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MODELING OF REAL COMBAT OPERATIONS

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Abstract: *This paper discusses the model of an offensive air-ground combat operation, a product of a specific modeling - operational planning and targeting process. The modeling was performed using approximate methods of numerical integration based on Lanchester's law of combat. Operational factors and their implications for outcome are used for modeling a spetial but still current, offensive air-ground campaign/operation "Desert Storm". The model considers relevant factors, such as combat capabilities, the number of forces and consumption of resources and forces attrition in combat, in order to enable planning and prediction of the winning side in battle. The optimal solution of the problem as a useful strategic tool, based on the attrition model of combat has a practical benefit for making the right management decisions in operational planning process. This approach corresponds to project management, considering the same operational factors and the ability to manage them.*

Keywords: *Force attrition, combat operations, approximateive model*

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1. Introduction

Modeling of strategic operation (campaign) with various types of air, land, maritime and joint force operations and their variations (Doctrine, 2010) is based on operational planning and targeting process. The product of these complementary processes is a War plan for carrying out a strategic operation, based on military doctrine, which enables the use of forces and resources to achieve specific desired effects and operational objectives (Doctrine, 2021). The method of engaging the forces depends on the assessment of the operational situation and the operational capabilities of the forces and combat systems. This is an experiential, expert method, based on accumulated knowledge from case studies and analysis of warfare in past battles.

The aim of the research in the paper is to consider the possibility of applying modeling and simulation using numerical integration methods, based on the Lanchester's equation method, in the operation planning of the use of forces and predicting the outcome of complex military operations. The purpose of their application is predictive warfare, with the possibility of optimal programming of a feasible plan. The expected results are the improvement of the efficiency of the

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operational planning process and the possibility of validating of military doctrines, by checking the effectiveness and efficiency of the armed forces in modeled combat operations.

In the second part of the paper, the theoretical foundations of the used method are given, in accordance with attrition law of combat as the basis of the simulation model, and a brief historical overview of its beginning and evolution.

The third part presents the experimental modeling. The paper discusses the modeling of Operation "Desert Storm" according to the available historical data, in order to verify the model. After that, the method of engaging the forces of both adversaries in two hypothetical operational variants is considered, but from the perspective of Iraq, in order to emphasize the importance of applying scientific methods in the process of operational planning. In the first case (elusive defense or maneuver attack), the way of engaging the forces implies achieving a goal, delaying the battle or creating a certain effect, such as achieving air superiority in the area of operations. The second case (decisive attack or positional defense of the opponent) means an attack, a direct fight until the neutralization or destruction of the defending opponent or the defense of the assigned space with fixed forces until the neutralization of the enemy's attack. Both cases have different effects on combat capability and the rate of expenditure of forces in attack or defense.

The fourth part of the paper contains the validation of the model based on comparison of historical data and experimental results. After validation, a discussion of the results for a real operation and two hypothetical scenarios, each according to different modes of force engagement and combat effectiveness, was performed. This part also analyzes correlation of war facts and results in relation to Lanchester's principle of concentration of forces.

2. Methodology

One of the first attempts to scientifically describe the process of armed combat is Lanchester's (1916) mathematical model, widely known as Lanchester's equations or the law of combat. Originally, Lanchester's work represents two mathematical laws of combat which explains two historically modes of combat. The first law is linear and is immanent to ancient and medieval battles. The second, quadratic law is modern warfare, since the beginning of the appearance of firepower and assumes the decisive influence of the concentration of force (MacKay, 2002). If the rate of depletion (α) and (β) does not change over time in the battle of homogeneous forces of two opponents, the quadratic law can be expressed as a system of ordinary differential equations (Washburn, 2000):

$$\frac{dX}{dt} = -\beta * Y \qquad \Lambda \qquad \frac{dY}{dt} = -\alpha * X \qquad (1)$$
$$for \quad X > 0 \quad \Lambda \quad Y > 0$$

The winner is the side with a better concentration of force at the right time and place or the one with a larger army. It also means that a numerically inferior force must possess much greater combat capability to achieve equality of power (Lanchester, 1916). But Osipov (1915), Lanchester's contemporary, who published his work at the same time and independently, came to the conclusion that in addition to the numerical strength of the opponent, there are other factors that affect the speed of inflicting losses, and ultimately victory.

Lanchester's differential equations are the basis for the application of the more complex Deitchman's (1962) law of mixed combat, which enables the simulation of the combat dynamics of qualitatively different opponents. For example, the warfare of two opponents in guerilla or conventional style combats. This problem could be solved by combination of quadratic and linear law (Darcom Pamphlet, 1979). There were a lot of papers published about historical battles like: Ardennes Campaign (Fricker, 1997), the battle of the Iwo Jima (Engel, 1954) and Kursk (Lucas, 2004).

The shortcoming of the methodology of the basic Lanchester's model, as Osipov noted, is that it considers a constant rate of force attrition and does not consider the influence of other different parameters such as: maneuver, tactical decisions, logistics, firing process, operational situation factors (weather, geography, etc.). These factors affect the rate of attrition and the manner in which forces are used (Tactic), and ultimately the law of battle. Given these facts, it is understandable why the basic model is not suitable for modeling a real war combat, but an attempt was made to eliminate the shortcomings of the basic method.

Thus, Helmbold (1965) observed that the relative speed of spending the opponent's forces (combat capabilities) depends on the ratio of the size of the forces, but if this ratio is "extremely large" it will happen that they cannot all be in battle. It is also important to mention the work of Bonder- Farrell (1970), who emphasized the influence on the coefficient of force attrition in the quadratic model as a function of time at a constant rate of change of the distance.

Confirmation of the general views on the concentration of forces can be seen in the works of McCartney M. (2023) and Kostić and Jovanović (2023). Of particular interest is the paper (Han et al., 2022) related to the application of Lanchester's equations to warfare in the cognitive domain, which may become key to winning future intelligent warfare, where one side may gain an exponential advantage in improving combat power.

More recent studies of combat operations include irregular warfare, insurgencies, terrorism, etc., with an emphasis on the role of target information and include multilateral situations in which several players are involved in the conflict, which is an example of the war in Syria (Kress, 2020). Lanchester's work on combat modeling inspired research on developing models to support decision making under conditions of uncertainty. But, due to their great simplification, they were relatively inadequate. In order to overcome the mentioned shortcomings, new models and techniques were developed. One of the proposed models was elaborated by Gerardo (2022), in his thesis "Automated Support for Battle Operational-Strategic Decision-Making"

The basic Lanchester's model considers only a homogeneous structure of forces. In reality, warfare is a struggle of different types and branches of armed forces (air force, navy and ground forces, artillery, armor etc.). Modeling a real battle implies a combat structure of forces, which is always heterogeneous. An illustrative description of the problem can be seen when considering a fight between two opponents, whose forces are heterogeneous.

In this combat model there are several assumptions that must be considered: a)

- The effects of force attrition are additive for each specific combat element, without mutual support and synergistic effects;
- The attrition efficiency of any combat system is proportional to the number of elementary units of that type and
- Each part of the force will, in accordance with its combat capabilities, cause losses to all available elements of the enemy.

Opposing forces consist of different types of combat systems and elementary units of forces (X_i) that form an order of battle or task force (Xij) for a special combat action such as air bombardment or surface attack by ground forces on certain parts or combat systems of the opposing forces (Y_j) and vice versa.

This represents the distribution of fire, which is according to Taylor (1980a), the allocation factor:

$$\boldsymbol{\psi_{ij}} = \boldsymbol{Y_{ij}}/\boldsymbol{Y_j} \qquad \Lambda \qquad \boldsymbol{\phi_{ij}} = \boldsymbol{X_{ij}}/\boldsymbol{X_j}, \tag{2}$$

$$where: \quad \sum_i \psi_{ij} = 1 \qquad \Lambda \qquad \sum_j \phi_{ij} = 1$$

According to assumptions and heterogeneous force's structure the final model is:

$$\frac{dX_i}{dt} = -\sum_{j=1}^{n} \psi_{ij} * b_{ij} * Y_j \quad \Lambda \quad \frac{dY_j}{dt} = -\sum_{i=1}^{m} \phi_{ij} * a_{ij} * X_i, \tag{3}$$

with: $X_i(0) = X_i^0 \;\; \Lambda \;\; Y_j(0) = Y_j^0 \quad \Lambda \quad X_i > 0,\; Y_j > 0 \;\; \Lambda \;\; 0 < \phi_{ij}, \psi_{ij} < 1$

If we redefine the coefficient of loss in (4) by absorbing the allocation factor into it, we get, according to Taylor (1980) and Westbourne (Caldwell et al., 2000), a linear model (5):

$$\psi_{ij} * b_{ij} * Y_j = b_{ij} * Y_{ij} \quad \Lambda \quad \phi_{ij} * a_{ij} * X_j = a_{ij} * X_{ij}\,, \tag{4}$$

$$\frac{dX_i}{dt} = -\sum_{j=1}^{n} B_{ij} * Y_j\,, i = 1, \dots, m \quad \Lambda \quad \frac{dY_j}{dt} = -\sum_{i=1}^{m} A_{ij} * X_i\,, j = 1, \dots, n \tag{5}$$

These are the analytical equations of the combat model (Taylor, 1980) where two adversaries have a heterogeneous force structure such as: armor, artillery, combat aircraft, combat helicopters, etc. In such complex models of the battle of heterogeneous forces, by dividing the depletion process into several different sub-processes, the attrition rate of forces (Aij) and (Bji) approaches reality. The reason for that is it takes into account complex functions such as: combat capabilities of the weapon system, targeting process, distribution of weapons, etc. On the other hand, the solution to the problem seems very simple, but it is an illusion, because the real solution is very complex (Petric, 1974), even impossible to solve (Taylor, 1980a). This becomes evident in combat model of two opposing forces with three or more combat elements (Hsiao and Guu, 2004). As Taylor (1980a) said„ for small-scale operations it may be possible to reasonably represent force interactions and attendant attrition rates with a few differential equations, but for large-scale operations of conventional armed forces the same approach might well involve hundreds (and possibly even thousands) of differential equations tied together through battlefield operations ". This statement is confirmed by Petric (1974) and Milovanovic (1988). Based on these arguments and considering the complexity of the methodology for practically solving this problem, Taylor (1980a) pointed out that only a few useful analytical models have been developed. For modeling large-scale combat operations, such as strategic land-sea-air operation or a campaign, the Aggregated Force model (Kostic et al., 2023) and the Detailed Lanchester model are more suitable (Darcom Pamphlet, 1979). Monte-Carlo simulation is more suitable for small scale combat model (bellow battalion force level). It is interesting to see the comparison of the difference in results between these two methods (Yildirim et al., 2009), obtained by applying the simulation in the "JANUS" simulator for a small-scale combat model. Regardless of the stochastic and deterministic nature of these methods, many authors consider both models quite similar in terms of results, but the deterministic model is more practical to use (Taylor & Brown, 1982).

A modern approach to the methodology of solving Lanchester-type models can be found in Caldwell et al. (2000) "Enrichment of Lanchester-type models". This methodology is based on developed procedures for the numerical solution of Lanchester-type models. One of the simplest numerical methods (Euler-Cauchy) works very efficiently in both cases. A significant contribution to the development of this method was made in the works: Washburn and Kress (2009) and Caldwell et al. (2000). Numerical methods allow solving complex problems where analytical methods cannot help (Radunovic, 2003). They allow solving complex problems, for which the solutions satisfy a certain degree of accuracy, which means that there is a certain "error" that is within the limits of tolerance in relation to the analytical solution. As mentioned earlier, the simplest numerical method, which confirms this claim and which has proven to be very good in the practical application of conflict description, is the Euler-Cauchy method.

The application of this method in the combat model of heterogeneous forces with a sufficient degree of accuracy describes the combined linear and quadratic law. Lanchester's equations, according to applied mathematics, behave very well compared to other differential equations, which are applied in science and engineering. The reasons for this are the factors that are immanent to them: monotony, no singularity and they have stable solutions (Washburn and Kress, 2009). This is a "lucky" advantage, because the simulation method, which applies a simple time step, is easy to implement in a combat model. The essence is to implicitly replace the model of Lanchester's quadratic law, described by differential equations, with the Euler-Cauchy method of recurrent equations, based on a simple, finite approximation (Caldwell et al., 2000): $\left(\frac{dX}{dt}\right)$ or $\left(\frac{dY}{dt}\right)$, where:

$$\frac{dX}{dt} \cong \frac{x(t+\Delta t)-x(t)}{\Delta t} = -B\,(\beta, x, y, t..) \quad \Lambda \quad \frac{dY}{dt} \cong \frac{y(t+\Delta t)-y(t)}{\Delta t} = -A\,(\alpha, x, y, t..) \qquad (6)$$

Although there are more efficient methods, according to Washburn (2000), only Euler's numerical method is applicable, where time is discretely determined and differential equations are replaced by difference equations. If the time increment is determined by (Δt), the difference equations, which replace the differential, are (Washburn and Kress, 2009):

$$x(t+\Delta t) = x(t) - \beta\Delta y(t) \quad \Lambda \quad y(t+\Delta t) = y(t) - \alpha\Delta x(t) \qquad (7)$$

Since the initial values of the forces: x(0) and y(0) are already given, equations (10) determine x(Δ) and y(Δ) as the next terms of the series (Stevanovic et al., 2004). The final state can be defined by the expiration of time or total and individual losses, that is, by reaching a certain level of strength of one of the opponents, after which his surrender or defeat occurs. If one of the values becomes negative during the calculation the value (0) is determined instead (Washburn and Kress, 2009). The theory of ordinary differential equations ensures that in the limits as (Δt) approaches (0), the solution of the recurrent equations approaches the exact solution of the ordinary differential equations. Although the time increment (Δ) is a conventionally "small" number, it should be understood that it is more appropriate to respect reality, which in this case means that large operations are conducted as a series of special events, smaller operations or tactical battles, which can also be considered as discrete events. As an example, historical naval battles can be used (Washburn, 2000), where a cannon salvo is considered as such a discrete event.

Effectiveness in practical application can be seen in the works of (Washburn, 2000) on the example of two models. The first model is the anti-submarine conflict in the Atlantic during World War II, based on Morse and Kimball (1950) which describes a battle in which German submarines sank Allied ships in convoys, protected by anti-submarine forces. The second model describes the struggle of two aviation elements, attack aviation and bombers, against air defense artillery and ground facilities.

Considering the stated facts about combat operations and modeling methods, stochastic models are obviously more applicable for the study of stochastic dynamic processes, such as combat operations of smaller forces, as well as combat operations in general, and therefore air operations. However, certain advantages imply the use of deterministic models, such as; contribution to a better understanding of real phenomena due to the theoretical description of real systems. It is important to emphasize that the behavior of real systems can be predicted with a certain probability and used as a tool for designing and managing processes. This means that in this paper, battle modeling is limited to a discrete, dynamic and deterministic model (Radenkovic, 1999), theoretically, structurally and functionally complex and partially limited to air/ground combat actions. Due to the nature of war conflicts and the limited availability of relevant facts, modeling was done, and then evaluation of findings based on the results of a case study, a representative historical example of a strategic air operation (campaign) "Operation Desert Storm "(Cordesman, 1994).

3. Experimental

The essential question is both complex and difficult to answer: whether the created combat model behaves consistently, in a way that corresponds to reality? The key is the assessment of the parameters that are an integral part of the model. By practical verification, on the example of a combat situation, a comparison can be made and the real applicability of the approximate method can be verified. According to the formal criteria for determining the validity of the model, the observed operation "Desert Storm" can be viewed as a real system, described at a higher level, while the created model is at a lower level of description. The essence of the formal model verification procedure consists in mapping (X) by means of which it is possible to move from each state of the basic model to the corresponding state of the simplified model. If such a mapping exists, then it might be concluded that the input-output behaviors of the basic and simplified models are the same for the given conditions (Radenkovic, 1999). In this case, it can be said that the simulation model is adequate to real model and can be used to describe reality. Validation was carried out by simulating the "Desert Storm" operation model, due to the possibility of using open sources with a wealth of statistical data such as: the number of flights performed (Survey I, 1993), the number and type of target objects (Survey IV, 1993b), ammunition and fuel consumption, expected effects of actions, etc. For the sake of simplicity of application and data processing, a certain approximation was made, which refers to the generalization of the forces and the determination of their combat capabilities.

The considered forces in the model are listed in table (1) and (2) (Survey V, 1993c), according to their affiliation and numerical strength status.[†]

Table 1.Comparative strength of Coalition (X) and Iraqi Air Forces (Y) at start of operation

X_F	X_B	X_{STELTH}	X_{EW}	$X_{F/A}$	X_{SEAD}	X_{AH}
205	420	40	59	2150	450	681
Y_{F1}	Y_{F2}	$Y_{F/A}$	Y_{EW}	Y_{SEAD}	Y_{RECC}	Y_{AH}
56	164	908	20	12	32	442

Source: (Survey V, 1993c)

[†] Meanings of variables are:

- X_F-fighter airplane,
- X_B-bomber,
- Y_{FB}- fighter-bomber,
- X_{STH}-STEALTH bomber (F-117A),
- X_{EW}- electronic warfare,
- X_{FA}-fighter attack,
- X_{SEAD}-suppression of enemy air defence,
- X_{AH}-attack helicopter,
- $X_{ADL(L,M,S)}$ - air defense (long, medium, short range),
- X_A-artillery,
- X_E-enfantry,
- X_{ARM}-armory; tanks - (T) and armored personnel carriers-(APC) and
- Y_{WMD}-weapons of mass distruction.

Table 2. Comparative strength of Coalition (X) and Iraqi Air Defense and Land Forces (Y) by types at start of operation

X_{ADL}	X_{ADM}	X_{ADS}	X_{WMD}	X_A	X_E	X_{ARM}
96	44	0	0	4550	1.110.000	8500/15000
Y_{ADL}	Y_{ADM}	Y_{ADS}	Y_{WMD}	Y_A	Y_E	Y_{ARM}
18	270	558	110	4140	1.106.000	7000/11200

Source: (Survey V, 1993c)

The method of numerical integration, (Euler-Cauchy method) is programmed with a set of analytical equations that describe the „attrition" or combat losses of each particular force element, according to the way it is used in operation or campaign.

When using this method, the following assumptions are made:

- The impact of the maneuver of the operational group is related to the speed of spending forces and has no other influence;
- The operation takes place successively in cycles of three days (72 hours) until completion (total of 14 cycles);
- Before each phase (sequence) of the operation, a „targeting" process is carried out, which implies a process of selecting and prioritizing targets. This process takes into account operational requirements, combat capabilities (reassessed for the impact of attrition, command, logistics, etc.), determines battle force compositions, and there are no changes until the end of a given stage or sequence.

The following system of numerical equations is a mathematical expression of the conceptual model of the conflict regarding attrition of special, dedicated forces in various combat actions: offensive air, combat support, strategic attack, air defense operations and other actions in the conflict such as ground force attack/defense and strike by weapons of mass destruction (Seung, 2013).

Blue force

$$\sum_{i=1}^{m} X_i^{t+1} = \sum_{i=1}^{m} X_i^{t} - \sum_{j=1}^{n} B_{ij}^{t} * Y_j^{t}, \qquad i \in \{1, \dots, m\}, j \in \{1, \dots, n\} \, \Lambda \;\; t \in [0, T\,], \qquad (8)$$

where: $X_i > 0,\;\; Y_j > 0 \;\; and \;\; 0 < \; B_{\text{ij}} < 1$

Blue (Coalition Force) has m = 3 types of combat forces, which are then grouped according to their type and purpose, and deployed in the appropriate order of battle**:**

- air force $\text{X}_{\text{I,}} \in \{\text{X}_{\text{F}}, \text{X}_{\text{B}}, \text{X}_{\text{STH}}, \text{X}_{\text{EW}}, \text{X}_{\text{FA}}, \text{X}_{\text{SEAD}}, \text{X}_{\text{AH}}\},$
- air defense $\text{X}_{\text{I,}} \in \{\text{X}_{\text{ADL}}, \text{X}_{\text{ADM}}, \text{X}_{\text{ADS}}\},$
- army force $\text{X}_{\text{I,}} \in \{\text{X}_{\text{A}}, \text{X}_{\text{E}}, \text{X}_{\text{ARM}}\}$

Red force

$$\sum_{j=1}^{n} Y_j^{t+1} = \sum_{j=1}^{n} Y_j^{t} - \sum_{i=1}^{m} A_{ij}^{t} * X_i^{t}, \qquad i \in \{1, \dots, m\}, j \in \{1, \dots, n\} \, \Lambda \;\; t \in [0, T\,], \qquad (9)$$

where: $X_i > 0,\;\; Y_j > 0 \;\; and \;\; 0 < A_{\text{ij}} \; < 1$

Red (Iraq Force) has m = 4 types of combat forces, which are then also grouped according to their type and purpose, and deployed in the appropriate order of battle:

- air force $\text{Y}_{\text{j,}} \in \{\text{Y}_{\text{F1}}, \text{Y}_{\text{F2}}, \text{Y}_{\text{FB}}, \text{Y}_{\text{EW}}, \text{Y}_{\text{SEAD}}, \text{Y}_{\text{Recon}}, \text{Y}_{\text{AH}}\},$
- air defense $\text{Y}_{\text{j,}} \in \{\text{Y}_{\text{ADL}}, \text{Y}_{\text{ADM}}, \text{Y}_{\text{ADS}}\},$

- army force $Y_{j,} \in \{Y_A, Y_E, Y_{ARM}\}$,
- tactical ballistic missiles $Y_{j,} \in \{Y_{WMD},\}$

Based on the works of Washburn (2000) and Morse and Kimball (1950) as well as the operational facts of the Desert Storm campaign an estimate of possible forces attrition rates for Blue and Red was created. This attrition rate represents the mutual conditioning of combat capabilities expressed by the probability of destruction of a red combat system by a blue combat system and vice versa, the number of probable target objects or combat elements (units) that can be acted upon in one cycle of actions and the state of forces, as a consequence of the operational situation (Gaver and Jacobs, 2000).

According to the Bonder-Farrell (1970) technique, depending on the achieved effects and the assessment of the state of forces after the completion of each stage or cycle of the operation, there is a change in the efficiency of the rate of inflicting losses. Due to the realistic modeling of reality and the influence of the state of forces operational capabilities at the end of each stage of the operation, it is necessary to perform a new assessment of effectiveness (Davis, 1989). For this purpose, meta-model was used, which consider the change in the "state of forces" at the end of each operational cycle (Taylor, 1980a). This time period was taken into account, based on the empirical assumption that the execution of the planned tactical actions, which make up the whole of the operation, will not be significantly disturbed by the influence of the change in the state of forces. The change will have an impact in the next cycle. This change is given by the following, general relations (Yunzheng et al., 2022):

$$A_{ij}^t = m_{ij} * k_{ij} * p_{ij} \quad \Lambda \quad B_{ij}^t = n_{ij} * z_{ij} * q_{ij}, \tag{10}$$
$$i \in \{1, \dots, m\} \quad \Lambda \quad j \in \{1, \dots, n\} \quad \Lambda \quad t \in [0, T]$$
$$\text{where:} \quad 0.3 \le k_{lj}, p_{ij} \le 1 \quad \Lambda \quad 0.3 \le z_{lj}, q_{ij} \le 1$$

- m_{ij}/ n_{ij} - the average number of expected combat actions during one operational cycle in relation to the size of the task forces defined by the probable order of battle required for the execution of the mission;
- k_{ij} / z_{ij} - blue and red "strength status" characteristics, which represent the degradation of operational capabilities and capabilities of special elements of the battle order after each stage or operational cycle and
- p_{ij} / q_{ij} - the combat capabilities of elementary units of the task force for the destruction of each special element of the enemy forces that can be combated.

Experiment will include real course of action as (CA 1), during the real campaign and two more. Course of action (CA 2) has been considered as a case study if Iraqi forces attempted to fight defensive operation. Course of action (CA 3) was considered as an even more aggressive approach where Iraqi forces attempted to continue offensive operation as strategic attack operation into Saudi territory. Overviews of actual flight efforts of various combat missions and actual ammunition consumption during the campaign are given in Table (3) and Table (4). Real losses during 42 days of "Desert Storm" are presented in table (5) and (6).

Table 3. The real number of completed flight missions by Coalition Force

Strategic attack and interdiction	38277
Air support	6128
Offensive or defensive counterair	19419
Suppression of Enemy Air Defense (SEAD)	4326
Electronic warfare (EW)	2918
Reconnaissance	3236
Combat flights	68150

Operation support	51421
Overall flights	119571

Source: (Survey V, 1993c)

Table 4. The real expenditure of air weapons by Coalition forces

Targeting / Combat cycles	14
Combat mission flights	68150
Air to air missiles	174
Air defense missiles	360
Air bombs (dumb)	210004
Guided /cruise missiles and bombs	15605
Anti-radiation missiles	2039
Overall munitions consumption	228182

Source: (Survey V, 1993c)

Table 5. The real combat losses of Coalition and Iraqi Air Forces at the end of operation

X_F	X_B	X_{STELTH}	X_{EW}	$X_{F/A}$	X_{SEAD}	X_{AH}
1	21	0	1	28	1	23

$Y_{F1/F2F/A/EW/SEAD/RECC}$	Y_{WMD}	Y_{AH}
442	110	188

Source: (Survey V, 1993c)

Table 6. The real combat losses of Coalition and Iraqi Air Defense and Land forces

X_{ADL}	X_{ADM}	X_{ADS}	X_A	X_E	X_{ARM}
-	-	-	1535	153 / 16200	62/ 1050
Y_{ADL}	Y_{ADM}	Y_{ADS}	Y_A	Y_E	Y_{ARM}
-	-	-	2917	32.340 dead/ 75.000 wound/ 220.000 captured	3847/2880

Source: (Survey V, 1993c)

The final results of the "Desert Storm" simulation modeling are given in the overview of the state and comparative attrition of forces for Coalition and Iraq as a course of actions (CA1). The estimated dynamics of ammunition consumption and estimated number of flights are presented as course of actions (CA1) in tables from (7) to (9). Operational-tactical units of the ground army are organized into divisions and brigades, and the representation of the strength of the forces in the tables of the courses of action represents the number of elementary parts of the aggregate forces, not the actual forces of the army. The real numerical composition consisting of: tanks, armored fighting vehicles, artillery and infantry is obtained by recalculation.

Table 7. Estimated expenditure of air weapons by Coalition forces CA1

Targeting / Combat cycles	14
Combat mission flights	64681
Air to air missiles	276
Air defense missiles	369
Air bombs (dumb)	199826
Guided /cruise missiles and bombs	14300
Anti-radiation missiles	2648
Overall munitions consumption	217419

Source: Author's processing with the support of EXCEL 2010

Table 8. Comparative attrition loss estimates of Coalition and Iraqi air forces for CA1

Combat stage	X_F/Y_F	$X_{F/A}/Y_{F/B}$	X_{STLH}/Y_A	X_{EW}/Y_{EW}	X_B/Y_{RECC}	X_{SEAD}/Y_{SEAD}	X_{AH}/Y_{AH}
1	205/56	2150/164	40/908	59/-	420/32	450/-	681/442
2	205/49	2142/139	40/878	59/-	408/30	442/-	681/417
3	205/43	2138/116	40/856	59/-	404/28	438/-	681/401
4	205/36	2137/100	40/820	59/-	400/26	432/-	681/400
5	205/28	2137/90	40/783	59/-	400/24	430/-	681/399
6	205/26	2137/80	40/773	59/-	400/22	430/-	681/367
7	205/24	2137/78	40/763	59/-	400/20	430/-	681/335
8	205/22	2137/76	40/753	59/-	400/18	430/-	681/315
9	205/20	2137/74	40/743	59/-	400/16	430/-	681/295
10	205/18	2137/72	40/733	59/-	400/14	430/-	681/275
11	205/16	2137/70	40/723	59/-	400/12	430/-	681/255
12	205/14	2137/68	40/713	59/-	400/10	429/-	681/253
13	205/14	2137/67	40/703	59/-	400/8	429/-	681/251
14	205/14	2137/66	40/693	59/-	399/6	429/-	680/249
Completion	205/14	2137/65	40/683	59/-	399/4	429/-	680/247
Losses	0/42	13/99	0/225	0/-	21/28	21/-	1/195

Source: Author's processing with the support of EXCEL 2010

Table 9. Comparative attrition loss estimates of Coalition and Iraqi Air defense and Army CA1

Combat stage	X_{ADL}/Y_{ADL}	X_{ADM}/Y_{ADM}	X_{WMD}/Y_{WMD}	X_A/Y_A	X_E/Y_E	X_{ARM}/Y_{ARM}
1	96/18	44/270	-/110	4550/4150	486400/330000	20876/11110
2	96/11	44/263	-/103	4550/4150	486400/329250	20876/11107
3	96/5	44/257	-/96	4550/4149	486400/329853	20876/11105
4	96/0	44/224	-/92	4550/4148	486400/329800	20876/11103
5	96/0	44/151	-/89	4550/4148	486400/329782	20876/11102
6	96/0	44/149	-/67	4550/4128	486400/328287	20876/11049

7	96/0	44/147	-/60	4550/4110	486400/326911	20876/10999
8	96/0	44/145	-/10	4550/4092	486400/325638	20876/10950
9	96/0	44/143	-/5	4550/4071	486400/324002	20876/10895
10	96/0	44/142	-/0	4550/4051	486400/322470	20876/10840
11	96/0	44/140	-/0	4542/3734	485524/298470	20837/9980
12	96/0	44/138	-/0	4533/3218	484591/268470	20795/8840
13	96/0	44/111	-/0	4525/2902	483820/238470	20761/7700
14	96/0	44/84	-/0	4519/2486	483227/208470	20734/6560
Completion	96/0	44/58	-/0	4515/2070	482803/178470	20715/5420
Losses	0/18	0/212	-/110	35/2080	3597/151530	161/5690

Source: Author's processing with the support of EXCEL 2010

After determining the validity of the model, two experiments were performed for two hypothetical scenarios. In the first experiment (CA2), the assumption is that Iraq decided for a decisive defense operation supported by air power, after defensive air operations.

In the second experiment (CA3), the assumption is that Iraq launched joint offensive operation, after the Coalition forces began to prepare for the campaign. An overview of the comparative losses due to the attrition of forces in the battle is given in tables from (8) to (11).

Table 10. Comparative attrition loss estimates of Coalition and Iraqi Air Forces CA2

Combat stage	X_F/Y_F	$X_{F/A}/Y_{F/B}$	X_{STLH}/Y_A	X_{EW}/Y_{EW}	X_B/Y_{RECC}	X_{SEAD}/Y_{SEAD}	X_{AH}/Y_{AH}
1	205/56	2150/164	40/908	59/-	420/32	450/-	681/442
2	204/50	2140/140	40/851	58/-	397/30	442/-	665/393
3	203/44	2133/117	39/798	57/-	377/28	437/-	660/360
4	202/42	2132/102	39/728	56/-	371/26	430/-	659/358
5	202/40	2127/87	39/666	56/-	371/24	423/-	658/357
6	202/38	2127/77	39/607	56/-	371/22	423/-	658/337
7	202/36	2127/10	39/551	56/-	371/20	423/-	658/317
8	202/34	2126/8	39/495	56/-	371/18	423/-	658/297
9	202/28	2122/8	39/439	56/-	368/16	422/-	657/266
10	202/21	2119/6	39/383	56/-	366/14	421/-	657/235
11	202/19	2118/4	39/327	56/-	365/12	421/-	656/204
12	202/14	2117/4	39/250	56/-	365/10	421/-	656/173
13	202/12	2117/4	39/205	56/-	365/8	421/-	656/143
14	202/12	2117/4	39/159	56/-	365/6	421/-	656/112
Losses	3/44	33/160	1/749	3/-	55/26	29/-	25/330

Source: Author's processing with the support of EXCEL 2010

Table 11. Comparative attrition loss estimates of Coalition and Iraqi Air defense and Army CA2

Combat stage	X_{ADL}/Y_{ADL}	X_{ADM}/Y_{ADM}	X_{WMD}/Y_{WMD}	X_A/Y_A	X_E/Y_E	X_{ARM}/Y_{ARM}
1	96/18	44/270	-/110	4550/4140	486400/330000	20876/11110
2	96/16	44/268	-/108	4550/4140	486400/329887	20876/11106
3	9614	44/266	-/105	4550/4140	486400/329778	20876/11102
4	95/0	43/248	-/104	4550/4140	486400/329683	20876/11099
5	95/0	43/229	-/78	4550/4140	486400/328703	20876/11102
6	95/0	43/228	-/56	4550/4128	486400/326979	20876/11002
7	95/0	43/227	-/22	4550/4110	486400/326097	20876/10970
8	95/0	43/225	-/0	4550/4092	486400/325239	20876/10939
9	95/0	43/224	-/0	4458/4071	476158/292347	20375/9635
10	95/0	43/222	-/0	4257/4051	455133/260689	19381/8379
11	95/0	43/221	-/0	4070/3734	435438/229168	18461/7128
12	95/0	43/201	-/0	3834/3318	413061/191166	17421/5630
13	95/0	43/181	-/0	3692/2902	398132/140592	16737/3846
14	95/0	43/162	-/0	3618/2486	390258/114078	16376/3183
Losses	1/18	1/108	-/110	932/1654	96142/215922	4500/7917

Source: Author's processing with the support of EXCEL 2010

Table 12. Comparative attrition loss estimates of Coalition and Iraqi Air Forces CA3

Combat stage	X_F/Y_F	$X_{F/A}/Y_{F/B}$	X_{STLH}/Y_A	X_{EW}/Y_{EW}	X_B/Y_{RECC}	X_{SEAD}/Y_{SEAD}	X_{AH}/Y_{AH}
1	205/56	2150/164	40/908	59/-	420/32	450/-	681/442
2	205/50	2141/140	39/850	59/-	411/30	442/-	664/417
3	204/44	2136/120	39/807	58/-	409/28	437/-	659/389
4	204/44	2136/120	39/730	58/-	409/26	430/-	659/389
5	204/44	2132/120	39/695	58/-	409/24	428/-	658/388
6	204/41	2132/108	39/665	58/-	409/22	428/-	658/371
7	204/37	2132/96	39/652	58/-	409/20	428/-	658/354
8	204/33	2132/84	39/638	58/-	409/18	428/-	658/346
9	204/29	2131/80	39/626	58/-	409/16	428/-	658/341
10	204/25	2131/76	39/613	58/-	409/14	428/-	658/326
11	204/21	2131/72	39/588	58/-	408/12	428/-	658/312

12	204/17	2129/61	39/563	58/-	407/10	427/-	658/298
13	204/13	2129/51	39/539	58/-	407/8	427/-	658/285
14	204/9	2128/39	39/513	58/-	406/6	427/-	658/272
Losses	1/47	22/125	1/395	1/-	14/26	23/-	23/170

Source: Author's processing with the support of EXCEL 2010

Table 13. Comparative attrition loss estimates of Coalition and Iraqi Air defense and Army Forces CA3

Combat stage	X_{ADL}/Y_{ADL}	X_{ADM}/Y_{ADM}	X_{WMD}/Y_{WMD}	X_A/Y_A	X_E/Y_E	X_{ARM}/Y_{ARM}
1	96/18	44/270	/110	4550/4140	486400/330000	20876/11110
2	96/16	44/268	/108	4074/4001	434289/319318	18781/10698
3	96/14	44/266	/105	3625/3860	386154/309317	16783/10313
4	95/0	43/224	/104	3506/3672	373382/295881	16253/9795
5	95/0	43/196	-/78	3087/3511	328274/284364	14396/9352
6	95/0	43/195	-/56	2742/3367	292040/274092	12688/8957
7	95/0	43/194	-/0	2674/3170	284924/260296	12348/8415
8	95/0	43/193	-/0	2496/2968	266314/246134	11457/7857
9	95/0	43/192	-/0	2353/2647	251440/223837	10749/6969
10	95/0	43/191	-/0	2163/2338	231368/202441	9815/6117
11	95/0	43/190	-/0	1975/2076	211597/183890	9808/5393
12	95/0	43/178	-/0	1815/1724	194877/159417	8135/4423
13	95/0	43/166	-/0	1682/1556	180695/146482	7486/3696
14	95/0	43/155	-/0	1576/1399	169541/134663	6975/3544
Losses	1/18	1/115	-/110	2974/2741	16859/195337	13901/7566

Source: Author's processing with the support of EXCEL 2010

4. Results and discussion

The planned combat distribution of forces for a three-day combat cycle, obtained by the targeting process, at first glance allows a simple calculation in the simulation. In the real world, a whole range of factors affect the execution of tasks, from weather conditions, through the correctness of the aircraft, to the specifics mission terms, target characteristics, topography and local tactical conditions in the area of operation, etc. In the case of applying a mathematically deterministic model, these situations can only be expressed by the probability of execution, that is, by the attrition rate coefficient. This means that we cannot use simple calculations for the precise, daily number of combat sorties and ammunition consumption because the number of possible or required actions is not symmetrical with the actually performed, but only probable. It is also important to note that the mathematical model is deterministic and discrete, with calibration performed for certain deviations that have appeared in relation to reality. However, these deviations can be considered acceptable for three reasons.

The first reason is that the model processes operational actions on the battlefield and in the operational depth, according to the doctrinal principles of use, but also considering the specific situation in this conflict. This means that it was practically difficult to project a real combat sortie and the availability of Iraqi aircrafts to act as targets, due to the atypical use, because the Iraqis decided to preserve their aviation, by disguising it and a part defected to Iran. The last fact is not entirely clear, what it means: whether it is intentional or desertion.

Second, the effects on strategic facilities, such as: communications, energy, industrial and economic or political infrastructure of Iraq, where some of the guided aerial bombs, missiles and cruise missiles were probably used, were not considered.

Third, given the stochastic nature of the actual process of armed struggle, certain interruptions and changes in planned actions, caused by various causes, must have occurred, which affected the change of action plans, increased the consumption of ammunition in reality and caused atypical use. For example, a large number of Iraqi aircraft were destroyed by guided missiles and aerial bombs on the ground, and Coalition air defense was only effective against "SCUD" missiles on the territory of Saudi Arabia and Israel, which is still secret and the number is only an estimate by declared SCUD detection.

Finally, evaluation of the model and determination of the representativeness of the output results of the simulation (verification of the output data, with data from the real world) was performed by comparing the parameters of "Desert Storm" with the results obtained by the simulation. Computer testing of the "Desert Storm" operation model was performed, according to the available data and certain deviations (errors) were considered. A Summary Report on Operation „Desert Storm ", based on an exceptional database from the Survey II (1993a), served to validate the model. This was a necessary condition, which could lead to a relatively reliable structure and functioning of the operation, as well as relatively reliable data (Radenkovic, 1999).

The model check was performed based on the evaluation of the correlation between the real and the constructed model. The results showed that the correlation is relatively significant, which potentially indicates that certain changes within the estimated model manifest themselves in almost the same way as changes in real world. However, there are certain deviations.

The overall estimated strength of Iraqi forces is given by characteristic periods and reflects losses throughout the campaign. The situation in January 1990 marks the period Operation "Desert Shield" and the situation in February-March 1991 includes the situation before and after Operation "Desert Storm". For more details, it is useful to consult Cordesman (1994), with a list of tasks for various combat missions, with the number of flights flown and the percentage of the total sorties performed. he total number of flights during the anti-aircraft and strategic attack within the campaign was about 68,000, and in the phase of air support and isolation of the battlefield about 15,000 (Engelhard, 1991). An analysis and comparison of real statistical data, based on Cordesman (1994) and data from modeling results was performed by Survey V (1993c). Certain deviations were observed, and the results are presented comparatively as available statistical data / data obtained by the simulation process:

- 75 aircraft (airplanes and helicopters) of Coalition forces were shot down, and 141 were damaged / 56 were destroyed in the model;
- 400 planes out of 724 Iraqi Air Force were destroyed (121 defected to Iran and were later confiscated by Iran) / in the model 394 planes and 195 helicopters;
- destroyed Iraqi armored forces were about 3847-4280 tanks (T) and 1450-2870 armored fighting vehicles (AFV) / 3297 T and 2393 AFV in the model;
- Iraqi artillery forces destroyed were 2917/2080 in the model;
- destroyed about 100 Weapon of Mass Destruction (WMD) launchers/ 110 in the model:
- spent all types of ammunitions were: 228182/ 217419 in model;
- unguided bombs: 210004 / 199826 in model;
- guided aerial bombs and missiles for all surface targets other than aircraft: 15605 / in model 14300;
- air to surface anti-radiation missiles: 2039 /2648 in the model;
- air to air anti-aircraft missiles: 174 / in model 276;

- air defense Coalition and Israeli ADF missiles estimated at about 140-190 / 369 in the model;
- the actual number of completed combat flights of Coalition air force is about 68150 / 64681 in the model.

It should be noted some facts relevant for objectivity of modeling:

- Iraqi planes defecting to Iran would probably have been destroyed if they had participated in the battle. Iran never recovered these planes;
- In the operation model, air operations were considered by available Iraqi aircraft that could be detected on the ground or in the air. Due to methodological limitations, the model, in this case, simultaneously calculates the probable average expenditure of precision-guided air-to-air and air-to-ground missiles;
- the model implies the possibility that part of the forces in the conflict will be out of combat and that part of the force cannot be acted upon, e.g. ammunition and equipment in shelters;
- the most important problem was the deviance in the way of fighting by Iraq. The operation was outside doctrinal and strategic principles. Iraqi forces were passive, buried in desert dust outside the towns, with weak air defense and the will to fight offensively.

It is easy to see the correlation of the results between the losses inflicted on Iraqi air and ground forces and the number of combat sorties in the model and in reality. The results differ somewhat in munitions consumption, where there are some minor deviations in unguided and precision-guided weapons on surface targets, such as army force, operational support, logistics infrastructure and commands. Somewhat larger deviations are observed in actions against aircraft and WMD (SCUD) of Iraq, air defense missiles. The reason for this is the availability of official data on the effectiveness of the "Patriot" air defense system, which has not been published to date, due to the protection of confidentiality. The results shown are the estimated number of missiles fired at the incoming SCUD based on probabilistic calculations.

In addition to the actual war case study, two different hypothetical scenarios are introduced to further develop courses of action for Iraq Coalition forces. In the case (CA2), Iraq could hypothetically decide on a mode of combat by conducting an evasive-elastic defensive operation against Coalition forces in the cities of Kuwait. Both sides would suffer heavy losses, but in the case of Coalition forces this effect could imply mission termination. However, the modeling (CA3) produced very intriguing results for the hypothetical case that Iraq decided to launch an offensive operation, during the preparations for „Operation Desert Storm ", which was named „Operation Desert Shield ". Despite the heavy losses of the Iraqi Air Force in this, as in other cases, the result of combat for (CA3) shows a shorter time of active fight and losses on both sides, but quite heavy for Coalition forces. If Iraq had decided to launch an offensive operation, Coalition forces would probably not have been able to sustain the losses politically. History could have been different, but Saddam Hussein chose to fight in the worst way - to be passive and leave the initiative to the opponent.

5. Conclusion

By comparing empirical facts and data obtained by experimenting on the model with a change in parameters, its verification was carried out and its validity was confirmed on the historical example from the case study. According to the methodological criteria for determining the validity of simplified models, given that the creator of the model is methodologically allowed to determine the maximum degree of deviation, it can be concluded that it is adequate. As such, the created simulation model can be used as a tool for experimentation in further research to obtain reliable knowledge about the laws of armed combat and the phenomenology of combat operations. Also, it has been verified as a reliable tool in the process of operational planning and targeting for obtaining relevant data, which can be used in the assessment of the operational situation.

Considering the data obtained by experimenting with different ways of using forces (courses of actions), Lanchester's quadratic law and the importance of the number of forces and the expression of the center of gravity in battle were confirmed. However, it is important to emphasize that the victory of the mass of forces can also be achieved by the application of operational skills. With the operational maneuver of faster, more agile, more operationally capable forces, supremacy can be created in a certain area, by concentrating forces and efforts in battle, regardless of the overall balance of forces on the battlefield, which brings victory. These results prove the claims of American strategists who conducted operational simulations before the start of the battle and planned the campaign in accordance with the facts obtained. This is the likely reason for the deviation from the then current doctrine of air-land battle and the execution of a strategic attack in the air campaign and air isolation of the battlefield, which broke the military power and the will of Iraq to continue the fight.

This also confirms the premise of a misunderstanding of the planning and decision-making of such undertakings by top officials as was the case in Iraq. The problem with the entire history of warfare is that most political actors who decide to enter a conflict "believe" that they can predict the outcome of a conflict based only on military knowledge of the history of previous wars and personal experiences, and that they are confident in their abilities to plan and execute combat operations, which often leads to disasters.

Further work will be carried out in the direction of research into more complex problems of optimizing the use of forces and determining the objects of action in the process of operational planning. The solution that currently exists is the application of multi-criteria optimization, using multi-objective programming and multi-attribute decision making. Also, it is necessary to examine changes to the existing algorithm of the operational planning and targeting process in order to improve it using software tools.

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