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PEDAGOGICAL APPROACHES THAT CONNECT MUSIC AND MATHEMATICS

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Summary: The starting point of the publication is considering methods of music education that ephasize an early music stimulation and the possibility of their implementation and integration in the formation and development of initial mathematical concepts. Aspiration to connect two seemingly incompatible disciplines, music and mathematic, opens questions whose goal is to find adequate pedagogical approaches that would be in the function of music and mathematics education and achievement, but also the overall development of children. Applying the method of theoretical analysis, a comparison was made among different conceptual approaches, the results of the conducted research and the authors' conclusions. Considering the aforementioned questions, it was concluded that the considered methods emphasize the importance of an early development in the formation of a person and start from the premise that the potential and the environment enable the children to build themselves. Organizing an environment that allows children the freedom by acting through organized and spontaneous activities in kindergarten and school can contribute to children's musical and mathematical development. Connecting musical and mathematical contents with each other, as well as connecting them with everyday activities and setting learning situations in a context that is close and familiar to children, and with the contents of other areas of education and upbringing, provides the basis for successful learning of music and mathematics. implications Pedaaoaical are contained in aforementioned recommendations for a proper approach to connecting musical and mathematical content and creating a stimulating environment. Future research should be focused on examining and improving the competencies of professionals – teachers and pre-school teachers, which would be in the function of music and mathematics education and achievement, but also the overall development of children.

Key words: musical abilities, initial mathematical concepts, learning methods, early stimulation, stimulating environment.

This paper considers the presentation of musical educationmethods, which emphasize early musical stimulation and the importance of early development in the formation of a person, the possibilities of their implementation and integration in the formation and development of initial mathematical concepts.

Although music and mathematics seem incompatible, looking at historical development, philosophical viewpoints, as well as numerous researches, there is an inextricable connection between these disciplines. Ancient Greek philosophers researched the connections between these areas and realised the relatedness. "In the Pythagorean school, music was on the same level as arithmetic, geometry and astronomy" (Beer, 1998). Developing the theory of numbers, Pythagoras points out the close connection between numbers and musical tones. They believed that the science of numbers is connected with musical harmony (Adžaga, 2012: 102). The Pythagorean music theorist, Archytas, pointed out the division of intervals in the tonal scale by means of appropriate proportions (Gika, 1987 According: Rajić 2020). Pythagoras' student Philolaus is considered the founder of the tetrachord theory (Barker, 2007). Overtime, a different division emerged, that has remained till these days, mathematics and science were on one side and language and the art on the other. Interest in the connection between mathematics and music still exists and a large number of researchers are trying to discover how musical laws can be represented mathematically, how music can contribute to the development of mathematical thinking and creativity. how mathematics can contribute to the development of musical thinking (Parker. 1973; Rauscher, 1995.; Tajčević, 1997; Milić and Vukićević, 2011; Maričić and Ćalić, 2015; Savić, 2018).

Analyzing the determinants of the development of musical and mathematical abilities, it was noticed that there are contrary points of view in the literature. A certain number of researchers support the thesis of innate components that manifest and function from early childhood (McPherson & Williamon 2006; Haroutounian 2002; Radoš 2010; Dejić and Mihajlović 2014) while the others point out that, besides hereditary factors, there is also the interaction with the stimulating environment, which is more important factor in the development of potential than the hereditary factor. Based on the current knowledge about the nature of musical and mathematical abilities, the aforementioned dilemmas lose their basis. Most

psychologists today agree with the opinion of Farnsworth who emphasizes that both determinants, genetics and environment, are important in shaping abilities and what differentiates them is which factor is considered more important (According to: Matić & Mirković Radoš, 1986: 35).

It is evident that music and mathematics include certain common abilities. such as perception, memory, insight, creativity and others, which can be innate but also learned. Through music, children acquire melody, rhythm, pitches, harmony, tempo, meter, dynamics, while in mathematics they acquire initial arithmetical, algebraic, geometric content, then measures, measurements and others (Radoš, 2010). Numerous studies indicate that connecting these two disciplines can have a positive effect on the general development of the child. In the researches (Rauscher et al., 1995; Chan et al., 1998; Rauscher & Zupan, 2000; Vaughan, 2000; Hetland, 2000; Schellenberg, 2004; Fauvel et al., 2006) in which the influence of music education on cognitive development was examined., respondents who had a certain musical education showed more successful results on various tests. The tests examined mathematical, verbal and non-verbal abilities. According to the aforementioned research, it was concluded that more successful results of musically educated subjects derive from general intelligence. which is a common component, on which one external factor can have a great influence. In Wang's research (Wang, 1989), the correlation among two tests of musical achievement and tests of mathematical and visual - spatial abilities was examined, and spatial abilities shown to be a significant prerequisite for musical success. Parker (Parker, 1973) points out that cognitive skills of classification, serialization, understanding the spatial relationships can be developed by guided listening to music. Areas of mathematics that require spatial-temporal reasoning are geometry and certain aspects of calculation, and in further mathematics the ability to perform mathematical proofs. Seeing the rhythm as aflow of sounds or tones of different time duration closely connected and interdependent with the musical meter (Tajčević, 1997), in mathematics we can connect it with proportions, the relation between the part and the whole.

A large number of researches focused on the development of abilities indicate that the period of early childhood is a "critical period" for learning, and therefore for the development of all abilities (Radonjić, 1992; Baumcal, 2012). The development of musical and mathematical abilities in the preschool period is the most intensive and this period

forms the basis of further development. The ability to accept new knowledge in this period is really great and it should be used in the best way, because reaching biological potential depends on proper stimulation (Rajović, 2012). Musical and mathematical activities in the integrated preschool programme cannot be clearly separated from other types of children's experiences and are more closely related to them than in any other period of their development (Duh 2009; Slunjski 2011). The current conception of the preschool programme "Years of ascent" implies the realization of the activities in relation to children's interest in certain contents, therefore adequate support of adults, parents and teachers is of key importance (Regulations on the basics of the programme of preschool upbringing and education -Years of ascent 2018). "The holistic development of children must be methodically supported in accordance with age and individual capabilities. The activities and the complexity of tasks are psychophysical differentiated depending differences in on development" (Popović and Rašković, 2022).

Therfore, this publication will present methods important for early musical stimulation and the possibilities of their implementation and integration in the formation and development of initial mathematical concepts. The aim of the publication is to guide the adults – parents, pre-school teachers and teachers to realize the possibility of connecting music and mathematics and to find adequate pedagogical approaches that would be a function of music and mathematics education and achievements, but also the overall development of children.

LEARNING METHODS THAT ENHANCE EARLY MUSICAL AND MATHEMATICAL STIMULATION

In music education in preschool institutions, as well as in other educationalinstitutions, various pedagogical approaches are increasingly being introduced. They contribute to the development of creativity, the integration of children in music education and musical activation, as well as the creation of future music lovers. Contemporary developmental psychology indicates that learning through experience and the concept of active learning, which intensify the learning process and activate the individual learner, should be prioritized in the educational process (Matijević and Radovanović,

2011). For this reason, it is important to point out that by properly combining methods, content, forms of work, approaches, it is possible to influence the development of children's biological potential (Milić, 2020). The teacher's task is to carefully select and adapt the educational approach to the children's musical and psychophysical abilities, strengthening the creative potential in accordance with the interests and prior knowledge of the children.

In order for a child to successfully develop the abilities necessary for learning mathematical content, it is necessary to lay the foundations that are reflected in the successful formation of mathematical concepts both in preschool and early school age. Taking into account the cognitive abilities of children, constant action through organized and spontaneous activities in kindergarten and through lessons at school can significantly contribute to the development of mathematical concepts. Connecting mathematical contents with everyday activities and setting learning situations in a context that is close and familiar to them, as well as with the contents of other areas of education and upbringing, provides the basis for successful learning and acquisition of mathematical concepts. Kamenov defines learning as an important part of the process of acquiring mathematical concepts as "the processing and shaping experience, discovery, insight into the essence of phenomena, reconstruction of existing knowledge and establishment of associations between different kinds of knowledge" (According: Marendić, 2009: 4).

In order for children to successfully develop the abilities necessary for acquiring musical and mathematical concepts, it is necessary to organize learning so that children gain both motor and sensory experiences. This contributes to choosing an adequate learning method, connecting different learning contents, and all of this must take place in a stimulating educational environment. Mathematical activities that can be combined with music, verses, art and other content can significantly contribute to the acquisition of mathematical concepts such as the concept of a set. Preschool children can successfully adopt the properties of geometric figures if we present them through certain musical content, such as songs and chants (Paunović et. al., 2018).

Many researchers attach great importance to early musical stimulation and its importance in activating the child's personality and enabling a healthy approach to their psychophysical characteristics. We will present some of the most famous methods of music education

in the twentieth century that emphasize early musical stimulation, namely: Montessori, Dalcroze, Willems and Martenot method. In addition to the importance for the development of musical abilities, the mentioned methods can also represent the basis for the development of initial mathematical concepts. Each of the mentioned methods can be of great importance in improving and connecting learning in the field of music and mathematics.

Maria Montessori (1909), a scientist who devoted her whole life to researching the importance of early development in the formation of a person, starts from the premise that the child is not determined by heredity and that the potential and environment allow the child to build itself. The education she designed allows the child to understand the world around it in its own way through sensory education. Although children go through the same developmental rules, this does not condition the same individual development. Without an appropriate stimulating environment in a certain developmental period important for learning, the development of important potentials will remain neglected (Bašić, 2011).

Montessori believes that sensorimotor triggers are activated between the ages of three and six and points out that the activation of melodic and rhythmic sensations and movement reactions in children should first start spontaneously, then coordinated with listening to music, and only then we can talk about making music (Montessori, 1909, According to: Terzić, 1998). For the purposes of Maria Montessori's research, Elise Brown Barnett composed a collection of piano pieces. Music corresponds to walking, running, jumping, slow walking, waltz, polka. In the initial phase, students react spontaneously, only to be corrected later (Brown Barnett, 1973).

To practice perceptual abilities, Montessori made several children's instruments. She filled the wooden cylinders with different materials that gave six different volumes when shaken. She solved melodic problems by making and playing melodic instruments. She used double bells placed in one parallel row, suitable for comparison, then monochord, whistles, wooden cymbal, children's organ. In her exercises, she paid a lot of attention to silence, starting from it, she taught children to listen, compare, distinguish (Terzić, 1998: 23).

In addition to the importance for the development of musical abilities, the entire work of Maria Montessori represents the basis for the development of initial mathematical concepts. Maria Montessori method is based on encouraging children's development, organizing

an environment in which children have complete freedom to choose the means for their activities (Kopas Vukašinović, 2018: 176). Children's play has a key role in this process, as a spontaneous and free child's activity. The time a child spends in an environment that supports its development will create the basis for independent learning and discovering its own possibilities. In this way children independently acquire knowledge from their environment and thus education becomes a natural process that takes place spontaneously in accordance with the child's capabilities.

When it comes to children's ability to learn mathematical concepts. Maria Montessori believed that every child has the ability to think abstractly, i. e. possesses a "mathematical mind" that children develop from their birth. Apart from the fact that mathematical concepts are abstract, her method implies that mathematics should be made as concrete as possible so that children can understand it. Some of the ways to do this are to connect mathematics with everyday life, whereby one should always start with familiar and concrete material that significantly contributes to the development of mathematical thinking. She believes that mathematical content should start in kindergarten, through which children should be enabled to work independently through research, encouraging cooperation, with the use of various didactic material, while respecting the child as an individual and preparing children for life in the society. The importance of the Montessori method is discussed by Milinković and Bogavac, who believe that this method, if applied in preschool age, creates the basis for an integrative approach to learning at school and that this approach is particularly suitable for learning mathematical contents (Milinković and Bogavac, 2011).

Jacques - Dalcroze Emile is a Swiss composer and pedagogue whose name is inextricably linked to rhythmic gymnastics. The staring basis of his upbringing is the belief that rhythmic certainty will be best developed through large body movements. That is why he tried to create presentations of rhythm in children through various gestures, beating, walking and other large body movements (Rojko, 1982). He devised a method that involves musical activities for children at the age of four. The syncretistic of children's behavior in this period is significant, and that is why the conditions for activities of this kind are extremely favorable. Dalcroze devised that rhythmic movements begin with walking and in this way one starts learning the rhythm. Along with walking, there is an improvisation on the piano, which, in the initial phase adapts to the child's movements, and later takes the leading role, so that the child coordinates the body with the music and creates harmony with it (Terzić, 1998: 25). The elaborate system of exercises also includes those related to changing time signatures, syncope, phrasing, dynamics. There are also concentration exercises in which movements are interrupted by an unexpected command (stop) and changed. Expressing the rhythm with the body and participating in the musical expression leads to the development of spiritual and physical harmony, thus achieving contact with art. Dalcroze's method affects the development of children's intelligence, concentration and the ability to quickly react to changes in music and to changes in general. These qualities have a positive effect on children's development because they force them to choose before acting and to react quickly. The rising of feelings for the collective is emphasised because in the realisations of this method the child must adapt to improvisations in the group.

While Dalkroz indicates the importance of movement for the acquisition of rhythm, Stanković and Aleksić Veliković pointed out the importance of physical activity and motor skills for the acquisition of mathematical concepts. They point out that children's physical activity contributes significantly to the learning of basic mathematical concepts, which includes the activity of the whole body through various movements (Stanković and Aleksić Veljković, 2021). It is of special importance to show children from preschool age that mathematics does not exist only on its own, but should be connected with other activities and subjects and the space that surrounds children. Movement plays a key role in getting to know the concepts that surround children. The experience that a child gains through movements and physical activities creates the basis for the later acquisition of certain mathematical concepts. This is confirmed by numerous studies that have shown that there is a connection between the development of fine motor skills in early life and later success in mathematics, reading and science (Lopes et al., 2013; Senturk at al., 2015; Stanković, 2017). Hence the need to find adequate methods that will encourage the development of motor and cognitive skills in children of preschool age, and these skills will create a basis for learningmathematical concepts. Children's physical activity through movement, play and other activities significantly contributes to the development of motor skills in children, which can be the basis for later successful reading, writing, mathematics and more (Stanković

and Aleksić Veljković, 2021). This is also confirmed by research carried out in Norway, which included three groups of children with differently developed motor skills (well, average and badly). Each group had to solve a certain mathematical task (to show their age using their fingers, to distinguish objects by shape, colour and size, or to use a number in a sentence. It was discovered that the children who have well-developed motor skills are more successful in solving these mathematical tasks, thanthe children whose motor skills are average and poor (Beck et al., 2016, According to: Stanković and Aleksić Veljković, 2021).

The new approach to learning with children of preschool age is aimed at the complete development of the child's personality. That is why it is necessary to connect the children's learning process with their daily activities and interests. The key activity for children is play, which includes certain physical activity and movement, and they contribute to the development of thinking, vocabulary enrichment, as well as the abilities necessary to solve certain problems. As Stanković and Aleksić Veliković point out, "pace and speed of movement, directionality and coordination of the whole body form a well - organized psychomotor system of a child" (2021: 247). That is why the goals of mathematics education are focused on the active position of children in everyday, real life situations. A stimulating and well organized environment will contribute to this. The child's interaction with the stimulating environment will significantly contribute to the acquisition of initial mathematical concepts related to space, relations, shapes, quantities, time. number and other mathematical concepts. The child's environment can represent a good starting point for the acquisition of mathematical terminology, which will represent the basis for later acquisition of new mathematical concepts.

It can be concluded that the active position of the child significantly contributes to the development of logical-mathematical abilities, as well as spatial-visual, musical and many other abilities. That is why it is necessary to enable children to acquire mathematical content through activities that involve the active position of the child through movements and manipulation. In order to achieve this, "we must encourage children's physical activity, gross and fine motor skills, complex movements and thereby help the development of the child's overall abilities, and therefore also mathematical" (Stanković, Aleksić Veljković, 2021: 249). This entire process will significantly contribute to the understanding of basic mathematical concepts.

Edgar Willems, a music psychologist and philosopher of Flemish origin, directs his method of learning music to continuous music education. Willems points out that children between the ages of two and three should be included in music education, organized and systematic, and their education should be continued during the school period. Of great importance is his reform at the academic level, which strengthens the competencies of future music teachers and artists in imparting knowledge in the field of music. Starting from the philosophical understanding that all children's knowledge is conditioned by life experiences, he connects his method of music education with the stages of learning the mother tongue. When connecting the phases of learning the mother tongue and learning music, it follows the psychological characteristics and natural phases of the child's development. In the sequence of natural events, it follows the sequence of learning the mother tongue in phases: hear speech listen to music, sounds, tones, look at the source of sound; accept the sound of speech and musical tones; remember words (tones), rhythms, songs; learn to speak and read (in music, as well); to become a writer (composer) (Terzić, 1998: 30).

In his most significant work, the Psychological basis of musical education (Bases psicologicas de la educación musical, 1961). Willems presents a method based on song, which is the synthesis of melody, rhythm and harmony as a means that enables the development of "inner knowledge, i.e. the key to real musicality" (Ivanović, 2019: 49). In music lessons, twenty minutes are spent singing, and twenty minutes are devoted to body movements. With exercises aimed at developing a sense of movement, it encourages motor imagination as the basis of musical and any other rhythm (Willems, 1964: 27). With precise didactic instructions, it enables the achievement of goals. In the first phase of musical education, the Willems method begins with "listening". Various percussion instruments. flutes. bells. metallophone, etc. are used, and the child listens and reacts with movement. As Terzić states, "the child's education goes even into the spheres of avant-garde music, where intratonal chimes (up to 1/16 tone) take the listener out of the tempered system of tones" (Terzić, 1998: 30).

Children learn through interaction with the environment in which they find themselves and through the experiences they acquire in that interaction. A stimulating environment, as pointed out by Stanković and Aleksić Veljković (2021), can represent the basis for learning and

developing skills on which mathematical concepts and other abilities (visual-spatial, musical) are formed. The basic content for learning mathematical concepts and developing certain musical abilities can be a song (Paunović and others., 2018).

Maurice Martenot, a French composer, pianist and creator of a new musical education system related to learning an instrument, presents some teaching principles that are interesting for children's development in general. He believes that music should not belong only to professionals but to everyone. It should enable enjoyment and provide creative possibilities (Martenot, 1993). The Martenot method is focused on the child and its experiences and feelings, which very often neither they nor their teachers know. Art and music should provide the child with balance and security in which it will be able to express itself freely. The author emphasizes the role of the teacher who will initiate an interdisciplinary approach connecting music and art in general.

The first phase of the work on theMartenot s method begins with sensory experiences, after which they become conscious. The next phase that the author emphasizes is listening to music. The author believes that children's hearing can be "passive (listening to music in the background), active (conscious, listening to unknown music or sounds) and very active (when a child listens to a familiar story or song)" (Martenot, 1979, According to: Tanasković, 2021).

Developing self-confidence in children is the basis that stands out in its upbringing and education, and all musical content is subordinated to it. It starts from natural rhythm, includes meter, modal scales and traditional notation. It works for a long time with a small number of tones so that children reach a high level of safety. After a certain time of singing by imitation, it moves to the synthesis of music and written signs, which allows the student to hear the music by reading the notation. Its interpretations will be personal and original and not an imitation of the teacher's interpretation. The author insists on selfcontrol of inner peace - silence (Terzić, 1998: 31).

Listening to music does not only and necessarily contribute to the development of musical abilities. It was confirmed in the researches that success in mathematics increases with the duration of active listening to music (Maričić, Ćalić, 2015). Thus, in the first grade, a song can be used as an activity for developing mental representations of the concept of number (Maričić, Ćalić, 2015), where students visualize the text, thereby creating a good basis for creating a realistic context in

relation to sets. The concept of a set represents the basis on which the concept of number will be formed in children's minds.

Mathematical content can represent the basis for developing and understanding certain musical concepts. "The mathematical-musical connection can also be seen in the division of a whole into parts: in mathematics, a whole is divided into two halves, a half into two quarters, a quarter into two eighths, an eighth into two sixteenths, and so on. The same principle can be applied in music, if we look at the even division of a whole note: a whole note contains two halves, a half two quarters, a quarter two eighths, an eighth two sixteenths and so on to the smallest note duration, one hundred twenty-eighths" (Despić, 1997 According to: Rajić, 2019: 79). Mathematical presentation and understanding of the concept of a fraction, the denominator showing how many parts the whole is divided into and the numerator showing how many parts are separated, will create a basis for understanding the note duration of a beat in music. It can be concluded that in order to read the durations and ratios of note durations, knowledge of the mathematical concept of fractions is necessary. The introduction of mathematical concepts related to the fractions half and quarter starts already in preschool age (Dejić, 2016). Children observe the parts of the whole and their values through examples, and thus create the basis for acquiring note durations in music.

CONCLUSION

The publication presents an overview of the most important methods that encourage the development of musical abilities in children of preschool and early school age, and which are important for the acquisition of initial mathematical concepts. The possibilities and importance of connecting different areas, such as music and mathematics, and their importance for the overall development of the child's personality, which is what is striven for in the new concept, were pointed out.

It was pointed out that for the development of musical and mathematical abilities, in addition to innate abilities, it is necessary to encourage and develop abilities in children. In this process, the stimulating environment in which the child lives and learns, which can be a source of both musical and mathematical content and a place of

their application, plays a significant role. Because, "in order to experience mathematics, in order to even feel the emotional charge, as, for example, in music, it should not be presented blandly. Its beauties, which are reflected in its universality and durability, should be constantly emphasized" (Dejić, Mihajlović, 2014: 19). One of the ways that can contribute to this is precisely connecting mathematics and other fields and pointing out its connection and importance with the everyday environment.

It was concluded that physical activity and movement play a significant role in the development of musical and mathematical abilities. Various movements create the basis for the development of rhythm, while activities that include movements of the whole body play a significant role in the acquisition of mathematical concepts. It can be concluded that the active position of the child significantly contributes to the development of logical-mathematical abilities, as well as spatial-visual, musical and many other abilities. The experiences that children gain in this way significantly contribute to their later success in various fields. All those activities that are related to children's interests contribute significantly to this.It was also concluded that the methods based on a song that is a synthesis of melody, rhythm and harmony, in addition to influencing the development of musical abilities, can contribute to the learning of mathematical concepts. Listening to music, which begins with sensory experiences and then becomes conscious, in addition to influencing the development of musical abilities, also affects success in mathematics and can be used as an activity to develop mental representations of the concept of number. As already pointed out in the publication, mathematical content can also represent the basis for developing an understanding of certain musical terms. We can see a mathematical-musical connection in the division of a whole into parts, which can be connected with the division of a whole note into shorter note values. Also, a mathematical understanding of fractions can create a foundation for understanding musical meter.

Directing practitioners to see the possibility of connecting music and mathematics and to find adequate pedagogical approaches in learning is extremely important for education. Therefore, future research should be focused on examining and improving the competencies of professionals –nursery-school teachers and teachers, which would be in the function of musical and mathematical education and achievement, but also the overall development of children.

BIBLIOGRAPHY

Adžaga, E. (2012). Povezanost glazbe i matematike. Matka, (82), Zagreb.

- Barker, A. (2007). *The science of harmonics in classical Greece*. New York, NY: Cambridge University Press.
- Barnett Braun, E. (1973). *Montessori & music: Rhythmic activities for young children*. New York, NY: Shocken Books.
- Baucal, A. (2012). *Standardi za rani razvoj i učenje dece ranih uzrasta u Srbiji*. Beograd, Srbija: Institut za psihologiju Filozofskog fakulteta i UNICEF.
- Bašić, S. (2011). Modernost pedagoške koncepcije Marije Montessori. Pedagogijska istraživanja, 8(2), 205–216.
- Beck, M. M., Rune, L. R., Geertsen, S. S., Ritz, C. G., Lundbye-Jensen, J., & Wienecke, J. (2016). Motor-enriched learning activities can improve mathematical performance in preadolescent children. *Frontiers in Human Neuroscience*, 10, 1–14.
- Beer, M. (1998). *How mathematics and music relate to each other?*. Brisbone: East Coast College. Retrieved from <u>https://www.scribd.com/document/275837356/Beer-How-Do-</u> Mathematics-and-Music-Relate-to-Each-Other
- Dejić, M., & Mihajlović, A. (2014). *Matematička darovitost*. Beograd, Srbija: Učiteljski fakultet.
- Dejić, M. (2016). Predškolac u svetu matematike. Beograd, Srbija: Kreativni centar.
- Duh, M. (2009). *Interdisciplinarni pristup učenju: put ka kvalitetnijem obrazovanju deteta*. Rijeka, Hrvatska: Grafika Zambelli. Retrieved from https://www.croris.hr/crosbi/publikacija/knjiga/7120
- Fauvel, J., Flood, R., & Wilson, R. J. (2006). *Music and mathematics: From Pythagoras to fractals*. Oxford, UK: Oxford University Press.
- Haroutounian, J. (2002). *Kindling the spark: Recognizing and developing musical talent*. Oxford, UK: Oxford University Press.
- Hetland, L. (2000). Learning to make music enhances spatial reasoning. *Journal of Aesthetic Education*, *34*(3/4), 179–238.
- Ivanović, M. (2019). Sistem perceptivnih aktivnosti u unapređenju muzičke percepcije i recepcije kod dece (Doktorska disertacija). Univerzitet u Novom Sadu, Novi Sad, Srbija.
- Kopas Vukašinović, E. (2018). Predškolski programi u Srbiji kao pretpostavka ostvarivanja kvaliteta vaspitne prakse. *Unapređivanje kvaliteta i dostupnosti obrazovanja u Srbiji*, 69–83.
- Lopes, L., Santos, R., Pereira, B., & Lopes, V. P. (2013). Associations between gross motor coordination and academic achievement in elementary school children. *Human Movement Science*, 32(1), 9–20. <u>https://doi.org/10.1016/j.humov.2012.05.005</u>
- Marendić, Z. (2009). Teorijski okvir razvijanja početnih matematičkih pojmova u dečjem vrtiću. *Metodika*, *18*(10), 129–141. Retrieved from <u>https://hrcak.srce.hr/file/63977</u>

- Maričić, S., & Ćalić, M. (2015). Integracija razvoja matematičkih pojmova i muzičkog vaspitanja u predškolskom vaspitanju i obrazovanju posredstvom pesama za pevanje. *Godišnjak Učiteljskog fakulteta u Vranju*, *6*, 317–326. <u>https://doi.org/10.5937/gufv1506317M</u>
- Martenot, M. (1993). Principios fundamentales de formación musical y su aplicación. Madrid: Rialp.
- Matić, E., & Mirković, K. (1986). *Muzika i predškolsko dete.* Beograd: Zavod za udžbenike i nastavnasredstva. <u>https://books.google.rs/books/about/Muzika_i_pred%C5%A1kolsko_dete</u> .<u>html?id=EDhCOgAACAAJ&redir_esc=y</u>
- Matijević, M., & Radovanović, D. (2011). *Nastava usmjerena na učenika*. Zagreb: Školske novine. ISBN 978-953-160-246-4
- McPherson, G., & Williamon, A. (2006). Giftedness and talent. U G. McPherson (Ur.), *The Child As Musician: A Handbook of Musical Development* (str. 239– 256). Oxford University Press. <u>https://psycnet.apa.org/doi/10.1093/acprof:oso/9780198530329.003.00</u> 12
- Milić, I., & Vukićević, N. (2011). Matematički pojmovi u funkciji upoznavanja ritmičkih trajanja u radu sa decom predškolskog uzrasta. Zbornik radova sa drugog međunarodnog naučno-stručnog skupa Metodički aspekti nastave matematike, 289–300. Jagodina: Pedagoški fakultet.
- Milić, I. (2020). Metodički aspekti primene animiranih filmova u muzičkom vaspitanju. U B. Mandić & J. Atanasijević (Ur.), *Ekspresivnost i intimnost u muzici & Tako male stvari: intimno u umetnosti i kulturi, Knjiga III, Zbornik radova sa XIV međunarodnog naučnog skupa Srpski jezik, književnost, umetnost,* 25–27. oktobar 2019 (str. 97–108). Kragujevac: Filološkoumetnički fakultet.
- Milinković, J., & Bogavac, D. (2011). Montessori method as a basis for integrated mathematics learning. *Metodički obzori*, 6(1), 135–143. <u>https://doi.org/10.32728/M0.06.1.2011.11</u>
- Parker, J. J. (1973). Discriminative Listening as a Basis for Problem Solving Among Four-Year-Olds. *Dissertation Abstracts International*, 33(8A), 4460–4461.
- Paunović, L., Rašković, B., & Gajtanović, Z. (2018). Primena muzičkih sadržaja u usvajanju pojmova o geometrijskim oblicima u predškolskom uzrastu. *Zbornik radova sa X naučnog skupa Nauka i nastava danas,* 77–95.
- Popović, D., & Rašković, B. (2022). Pedagoška podrška u razvijanju muzičkih sposobnosti kod dece predškolskog uzrasta. *DHS-društvene i humanističke nauke*, 1(18), 509–532. DOI: 10.51558/2490-3647.2022.7.1.509
- Pravilnik o osnovama programa predškolskog vaspitanja i obrazovanja Godine uzleta. ("Službeni glasnik RS", br. 88/17 i 27/18 dr. zakon). Beograd.
- Rajić, S. (2020). *Matematika i muzika u funkciji dečjeg razvoja*. Doktorska disertacija, Interdisciplinarne doktorske studije istorija i filozofija prirodnih nauka i tehnologije, Univerzitet u Beogradu.
- Rauscher, F. H., Shaw, G. L., & Ky, K. N. (1995). Listening to Mozart Enhances Spatial-Temporal Reasoning: Toward a Neurophysiological Basis. *Neuroscience Letters*, 185, 44–47.

Rauscher, F. H., & Zupan, M. A. (2000). Classroom keyboard instruction improves kindergarten children's spatial-temporal performance: A field experiment. *Early Childhood Research Quarterly*, 15, 215–228.

Radonjić, S. (1992). *Psihologija učenja*. Beograd: Zavod za udžbenike i nastavna sredstva.

- Radoš, K. (2010). *Psihologija muzike* (drugo dopunjeno izdanje). Beograd: Zavod za udžbenike.
- Rojko, P. (1982). Psihološke osnove intonacije i ritma. Zagreb: Muzička akademija.
- Rajović, R. (2012). *Kako uspešno razvijati IQ deteta kroz igru*. Novi Sad: Smart production.

Savić, J. M. (2018). Povezanost matematičkih i muzičkih sadržaja u nižim razredima osnovne škole. *Inovacije u nastavi - časopis za savremenu nastavu , 31*(3), 124-139. <u>https://doi.org/10.5937/inovacije1803124S</u>

Schellenberg, G. E. (2004). Music lessons enhance IQ. *Psychological Science*, 15, 511–514.

Senturk, U., Beyleroglu, M., Guven, F., Yılmaz, A., & Akdeniz, H. (2015). Motor skills in pre-school education and effects on 5-year-old children's psychomotor development. *Turkish Journal of Sport and Exercise*, 17(2), 42–47.

- Slunjski, E. (2011). *Kurikulum ranog odgoja-istraživanje i konstrukcija*. Zagreb: Školska knjiga.
- Stanković, S. (2017). Mogućnosti korelacije fizičkog vaspitanja i matematike u predškolskom uzrastu. *Uzdanica*, XIV(1), 197–205.
- Stanković, S., & Aleksić Veljković, A. (2021). Motoričke veštine, pokret i matematika. Zbornik radova Metodički aspekti nastave matematike IV, 240– 253. Fakultet pedagoških nauka u Jagodini.
- Tajčević, M. (1997). *Osnovna teorija muzike.* Beograd: Književno izdavačka zadruga Centar.

Tanasković, M. (2021). Uporedna analiza pristupa muzičkom obrazovanju i metodama Edgara Vilemsa i Morisa Martenoa. *Zbornik radova Pedagoškog fakulteta u Užicu,* br. 23. Univerzitet u Kragujevcu.

Terzić, E. (1998). Analiza i vrednovanje udžbenika muzičke kulture u osnovnoj školi u Mađarskoj i Srbiji. Beograd: Centar za usavršavanje rukovodilaca u obrazovanju.

Vaughn, K. (2000). Music and mathematics: Modest support for the oft-claimed relationship. *Journal of Aesthetic Education*, 34(3-4), 149–166.

Wang, C. C., & McCaskill, M. E. (1989). Relating musical abilities to visual-spatial abilities, mathematics, and language skills of fifth-grade children. *Canadian Journal of Research in Music Education*, 30, 184–191.

Willems, E. (1964). El ritmo musical. Buenos Aires: Eudeba.

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ПЕДАГОШКИ ПРИСТУПИ КОЈИ ПОВЕЗУЈУ МУЗИКУ И МАТЕМАТИКУ

Резиме: У раду се полази од разматрања метода музичког васпитања у XX веку које потенцирају рану музику стимулацију и могућности имлементације и интеграције истих у формирању и развоју почетних математичких појмова. Тежња за повезивањем наизглед неспојивих дисциплина музике и математике отвара питања чији је циљ усмерен на проналажење адекватних педагошких приступа који би били у функцији музичког и математичког образовања и постигнућа, али и укупног развоја деце. Примењујући методу теоријске анализе, извршено је упоређивање различитих концептуалних приступа и резултата спроведених истраживања и закључака аутора. Након разматрања наведених питања, закључено је да сагледане методе истичу значај раног развоја у формирању човека и полазе од поставке да потенцијал и окружење омогућавају детету да изгради себе. Организовање средине која детету омогућава слободу деловањем кроз организоване и спонтне активности у вртићу и школи може допринети дечјем музиком и математичком развоју. Такође, повезивање музичких и математичких садржаја међусобно као и повезивање са свакодневним активностима и постављање ситуација за учење у контекст који је деци близак и познате, као и са садржајима других подручја образовања и васпитања даје основу за успешно учење музике и математике. Педагошке импликације су садржане управо у наведеним препорукама за правилан приступ повезивања музичких и математичких садржаја и креирању подстицајне средине. Будућа истраживања потребно је усмерити на испитивање и могућности унапређивања компетенција професионалаца- васпитача и учитеља које би биле у функцији музичког и математичког образовања и постигнућа, али и укупног развоја деце.

Кључне речи: музичке способности, почетни математички појмови, методе учења, рана стимулација, подстицајна средина.