.3-1.4)  $\text{Mg}/\text{m}^3$  i.e. for low-ash particles and height content of coal - Fig.17, and 18. The Figure 21 shows the results of flotation tests for the selected particle size fraction (315-400) $\mu\text{m}$  as function of density fraction. The obtained results confirm the predictions that particles of low density floated better than the particles with a high gangue content i.e., high density and low carbon



**Figure 21.** Flotation kinetics for particles size 315-400  $\mu\text{m}$  for different density fractions. density fraction  $< 1.3$  [ $\text{Mg}/\text{m}^3$ ],  $e = 0.49(1 - e^{-0.24t})$ ,  $R^2 = 0.9936$ ; density fraction (1.3-1.4) [ $\text{Mg}/\text{m}^3$ ],  $e = 0.34(1 - e^{-0.237t})$ ,  $R^2 = 0.9979$ ; density fraction (1.6-1.8) [ $\text{Mg}/\text{m}^3$ ],  $e = 0.25(1 - e^{-0.20t})$ ,  $R^2 = 0.9898$ ; density fraction  $> 2.0$  [ $\text{Mg}/\text{m}^3$ ],  $e = 0.20(1 - e^{-0.20t})$ ,  $R^2 = 0.9979$

content. This is confirmed by the analysis results of the relationship between the ash content and the fraction density at a fixed particles size, which describes the proposed empirical model [32]

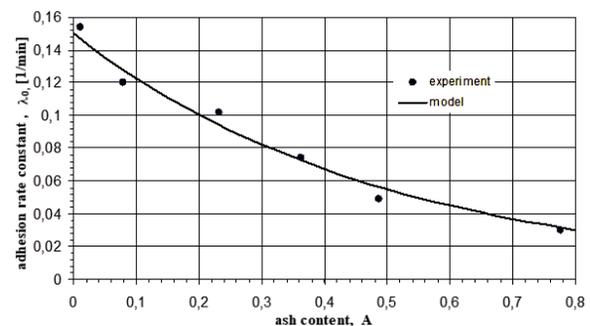
$$\lambda_0(A) = a e^{-bA} \quad (19)$$

where:  $a$  and  $b$  – empirical constants while  $a = \lambda_0(0)$  is the permanent adhesion rate constant for the ash free coal. It will depend on the coal type, its petrological composition, surface oxidation rate, particle size, etc., and physicochemical and dynamic conditions in the flotation chamber. The constant  $b$ , with the above factors, will depend first of all on the composition of coal mineral matter. For the sample of (200-315)  $\mu\text{m}$  particle size tested and the dependence  $\lambda_0(A)$  is expressed by the formula in Eq.20 and the graphical image is present Fig.22:

$$\lambda_0(A) = 0.15 e^{-2.4A} \quad (20)$$

The index of curvilinear correlation is 0.9999022 which proves good compatibility of the proposed model dependence with experimental values.

The quantitative development of the PSG model for particle adhesion in flotation process based on the analogy of the agglomeration process is satisfactory. However, please note that for example, at high energy dissipation rate, the rate of agglomeration will be high (Eqs. 7-9) so will be the kinetics of flotation for the particles of the same size. However, the drag and inertial forces for the agglomerates of that size will not be the same as finer agglomerates. This may give the impression that the authors are taking all positive forces in flotation kinetics, neglecting every negative effects that might cause detachment particle(s) from bubble. The nature of this detachment is not the identical for the particle that directly attaches to the bubble or through a group of agglomerates. It should be indicated here that the necessary condition for the useful substance floated to concentrate in foam flotation is the formation of a flotation aggregate (permanent adhesion of gas/particle). However, first of all, there must be a collision between particle and



**Figure 22.** The permanent adhesion rate constant as a function of ash content in the sample of particle size distribution (200-315)  $\mu\text{m}$ ;  $\lambda_0 = 0.15 e^{-2.4A}$  [13]



bubble and the kinetic energy of a particle should be within a certain range of values; on the one hand large enough to overcome the barrier potential of interaction between particle and bubble, and on the other small enough to make it a permanent connection. In other words, so that particle should not be detached from bubble.

Based on the stochastic model of flotation kinetics [13, 32] the particles attached permanently will be raised to the froth product. Whereas, the recovery results of a useful mineral in the froth product can be calculated from formula (16) for  $\varepsilon_\infty=1$  on the assumption that after the appropriately long time (theoretically infinitely long) the whole floatable mineral will be transferred to the foam (all particles connected with the air bubbles will find their way to the froth product). However in the technological condition exist the detachment process of particles from the bubble surfaces. Therefore in the case of the flotation model, a detachment process must be included in the following scheme of the process of adhesion as the balance of the number of particles in flotation:

$$\text{permanent adhesion} = \text{resultant adhesion} - \text{detachment} \quad (21)$$

Therefore accordingly, the recovery of particles raised to the froth product is analogical like the formula in Eq.16 and expressed as:

$$\varepsilon(t) = \frac{\lambda_o}{\lambda_o + \mu_o} \left[ 1 - e^{-(\lambda_o + \mu_o)t} \right] \quad (22)$$

Where  $\varepsilon_\infty = \frac{\lambda_o}{\lambda_o + \mu_o}$  and  $k = \lambda_o + \mu_o$ , and the constant  $\lambda_o$  denotes the permanent adhesion rate constant whereas the flotation rate constant  $k$  is the resultant adhesion rate constant and detached rate constant ( $\mu_o$ ).

Thus, for the value of flotation rate constant ( $k$ ) to remain unchanged while permanent adhesion ( $\lambda_o$ ) decreases with the increase in ash content in the coal sample. It means that the intensity of particles detachment from air bubbles increases. This leads to a low efficiency of the process (the equilibrium recovery decreases with increasing particle density and size).

## 5. Conclusions

The PSG method can be used for analysing coal flotation from the point of view of agglomerates formation due to effective collisions. It was assumed in the simulation, that collisions caused the formation of bigger particles not connected with chemical bonds forces, and their increase was associated solely with collisions. It was revealed that the number of particles with definite radius in

a given volume depended on density and remains unchanged over the agglomeration process. This population, however, was classified in particular size groups as the process continued. The suggested solution focused only on the dynamics of changes of the particle size for assumed mixing energy values  $\varepsilon=0.01 \text{ m}^2\text{s}^{-3}$ , and  $0.0001\text{m}^2\text{s}^{-3}$ . given particle sizes in group  $n_1$  (100, 200, 300, 400  $\mu\text{m}$ ) and agglomeration parameter  $\alpha$ : 1; 0.8; 0.5. The density of particles is a very important factor because we get the population of particles with a definite radius  $N_0$  for a constant mass of the sample 1 Mg. The effect of density and size of particle on the time of agglomeration was compared in Figure A1 and Figure A2. The conclusion from PSG modelling is, that particles with large radius and high density collide easier. This suggests that the most collisions are particles with a low content of volatile and combustible and a high content of gangue. It should be noted that the classical three-phase contact (necessary condition for creating a flotation aggregate) is formed on the surface of the particles with a large perimeter when the particles density is low with high coal content or with a smaller perimeter contact for low content of coal substance and a low rate of expositions [13]. The obtained experimental results of batch flotation shown, that particles of low density floated better than particles with a high gangue content, i.e. high density and low of carbon content. Nevertheless, creating a flotation aggregate isn't the same as taking it to the concentrate (foam). The PSG simulation results can be a quantitative measure of the collision probability under stated of process conditions. A thorough theoretical and experimental analysis of this issue will be carried out in the near future.

The duration of the process was definitely shortest for bigger particles, i.e. with radius 300 and 400  $\mu\text{m}$  regardless the assumed mixing energy. The efficiency of collisions was significantly influenced by agglomeration coefficient  $\alpha$  which accounted for among others viscosity of the medium. This is confirmed by the collective representation of simulation results for parameter  $\alpha$  in Figure A3 where the shortest agglomeration time  $t^*$  below 0.01 was obtained for particles with radius 400  $\mu\text{m}$ . Unlike particles with radius 100  $\mu\text{m}$  time  $t^*$  was over one order higher and totalled to  $t^*= 0.016$  and a certain number of particles belonging to group  $n_1$  remained in the solution and did not undergo agglomeration. Analogous results were presented for parameter  $\alpha=1$  (Figure A4)

The efficiency of collisions was significantly influenced by agglomeration coefficient  $\alpha$  which accounted for, among others, viscosity of the medium.



## Appendix A

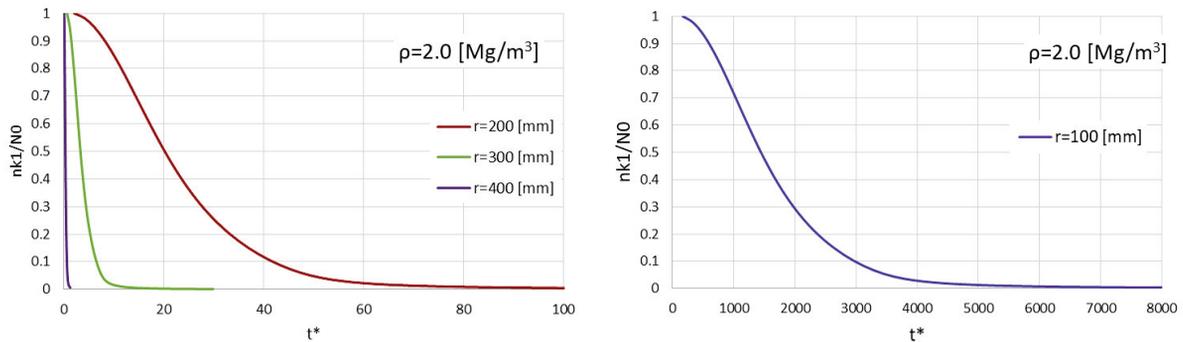


Figure 1. Effect of particle radius on agglomeration ( $n_i/N_0$ ) of particles with density  $2\text{Mg/m}^3$

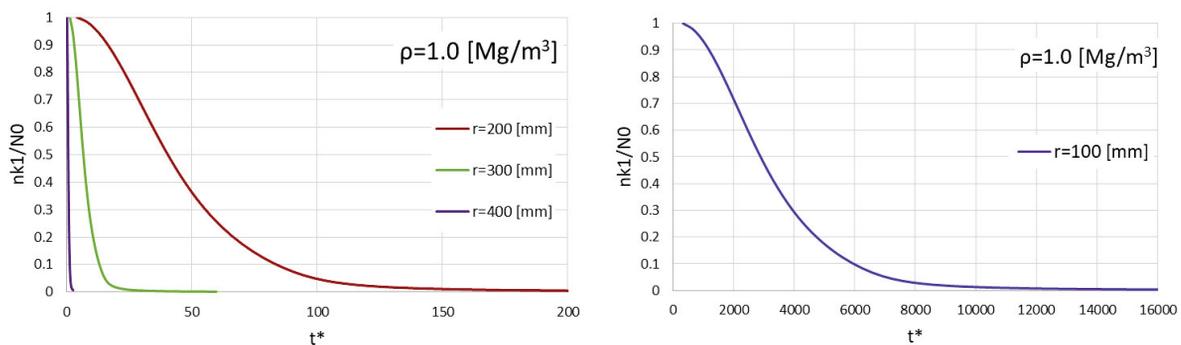


Figure 2. Effect of particle radius on agglomeration ( $n_i/N_0$ ) of particles with density  $1\text{Mg/m}^3$

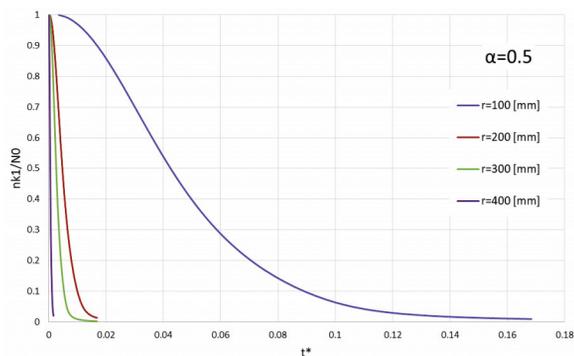


Figure 3. Effect of particle size on agglomeration of coal particles with radius  $r=100, 200, 300, 400 \mu\text{m}$  for  $\alpha=0.5$

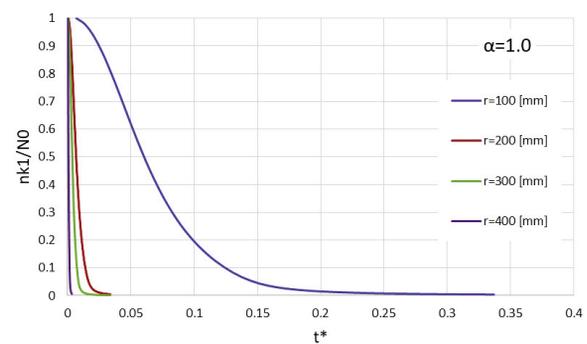


Figure 4. Effect of particle size on agglomeration of carbon particles with radius  $r=100, 200, 400 \mu\text{m}$  for  $\alpha=0.8$

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## METODA GRUPISANJA ČESTICA PO VELIČINI KAO KONTROLNI SISTEM EFIKASNOSTI PROCESA FLOTACIJE NA PRIMERU UGLJA

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### Apstrakt

*Aglomeracija čestica uglja tokom flotacije može se analizirati metodom grupisanja čestica po dimenzijama. Urađeni su brojni eksperimenti da bi se teoretski objasnio efekat aglomeracije čestica ugljenika, ali rezultati još uvek nisu potpuni. U ovoj studiji iskorišćen je PSG metod da bi se analizirale aglomeracione grupe čestica ugljenika početnih dimenzija 100-400 μm, zadržavajući ukupnu količinu čestica. Utvrđene su dimenzije čestica sa određenim radijusom i gustom za 1 Mg uglja. Analiziran je uticaj koji gustina i dimenzije čestica imaju na aglomeraciju uz date energije mešanja i parametar  $\alpha$ . Utvrđeno je da su dimenzije čestica imale uticaja na njihovu aglomeraciju. U analiziranim slučajevima bezdimenzijski parametar turbulencije kolizije  $t^*$  potreban za aglomeraciju čestica u grupama čestica određenih dimenzija bio je značajno kraći za čestice čija je početna veličina bila 300 i 400 μm. Promena energije mešanja nije imala uticaja na aglomeraciju čestica. Teorijska analiza zasnovana na kompjuterskim proračunima bila je dopunjena analizama procesa flotacije uglja, vodeni model. Eksperiment se sastojao od uvođenja sredstva za proizvodnju pene u obliku vodenog rastvora heksanola, a bez menjanja pH vrednosti pulpe, snižene vrednosti površinske napetosti, i povećanja disperzije vazduha u suspenziji. Eksperimentalni rezultati su predstavljeni u obliku kriva kinetike flotiranja. Ispostavilo se da su fine čestice dimenzija 100-200 μm bile najbolje za flotiranje, za razliku od grubih čestica dimenzija 400-500 μm.*

**Ključne reči:** Flotacija uglja, PSG metod, Aglomeracija, Modeliranje

