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MATHEMATICS CONCEPT BUILDING WITH DIDACTIC GAMES IN A GEOGEBRA-EMPOWERED ENVIRONMENT

***Summary**: In this study, we examine how GeoGebra software can be used for organizing a game-based learning environment and its impact on fraction concept building and the pupils' understanding of the concept. The learning/instruction process was organized in two groups of 9 to 10 years old pupils. The experimental group was offered GeoGebra game-like appletsand the learning process was based on them. The control group pupils learned the same subject material in a more traditional fashion. Pupils' understanding of the conceptwasassessed using results of regular tests they enrolled in the same environments they were learning. According to the results, the experimental group pupilsdemonstrated better understanding of the concepts, and the difference is statistically significant. This implies that GeoGebra softwarecombined with the game-based learning can givebetter results in building and understanding fractions at initial stages.*

***Key words:** GeoGebra, fractions, game-based learning, concept building.*

INTRODUCTION

A very important goal inthe early mathematics education is developing proper understanding of basic mathematical concepts. In achieving mathematics education goals students' motivation

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takes the key role. There are numerous ways and strategies to motivate students to learn mathematics – engagement (Martin, 2007;Martin, Way, Bobis, and Anderson,2015), fostering teacher-student interactions (Attard 2013, Cheeseman, 2009), using students' interest (Ainley, Hidi, and Berndorff. 2002; Flowerday, Schraw, and Stevens, 2004), or even introducing game-based learning (Tokac, Novak, and Thompson, 2019; Okur and Aygenc, 2018). Apart of boosting motivation, a proper use of didactic games in mathematics instruction can improve students' communication and collaboration skills (Ernest, 1986; Umbara, Munir,Susilana, and Puadi,2021), as well as their mathematical skills and achievements (Polycarpou, Krause, Rader, Kembel, Poupore, and Chiu, 2010; Bahrami, Rahimi Chegini, Kianzadeh, Emami, and Abdi, 2012).

Nowadays, the usage of Informatics-Communication Technologies (ICT) in education generally, and mathematics instruction in particular, is a commonplace. There are studies that suggest that ICT usage in the mathematics instruction can help in improving students' motivation, communication skills and achievements (Arbain, and Shukor, 2015; Brown, 2010; Hopper, 2009; Kokerić, Kreculj, and Vešović, 2016). On the other hand, some researchers (e.g. Oblinger,and Oblinger, 2005) suggest that the context is the key factor in understanding quality and efficiency of ICT impact on students' parameters.

All these studies indicate that didactic games, merged with ICT, might have a positive impact on many aspects of mathematics learning. For example, they help in developing a habit to explore mathematical content (Prensky, 2001). Ku, Chen, Wu, Lao, and Chan, (2014) claim that digital game-based learning can improve students' confidence in learning mathematics, which leads to improvements in their academic achievements. Besides, some studies (Heid, 1998; Drijvers, 2012) argue that ICT mathematics learning with emphasis on concept building do not hinder development of students' mathematical skills, thus supporting a concept-first approach. Reports on including remedial students in a game-based learning environment indicate that such students show improvements in motivation and mathematics achievements when exposed to a GeoGebra-empowered game-based learning environment (e.g. LimandLeong, 2016).

The results given in these reports encouraged us to organize and assess a GeoGebra-empowered learning environment with mathematical didactic games focused on concept building. As a subject material we chose fractions studied in lower grades of primary school.

FRACTIONSCONCEPT BUILDING IN PRIMARY SCHOOL

In the initial stages, in Serbian primary schools there are a total of 18 classes planned for building the concept of fractions. At first, pupils of the third grade (9 years old) learn about unit fractions ½, …, 1/10. It takes 7 classes to cover it. In the fourth grade (at the age of 10) they are supposed to learn reading and writing proper fractions, as well as comparing them using sketches and a number line. These topics are covered within 11 classes.
At the grades to follow pupils are expected to learn operations with fractions and building the notion of rational numbers. Therefore, it is of high importance to ensure deeper understanding of fractions. This mainly means to ensure students understand the meaning of denominator and numerator and the impact done by changes in their values.For that reason, special attention was paid at:

- (Geometrically) constructing and understandingunit fractions,
- Locating proper fractions on the number line and comparing them with the number 1,
- Estimating the fraction that corresponds to a part of a figure and vice versa.

Since the depth and quality of pupils' understanding of the fractions concept depends mainly on the way the concept was built, we organized activities that included GeoGebra based mathematical games.

GEOGEBRA AS A TOOL ININITIAL STAGES OF THE MATHEMATICS INSTRUCTION

GeoGebra is an open source educational software that combines algebra, calculus, geometry, vectors and other fields of mathematics (Ljajko, 2018). It offers several representations of the

objects dealt with, some of them being algebra and geometry windows. Teachers can easily adapt it as an appropriate tool for initial mathematics instruction. GeoGebraoffers different possibilities to represent a conceptand all of the representations can be used simultaneously. Very powerful tools are dynamic texts and the possibility to show/hide objects in the geometry window, which can enable teachers easily prepare interesting and meaningful dynamic worksheets that support students' activities similar to games. Through such activities, students can build concepts and knowledge that are more usable. This makes GeoGebra game-based activities a very appropriate tool for concept building. As an alleviation in planning and organizing these activities, one should know that there are many GeoGebra dynamic worksheets available online for all levels of mathematics education. It is a commonplace that wide numbers of teachers and students eagerly use GeoGebra, and their experiences and results are mainly positive (Hohenwarter et. al., 2008; Ljajko, Ibro, 2013). We used it as a tool to help students develop the concept offractions and summarized and estimated the effects that a game-basedlearning in a GeoGebra-empowered environment can have on students' understanding basic properties of fractions.

Study justification

One of the aims of the study is to specify ways of introducing GeoGebra based games in the fraction concept building in order to determine if they are accepted by students, and to assess their impact on the quality of the students' understanding and implementing the concept of fractions. The research results can also be used as a guideline for initial mathematics curriculum designers in Serbia to integrate or recommendGeoGebrabased didactic games in mathematics curriculum as supplementary tools for learning mathematics.

HYPOTHESIS OF THE STUDY

In order to confirm/refute assumptions given by other researchers, we organized a study to assess and compare understanding of the concept of fractions and academic achievements that experimental and control group students show

in different learning environments – with and without using GeoGebra based games. The null-hypothesis of the research was defined as:

H_0: The treatment significantly affects the achievement mean scores of the two groups regarding fractions.

METHOD AND TREATMENT PROCEDURE

Population and sample

As a sample we used a total of 43pupils of the fourth grade (9 to 10 years old) in two primary schools in Gora municipality – "Nebojša Jerković" in Dragaš (28 pupils, 15 of them included in the experimental, ad 13 in the control group) and "22. decembar" in Restelica (15 pupils, 7 of them included in the experimental, ad 8 in the control group).

Design of the study

The groups included in the study were formed out of different classes of both schools. Though we tried to form the groups in a way that the difference in their average marks in mathematics is as small as possible ($\overline{x_{exp}} = 3,45$, $\overline{x_{ctrl}} = 3,29$, mark 1 being the lowest, mark 5 the highest), we had to confirm it through a pretest. In the pretest, students of both groups were given the same problems (five of them, each of the problems could be scored with up to 20 points), that were aimed at assessing their understanding of the number line and division of natural numbers and its properties. The groups achievements and their statistical analysis are given in the table 1.

Table 1. The pretest statistical analysis for groups included in the study.

Group	N	Mean score	SD	Df	t-test for equality of means		
Exper.	22	67.27	17.57	41	t	Sig. (1-tailed)	Mean Diff
Contr.	21	64.52	17.74		0.60	0.05	2.75

Since, t = 0.60, we assumethat the groups show no statisticallysignificant difference, and the equivalent groups scheme was applied, figure 1.

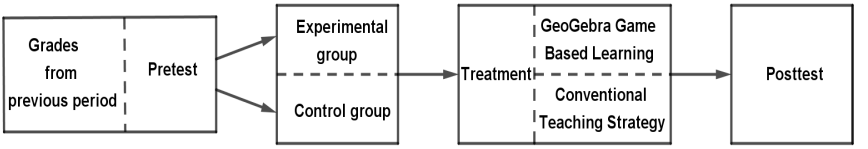


Figure 1. The research scheme

Sampling procedure

While forming the groups, we took into considerationif the technology (i.e. computers, internet connection, GeoGebra software) was available to the participants in the schools and at the home, the teacher's capacity and readiness to plan, organize and materialize a GeoGebra game-based learning, as well as the results pupils achieved in the pretest.

Instrument development and content validity

In order to measure the pupils' understanding and knowledge about fractions, after the lessons on the topic, we developed a posttest with 5 short problems and assignments covering unit fractions, reading and writing fractions and comparing fractions. Every of the problems/assignments could be marked with up to 20 points, thus making a total of 100 points.

In the process of choosing problems/assignments for the posttest we used standardized tests from textbooks, but also accepted suggestions by the teachers included in the instruction of both groups.

Survey management

There were a total of 43pupils included in the study. They were all attendantsof the fourth grade in two primary schools in Gora municipality. All the participants were separated into two groups: the one attending the GeoGebra game-based learning, and the other, that was learning the same subject materialin a conventional teaching fashion (i.e. without using ICT), table 2.

Table 2. The structure of the groups

Group	Teaching/Learning method	School/n. of students	Total
Experimental	GeoGebra game-based learning	Dragaš/15 Restelica/7	22
Control	Conventional	Dragaš/13 Restelica/8	21

The course of the experiment

According to Serbian mathematics curriculum for the fourth grade, there are11 lessons on fractions and we needed about two weeks – from May the 23rdto June the 3rd 2021. to teach them to students. The approach to the learning/teaching process was different in different instruction groups,not onlyconcerning the medium used for the subjectrepresentation, but also there were different student activities in both groups. The role of the teachers in the experimental group was remodeled towards the one supervising and directing pupils' activities, thus giving students the freedom to explore the content on their own.

Pupils of the experimental group learned about fractions through activities with five game-like GeoGebra dynamic worksheets available online at Fraction Activities – GeoGebra. They were as follows:

- *Unit fractions* – requiring the student to divide a rectangle into unit fractions as precisely as possible.
- *Where is the one whole located*–a fraction is given on a number line, and students are supposed to estimate where is 1 whole on the number line.
- *Fraction guesser* – the student has to guess and write down the fraction based on a visual representation of it.
- *Locating fractions on a number line* – proper fractions are given and the student should place them on their correct locations on the number line.
- *Proper fractions* – the student is given the task of dragging a fraction written in numbers to its proper location on the number line.

All the dynamic applets were very simple to use and closer to computer games than to worksheets, so thepupilsneeded almost nohelp/instruction about the way to use them. There were different levels of difficulty for every one of the applets. That was arranged through possibility of using hints when the solution was not so obvious.On the other hand, teachers' competency on game-based instruction was questionable. There are studies (e.g. Maričić and Milinković, 2022) indicating that teachers need support and guidelines to organize and implement a successful ICT instruction.Having that in mind, we organized a preparation phase, where the teachers had possibility to get familiar with the applets, their usage, and learn how to integrate them into the learning process.

In "Nebojša Jerković" school the classroom was equipped with 8 computers, so theexperimental group students worked in pairs. In "22. decembar" school the experimental group pupils could work individually on computers. In addition to that, students were able to use the game applets on mobile devices after the school activities. The very nature of the game applets imposed on students the way to use them – explore between symbolic and visual representations of fractions, thus deepening their understanding of the subject material and alleviating the concept building.

As for the control group, both classes that comprised it were included in a more traditional fashion of instruction. Both teachers and pupils were familiar with it and there were no additional requirements regarding the instruction process.

GeoGebra game-based educational activities

All the five GeoGebra game applets had their purpose, and activities were organizedin accordance with their purpose. Here we describe shortly their aims, ways pupils used them, and effects the students had in the building concepts of fractions.

Activities with the first applet (*Unit Fractions*) help students develop visual and symbolic representations of unit fractions as the first step in understanding meaning of denominator of fractions. Here, the notion of denominator is clearly connected to pupils' knowledge about the properties of division of natural numbers, and is widened through introduction of idea that there are ways to describe visually and numerically positive quantities smaller than 1.

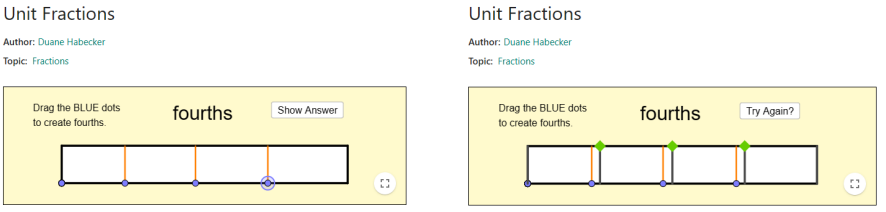
The concept of unit fractions is further widened to general proper fractions by using the second applet (*Where is the one whole located*). With this applet, pupils are introduced the notion of numerator, still with a limitation to proper fractions, i.e. the numerator has to be smaller or equal to the denominator. The main intention of this game-based applet is to develop the pupils' ability to compare fractions.

The third applet (*Fraction guesser*) is used in the next stage in developing pupils' ability to compare fractions. A part of a rectangle is shaded in red and pupils are required to estimate what is the fraction representing that part of the rectangle.

In the fourth applet (*Locating fractions on a number line*), students are given the task to find the correct locations of the given fractions on the number line. While doing so, the pupils strengthen their understanding of comparing fractions, but also adopt the idea of fractions being numbers. In addition, fraction greater than 1 are introduced.

Finally, using the fifth applet (*Proper fractions*) pupils are given an opportunity to estimate the value (quantity) represented by the fraction.

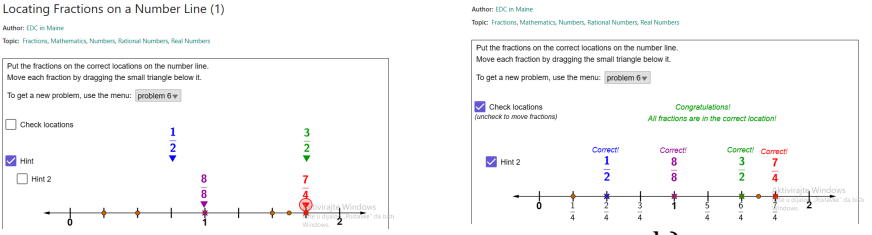
All the activities with the applets were oriented at visualizing main properties of the fractions, figure 2 a). Apart of them, teachers organized exercises in reading, writing and comparing fractions, thus connecting iconic and symbolic representations of the fractions.



a) b)

Figure 2. Visualizing and estimating unit fractions

All the applets were interactive, figure 2 b), and, if needed, offered help at different levels of difficulty, figures 3 a), b). In this process, the principle of the minimal help is met.



a) b)

Figure 3. Different levels of help in accordance with the minimal help principle

An important observation during the game-based activities is that teachers noticed and reported a significant increase in motivation among the experimental group pupils. They showed it in several ways: asking questions more frequently, stronger and more meaningful pupil-pupil and pupil-teacher communication and cooperation, more frequent attempts to solve problems on their own, etc.

Data analysis

After the posttest was done, a t-test was conducted in order to compare posttest achievements of experimental and control groups, table 3.

Table 3. The posttteststatistical analysis for groups included in the study.

Group	**N**	**Mean scores**	**SD**	**Df**	**t-test for equality of means**		
Exper.	22	70.68	17.01	41	**t**	**Sig. (1-tailed)**	**Mean Diff**
Contr.	21	59.52	17.82		1.97	0.03	11.16

From the Table 3one can see that the experimental group scored higher achievements (Mean = 70.68, SD = 16.59) than the control group did (Mean = 59.52, SD = 17.38), and the mean difference is 11.16. Also, for the t- and p-values we have: $t_{\text{calculated}} = 1.97 > 1.96$ and $p_{\text{calculated}} = 0.03 < 0.05$, so we accept the null hypothesis, which means that the mean difference is statistically significant.

DISCUSSION AND CONCLUSION

Having in mind the t-test results of the posttest, it is clear that a statistically significant difference between the means is disclosed at the 0.05 level. Also, according to the pretest statistical analysis, the groups were considered statistically equal. The only significant difference was the way students learned fractions. Therefore, one can draw a conclusion that introducing game-based GeoGebra applets into the learning process of fractions led to an increase in the academic achievements of experimental group pupils. This indicates better understanding of the fraction concept.

The overall conclusion is that the results are in line with the researches supporting the idea that the game-based learning in an ICT-empowered environment brings positive changes in different aspects of the learning process, such as: developing the habit of exploring subject material (Prensky, 2001), improving the pupils' confidence in learning (Ku et al, 2014), or improvements in motivation and achievements (Lim & Leong, 2016). Further, in addition to results given in Ljajko (2018) we suggest that in the initial stages of the mathematics education, ICT usage is not as effective as its combination with game-based activities.

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ИЗГРАЂИВАЊЕ КОНЦЕПТА МАТЕМАТИКЕ СА ДИДАКТИЧКИМ ИГРАМА У ОКРУЖЕЊУ КОЈЕ СЕ ОДНОСИ НА ГЕОГЕБРА

Сажетак: *У овом истраживању испитујемо ефекте употребе софтвера GeoGebra у организовању наставног окружења заснованог на дидактичким играма, као ињегов утицај на начин изградње и разумевање концета разломака код ученика. Процес учења/наставе је организован у двема групама ученика старости 9 до 10 година. Експерименталној групи су понуђени GeoGebra радни листови и процес учења је био заснован на њима. Ученици контролне групе су учили исту наставну материју на традиционалнији начин. Процену разумевања концепата код ученика смо извели коришћењем резултата редовних тестова које су ученици обавили у истом окружењу у ком су и радили. Према добијеним резултатима, ученици експерименталне групе показали боље разумевање концепата, а разлика је и статистички значајна. Ово указује да GeoGebra укомбинацији са учењем заснованим на игри може допринети бољим резултатима у изградњи концепта и разумевању разломака у почетним стадијумима учења.*

Кључне речи: *GeoGebra, разломци, учење засновано на игри, изградња концепата.*