ОРИГИНАЛНИ НАУЧНИ ЧЛАНЦИ
ОРИГИНАЛЫЕ НАУЧНЫЕ СТАТЬИ
ORIGINAL SCIENTIFIC PAPERS

CONTRACTIVE CONDITIONS IN *b*-METRIC SPACES

Tatjana M. Došenović[a], *Mirjana* V. Pavlović[b], *Stojan* N. Radenović[c]
[a] University of Novi Sad, Faculty of Technology, Novi Sad, Republic of Serbia,
e-mail: tatjanad@tf.uns.ac.rs,
ORCID iD: http://orcid.org/0000-0002-3236-4410,
[b] University of Kragujevac, Faculty of Sciences, Department of Mathematics and Informatics, Kragujevac, Republic of Serbia,
e-mail: mpavlovic@kg.ac.rs,
ORCID iD: http://orcid.org/0000-0001-6257-8666,
[c] University of Belgrade, Faculty of Mechanical Engineering, Belgrade, Republic of Serbia,
e-mail: radens@beotel.net,
ORCID iD: http://orcid.org/0000-0002-7417-1342

http://dx.doi.org/10.5937/vojtehg65-14817

FIELD: Mathematics (Mathematics Subject Classification: primary 47H10, secondary 54H25)
ARTICLE TYPE: Original Scientific Paper
ARTICLE LANGUAGE: English

Abstract:

The purpose of this paper is to consider various contractive conditions in b-metric spaces which have been recently published. Our results improve and complement many recent results from this field. Using the recently obtained result by R. Miculescu and A. Mihail (Miculescu & Mihail, 2017, pp.1-11) the authors of this article show that the proofs of the majority of known results in the context of b-metric spaces can be shortened.

Keywords: metric space, common fixed point, altering distance function, point of coincidence, weak compatibility.

ACKNOWLEDGMENT: The first author is grateful for the financial support from the Ministry of Education and Science and Technological Development of the Republic of Serbia (Matematički modeli nelinearnosti, neodređenosti i odlučivanja, 174009) and from the Provincial Secretariat for Higher Education and Scientific Research, Province of Vojvodina, Republic of Serbia, Project no. 142-451-2838/2017-01. The second author is grateful for the financial support from the Ministry of Education and Science and Technological Development of the Republic of Serbia (Metode numeričke i nelinearne analize sa primenama, 174002).

Introduction

It is well known that the Banach Contraction Principle (Banach, 1922, pp.133-181) states that, if a self-mapping T of a complete metric space (M, d) is a contraction mapping, then T has a unique fixed point (say u) and for each $v \in M$ the corresponding Picard sequence $\{T^n(v)\}$ converges to this fixed point u. In general, this principle has been generalized in two directions. On the one hand, the usual contractive condition is replaced by a weakly contractive condition. On the other hand, the action spaces are replaced by metric spaces endowed with an ordered or partially ordered structure or with some kind of generalized metric space (like cone metric space, G-metric space, partial metric space, fuzzy metric space, etc.).

In 1989 I. A. Bakhtin (Bakhtin, 1989, pp.26-37) and in 1993 S. Czerwik (Czerwik, 1993, pp.5-11) introduced a new distance on a non-empty set which is called a *b*-metric. A *b*-metric space is an attempt to generalize the metric space by replacing only the triangle inequality introducing one real constant. Their definition of this new kind of generalized metric space is the following.

Definition 1 (Bakhtin, 1989, pp.26-37), (Czerwik, 1993, pp.5-11) Let M be a (non-empty) set and $K \geq 1$ a given real number. A function $d_1 : M \times M \to [0, \infty)$ is called a *b*-metric on M if, for all $p, q, r \in M$, the following conditions hold:

(b1) $d_1(p,q) = 0$ if and only if $p = q$;

(b2) $d_1(p,q) = d_1(q,p)$;

(b3) $d_1(p,r) \leq K(d_1(p,q) + d_1(q,r))$.

In this case, (M, d_1, K) is called a *b*-metric space. If (M, $\preccurlyeq$) is still a partially ordered set, then $(M, \preccurlyeq, d_1, K)$ is called an ordered *b*-metric space.

Otherwise, for all other definitions of the notions in *b*-metric spaces such as *b*-convergence, *b*-Cauchy sequence, *b*-completeness, see (Abbas et al, 2016, pp.1413-1429), (Ansari et al, 2017, pp.315-329), (Bakhtin, 1989, pp.26-37), (Huang et al, 2015), (Jovanović, 2016), (Radenović et al, 2017a, 2017b), (Roshan et al, 2013), (Zhang et al, 2017, pp.1334-1344) and the reference therein.

Definition 2 (Khan et al, 1984, pp.1-9) A function $\varphi : [0, \infty) \to [0, \infty)$ is called an altering distance function if the following properties hold:

(1) φ is continuous and nondecreasing;

(2) $\varphi(t)=0$ if and only if $t=0$.

First, a very known (important) result from a *b*-metric space is the following:

Theorem 1 (Czerwik, 1993, pp.5-11, Theorem 1) Let (M,d_1,K) be a b-complete *b*-metric space and let $T:M\to M$ satisfy

$$d_1(T(p),T(q))\leq\varphi(d_1(p,q)),\ p,q\in M, \tag{1}$$

where $\varphi:[0,\infty)\to[0,\infty)$ is an increasing function such that $\lim_{n\to\infty}\varphi^n(t)=0$ for each fixed $t>0$. Then T has an exactly one fixed point v and

$$\lim_{n\to\infty}d_1(T^n(p),v)=0 \tag{2}$$

for each $p\in M$.

Lemma 1 (Miculescu & Mihail, 2017, Lemma 2.2.) Let $\{t_n\}$ be a sequence in a *b*-metric space (M,d_1,K) such that

$$d_1(t_n,t_{n+1})\leq\mu\cdot d_1(t_{n-1},t_n) \tag{3}$$

for some $\mu\in[0,1)$, and each $n=1,2,\ldots$. Then $\{t_n\}$ is a *b*-Cauchy sequence in (M,d_1,K).

Remark 1 In several published papers based on the *b*-metric concept, the authors assume that $\mu\in[0,\frac{1}{K})$ instead of $\mu\in[0,1)$, which is obviously weaker. Then under this weaker condition they show that the Picard sequence $\{t_n=T(t_{n-1})\}_{n=1,2,\ldots}$, $t_0\in M$ is a b-Cauchy. For the proof, the authors used the following clear inequality:

$$d_1(t_m,t_n)\leq Kd_1(t_m,t_{m+1})+K^2d_1(t_{m+1},t_{m+2})+\ldots+K^{n-m-1}d_1(t_{n-2},t_{n-1})+ K^{n-m}d_1(t_{n-1},t_n), \tag{4}$$

where $n,m\in N$ and $n>m.$

However, putting $\varphi(r)=\mu\cdot r,\ r\in[0,\infty),\ \mu\in(0,1)$ in (1), the proof of Theorem 1 from (Czervik, 1993, pp.5-11) follows that Picard sequence $\{t_n=T(t_{n-1})\}_{n=1,2,\ldots}$, $t_0\in M$ is a b-Cauchy.

Now, we can show that Lemma 2.2. from (Miculescu & Mihail, 2017) is an immediate consequence of the one part of Theorem 1 from (Czerwik, 1993, pp.5-11).

First of all, we give the next result:

Lemma 2 If $\{t_n\}_{n\in N}$ is an arbitrary sequence in the nonempy set M, then there exists at least one mapping $T: M \to M$ such that it is Picard sequence of T with t_1 as the beginning point.

Proof. We define $T: M \to M$ as $T(t_k) = t_{k+1}$ for $k = 1,2,3,...$ as well as $T(t) = v_0$ in case $t \in M \setminus \{t_1, t_2, ..., t_n, ...\}$ and $v_0 \notin \{t_1, t_2, ..., t_n, ...\}$. The last one is possible if $\{t_1, t_2, ..., t_n, ...\} \subseteq M$ and $\{t_1, t_2, ..., t_n, ...\} \neq M$.

Proposition 1 Lemma 2.2. from (Miculescu & Mihail, 2017) is an immediate consequence of (Czerwik, 1993, pp.5-11, Theorem 1).

Proof. Indeed, the $\{t_n\}$ is a Picard sequence of the mapping defined in Lemma 2. It is obvious that the mapping T satisfies the condition (1) where $\varphi(r) = \mu \cdot r$, $r \in [0, \infty)$, $\mu \in (0,1)$. Further (3) becomes $d_1(T(t_{n-1}), T(t_n)) \leq \mu d(t_{n-1}, t_n)$, $n = 2,3,4,..$ i.e. the sequence $\{t_n\}$ is a *b*-Cauchy according to the proof of (Czerwik, 1993, pp.5-11, Theorem 1).

Now, by (Czerwik, 1993, pp.5-11, Theorem 1) that is, by (Miculescu, Mihail, *2017*, Lemma 2.2.), the majority of already known results can be improved. Also, by using the same argument some known results can be made significantly shorter and nicer.

The first such result is the following:

Proposition 2 Let T be a self-map on a b-complete *b*-metric space (M, d_1, K) satisfying

$$d_1(T(p), T^2(p)) \leq \mu d_1(p, T(p)) \text{ for some } \mu \in (0,1), \tag{5}$$

either (**i**) for all $p \in M$, or (**ii**) for all $p \in M, p \neq T(p)$, and suppose that T has a fixed point. Then T has a property P.

Otherwise, if T is a map which has a fixed point v, then v is a also a fixed point of T^n for every natural number *n*. However, the converse is false. For, consider $M = [0,1]$, T is defined by $T(p) = 1 - p$. Then T has a unique fixed point at $\frac{1}{2}$, but $T^n = I$ for each $n > 1$, which has every point of $[0,1]$ as a fixed point. On the other hand, if $M = [0, \pi]$, $T(p) = \cos p$, then T is nonexpansive and every iterate of T has the same fixed point as T. Involutions are also examples where $F(T) \neq F(T^n)$. See, e.g. (Jeong & Rhoades, 2005, pp.71-105) and the references therein.

We shall say that a map T has a property P if $F(T) = F(T^n)$ for every $n \in N$.

Proof (of Proposition 2). The statement for $n = 1$ is trivial. Therefore, we shall assume that $n > 1$ is a given (fixed natural number). It is clear that $F(T) \subseteq F(T^n)$. Let $v \in F(T^n)$.

Case 1. Suppose that T satisfies (i). Then, using (5),

$$d_1(v, T(v)) = d_1(T^n(v), T^{n+1}(v))$$
$$= d_1(T(T^{n-1}(v)), T^2(T^{n-1}(v))) \le \mu d_1(T^{n-1}(v), T(T^{n-1}(v)))$$
$$= \mu d_1(T(T^{n-2}(v)), T^2(T^{n-2}(v)))$$
$$\le \mu^2 d_1(T^{n-2}(v), T(T^{n-2}(v))) \le \ldots \le \mu^n d_1(v, T(v)),$$

which implies that $v = T(v)$.

Case 2. Let now T satisfy (ii).

If $v = T(v)$, then there is nothing to prove. Suppose, if possible, that $v \neq T(v)$. Then a repetition of the argument for Case 1 again leads to $d_1(v, T(v)) \le \mu^n d_1(v, T(v))$, which implies that $v = T(v)$ and $F(T^n) = F(T)$.

Remark 2 Proposition 1.8. obviously generalize the corresponding result, Theorem 1.1. from (Jeong & Rhoades, 2005, pp.71-105), for standard metric spaces.

Corollary 1 Let T be a selfmap of a b-complete *b*-metric space (M, d_1, K) satisfying

$$d_1(T(p), T(q)) \le \mu d_1(p, q) \text{ for all } p, q \in M \text{ and for some } \mu \in (0,1). \quad (6)$$

Then T has a property P.

Proof. Indeed, condition (6) implies (5). Also, by (Czerwik, 1993, pp.5-11, Theorem 1) $F(T) \neq \emptyset$. Then the result follows according to Proposition 2.

The next is also generalization of one result from a metric to a *b*-metric space.

Proposition 3 Let T be a selfmap of a b-complete *b*-metric space (M, d_1) satisfying

$$d_1(T(p), T^2(p)) \le \mu d_1(p, T(p)) \text{ for all } p \in M \text{ and some } \mu \in (0,1). \quad (7)$$

Then $F(T) \neq \emptyset$, if T is a $b-$continuous.

Proof. Let $p_0 \in M$ be an arbitrary point and let $\{p_n\}$ be a corresponding Picard sequence. For each $n \in \{0\} \cup N$ we have

$$d_1(p_{n+1}, p_{n+2}) = d_1(T(p_n), T^2(p_n)) \le \mu d_1(p_n, T(p_n)) = \mu d_1(p_n, p_{n+1}). \quad (8)$$

Further, according to (Miculescu & Mihail, 2017, Lemma 2.2.) (see also (6)) follows that $\{p_n\}$ is a b-Cauchy sequence. Since (M,d_1) is a *b*-complete *b*-metric space there is $v\in M$ such that $p_n\to v$ as $n\to\infty$. The continuity of T implies that $T(v)=v$, i.e., $F(T)\neq\emptyset$.

Jungck's result in the concept of *b*-metric spaces:

Theorem 2 Let (M,d_1,K) be a *b*-metric space and $T,S:M\to M$, $T(M)\subseteq S(M)$ be self mappings such that for all $p,q\in M$.

$$d_1(T(p),T(q))\le \mu d_1(S(p),S(q)),\ \text{where}\quad \mu\in(0,1). \tag{9}$$

Also, assume that, at least one of the following conditions hold:

(i) $(T(M),d_1)$ or $(S(M),d_1)$ is *b*-complete;

(ii) (M,d_1,K) is *b*-complete, S is *b*-continuous and T and S are commuting.

Then T and S have a unique point of coincidence. Moreover, if T and S are weakly compatible (for case (i)) then they have a unique common fixed point in M.

Proof. First, we notice that if a point of coincidence of T and S exists, then it is unique. Indeed, if w_1 and w_2 are two distinct points of coincidence of T and S, then there exist two points $u_1,u_2\in M, u_1\neq u_2$, such that $T(u_1)=S(u_1)=w_1\neq w_2=S(u_2)=T(u_2)$. Now, by (9) we have $d_1(w_1,w_2)=d(T(v_1),T(v_2))\le\mu d_1(S(v_1),S(v_2))=\mu d_1(w_1,w_2)<d_1(w_1,w_2)$, which is a contradiction.

Further, the condition $T(M)\subseteq S(M)$ implies that there exists Jungck's sequence $j_n=T(v_n)=S(v_{n+1})$, where $\{v_n\}$ is a sequence in $M, v_0\in M$ is an arbitrary point. We shall prove that the sequence $\{j_n\}$ is a *b*-Cauchy. Indeed, for each $n\in\{0\}\cup N$ we have that

$$d_1(j_{n+1},j_{n+2})=d_1(T(v_{n+1}),T(v_{n+2}))\le\mu d_1(S(v_{n+1}),S(v_{n+2}))=\mu d_1(j_n,j_{n+1}),$$

i.e., for all $n\in\{0\}\cup N$ the sequence $\{j_n\}$ satisfies condition (3). This means that it is *b*-Cauchy.

Now, let (i) holds. Therefore, since $(S(M),d_1)$ is a *b*-complete *b*-metric space, it follows that there exists $v\in M$ such that $T(v_n)=S(v_{n+1})=j_n\to Sv$ as $n\to\infty$. We will prove that $T(v)=S(v)$. In order to prove this equality, we have

$$\frac{1}{K} d_1(T(v), S(v)) \le d_1(T(v), T(v_n)) + d_1(T(v_n), S(v)) \le \mu d_1(S(v), S(v_n)) + d_1(j_n, S(v))$$
$$= \mu d_1(S(v), j_{n-1}) + d_1(j_n, S(v)) \to \mu \cdot 0 + 0 = 0.$$

Hence, $T(v) = S(v) = w$ is a point of coincidence (unique) of the pair (T, S).

If $(T(M), d_1)$ is a *b*-complete the proof is very similar.

If (ii) holds, then since (M, d_1) is *b*-complete, there exists $v \in M$ such that $T(v_n) = S(v_{n+1}) = j_n \to v,$ as $n \to \infty$. Since both self-mappings T and S are *b*-continuous, we have when $n \to \infty$:

$$S(T(v_n)) \to S(v) \text{ and } T(S(v_n)) \to T(v) \text{ when } n \to \infty.$$

Since T and S are commuting, we again obtain that $T(v) = S(v) = w$ is a point of coincidence (unique) of the pair (T, S).

For both cases (i) and (ii), according to the known Jungck's result, it follows that w is a unique common fixed point of T and S.

The next is a common fixed point theorem of the Zamfirescu type in $b-$metric spaces.

Theorem 3 (Jovanović, 2016), (Khan et al, 1984, pp.1-9), (Rhoades, 1977, pp.257-290, Theorem 4.3.) Let (M, d_1, K) be a *b*-complete *b*-metric space and let $T : M \to M$ be a mapping and let there exist nonnegative numbers a, b, c such that for all $p, q \in M$ at least one of the following conditions:

$1^0\ d_1(T(p), T(q)) \le a d_1(p, q);$

$2^0\ d_1(T(p), T(q)) \le b[d_1(p, T(p)) + d_1(q, T(q))];$

$3^0\ d_1(T(p), T(q)) \le c[d_1(p, T(q)) + d_1(q, T(p))]$

holds.

If $a < \frac{1}{K}, b < \frac{1}{2K^2}, c < \frac{1}{2K^2}$ then T has a unique fixed point.

Remark 3 By using (Miculescu & Mihail, 2017, Lemma 2.2) the conditions for a, b, c can be relaxing, that is., we get $a < 1, b < \frac{1}{2}$ i $c < \frac{1}{2K}$ (for details see Theorem 2.2. below).

Main results

In this section, we shall consider several important as well as significant contractive conditions announced in the existing literature. Readers can compare all these conditions to the corresponding ones in the context of standard metric spaces, for more details see (Rhoades, 1977, pp.257-290).

Let Ψ_1 be the family of all nondecreasing functions $\psi_1 : [0,\infty) \to [0,\infty)$ such that $\lim_{n\to\infty} \psi_1^n(t) = 0$, for all $t > 0$.

It is well known that if $\psi_1 \in \Psi_1$ then $\psi_1(t) < t$ if $t > 0$ as well as $\psi_1(0) = 0$.

Our first result is the improvement of the proof in (Abbas et al, 2016, pp.1413-1429, Theorem 2.2.)

Theorem 4 Let $(M, \preccurlyeq, d_1, K > 1)$ be a partially ordered b-complete b-metric space and let T: M → M be an increasing mapping with respect to $\preccurlyeq$ such that there exists an element $p_0 \in M$ with $p_0 \preccurlyeq T(v_0)$. Assume that

$$K \cdot \frac{1 + K \cdot d_1(p,q)}{1 + \frac{1}{2} d_1(p, T(p))} \cdot d(T(p), T(q)) \le \psi_1(M_1(p,q)) + L_1 \cdot N_1(p,q) \tag{10}$$

for all comparable elements $p, q \in M$, where $L_1 \ge 0, \psi_1 \in \Psi_1$,

$$M_1(p,q) = \max\left\{ d_1(p,q), \frac{d_1(p,T(p)) d_1(q,T(q))}{1 + d_1(T(p),T(q))} \right\} \tag{11}$$

and

$$N_1(p,q) = \min\{d_1(p,T(p)), d_1(p,T(q)), d_1(q,T(p)), d_1(q,T(q))\}. \tag{12}$$

If T is continuous, then T has a fixed point.

Proof. If $p_0 \ne T(p_0)$ then $p_0 \prec T(p_0)$. Further, for the Picard sequence we can assume that $d_1(p_n, p_{n+1}) > 0$ for all $n \in \{0\} \cup N$. Now, we will prove that

$$d_1(p_n, p_{n+1}) \le \frac{1}{K} d_1(p_{n-1}, p_n), \text{for all } n \in N. \tag{13}$$

Indeed, since

$$\frac{1+Kd_1(p_{n-1},p_n)}{1+\frac{1}{2}d_1(p_{n-1},T(p_{n-1}))}=\frac{1+Kd_1(p_{n-1},p_n)}{1+\frac{1}{2}d_1(p_{n-1},p_n)}>\frac{1+d_1(p_{n-1},p_n)}{1+\frac{1}{2}d_1(p_{n-1},p_n)}>1,$$

then by using (10) with $p = p_{n-1}, q = p_n$, we obtain

$$Kd_1(p_{n+1},p_n)=Kd_1(T(p_n),T(p_{n-1}))\le \psi_1(d_1(p_n,p_{n-1}))+L_1\cdot N_1(p_n,p_{n-1}).$$

Because $M_1(p_n,p_{n-1})=d_1(p_{n-1},p_n), \psi_1(d_1(p_n,p_{n-1}))<d_1(p_n,p_{n-1})$ and $N_1(p_n,p_{n-1})=0$ we obtain that (13) holds.

This means that the sequence $\{p_n\}$ is a *b*-Cauchy, according to Lemma 2.2. from (Miculescu & Mihail, 2017) which then converges to some $u\in M$. The continuity of T implies that u is a fixed point of T.

Remark 4 *All that shows that our approach gives a much shorter and nicer proof than the ones in (Ansari et al, 2017, pp.315-329). Also, by the same method, the proofs of all results in (Ansari et al, 2017, pp.315-329) can be improved.*

In fact, the main (important) question is the following: Does some given contractive condition in the framework of any class of generalized metric spaces imply (give) that the corresponding Picard sequence is a Cauchy (in this class)? The previously contractive condition is such. We proved that for it holds $d_1(p_n,p_{n+1})\le \mu d_1(p_{n-1},p_n)$ for all $n\in N$ and some $\mu\in(0,1)$. Since $K>1$ and $\mu=\frac{1}{K}$ then the result follows by (Miculescu & Mihail, 2017, Lemma 2.2.).

In the framework of $b-$metric spaces, the following two results are specific.

Theorem 5 *Let* (M,d_1,K) *be a b-complete b-metric space and let* $T:M\to M$ *be a b-continuous mapping. Also let*

$$d_1(Tp,Tq)\le ad_1(p,Tp)+bd_1(q,Tq), \text{for all } p,q\in M, a,b\ge 0, a+b<1$$

that is

$$d_1(Tp,Tq)\le ad_1(p,Tq)+bd_1(Tp,q), \text{for all } p,q\in M, a,b\ge 0, a+b<\frac{1}{K}$$

In each of the given cases, T has a unique fixed point (say v) and for any $u\in M$ the sequence $\{T^n(u)\}\to v$ as $n\to\infty$.

Proof. In the first (Kannan) case, we obtain that $d_1(p_{n+1}, p_n) \leq \frac{a}{1-b} \cdot d_1(p_n, p_{n-1})$, while in the second one (Chatterjea), we have $d_1(p_{n+1}, p_n) \leq \frac{(a+b)K}{2-(a+b)K} \cdot d_1(p_n, p_{n-1})$. According to (Miculescu, & Mihail, 2017, Lemma 2.2.) it follows that in both cases that the Picard sequence $\{T^n p_0\}_{n\in\{0\}\cup N}, p_0 \in M$ is a *b*-Cauchy. Since T is *b*-continuous, the result follows.

Conclusion

Based on the previous discussion, we can conclude that the proofs of the majority results in the existing literature for the concept of b-metric spaces can be significantly shortened by using (Miculescu & Mihail, 2017, Lemma 2.2.).

All these results are in the following papers (Aghajani et al. 2014, pp. 941-960), (Allahyar et al, 2014), (Chandok et al, 2017a), (Chandok et al, 2017b, pp.331-345), (Demma & Vetro, 2015), (Ding et al, 2016, pp. 151-164), (Dung & Hang, 2016, pp. 267-284), (Harandi, 2014, pp. 351-358), (Kaushik et al, 2017), (Khamsi & Husain, 2010, pp. 3123-3129), (Kir & Kiziltunc, 2013, pp. 13-16), (Kumam et al, 2015), (Latif et al, 20915, pp. 363-377), (Liu & Gu, 2016, pp. 5909-5930), (Ozturk & Ansari, 2017, pp. 45-52), (Parvaneh et al, 2013), (Petrusel et al, 2017, pp. 199-215), (Piri & Kumam, 2016), (Roshan et al, 2014a, pp. 725-737), (Roshan, et al, 2015), (Roshan et al, 2014b, pp. 613-624), (Sarwar et al, 2017, pp. 3719-3731), (Sarwar & Rahman, 2015, pp. 70-78), (Sintunavarat, 2016, pp. 397-416), (Zabihi & Razani, 2014).

References

Abbas, M., Chema, I.Z., Razani, A., 2016. *Existence of common fixed point for b-metric rational type contraction.* Filomat, 30(6), pp.1413-1429. Available at: http://dx.doi.org/10.2298/FIL1606413A.

Aghajani, A., Abbas, M., Roshan, J.R., 2014. *Common fixed point of generalized weak contractive mappings in partially ordered b-metric spaces.* Math. Slovaca, 64(4), pp.941-960. Available at: https://doi.org/10.2478/s12175-014-0250-6.

Allahyar, R., Arab, R., Haghighi, A.S., 2014. A generalization on weak contractions in partially ordered b-metric spaces and its application to quadratic integral equations. J. Inequal. Appl., 2014:355. Available at: https://doi.org/10.1186/1029-242X-2014-355.

Ansari, A., Razani, A., Hussain, N., 2017. Fixed and coincidence points for hybrid rational Geraghty contractive mappings in ordered b-metric spaces. Int. J. Nonlinear Anal. Appl., 8(1), pp.315-329. Available at: http://ijnaa.semnan.ac.ir/article_453.html.

Banach, S., 1922. Sur les opérations dans les ensembles abstraits et leur application aux équations integrals. Fundamenta Mathematicae, 3(1), pp.133-181.

Bakhtin, I.A., 1989. The contraction principle in quasimetric spaces. Funct. Anal., 30, pp.26-37.

Chandok, S., Huang, H., Radenović, S., 2017a. *Some fixed point results for generalized F-Suzuki type contractions in b-metric spaces,* to appear in Sahad Comunications in Mathematical Analysis.

Chandok, S.C., Jovanović, M.S., Radenović, S.N., 2017b. Ordered b-metric spaces and Geraghty type contractive mappings. Vojnotehnicki glasnik/Military Technical Courier, 65(2), pp.331–345. Available at: http://dx.doi.org/10.5937/vojtehg65-13266.

Czerwik, S., 1993. Contraction mappings in-metric spaces. Acta Math. Inform. Univ. Ostrav, 1, pp.5-11.

Demma, M. & Vetro, P., 2015. *Picard sequence and fixed point results on b-metric spaces.* J. Funct. Space, Vol.2015. Available at: http://dx.doi.org/10.1155/2015/189861.

Ding, H.S., Imdad, M., Radenović, S., Vujaković, J., 2016. *On some fixed point results in b-metric, rectangular and b-rectangular metric spaces.* Arab J. Math. Sci, 22(2), pp.151-164. Available at: https://doi.org/10.1016/j.ajmsc.2015.05.003.

Dung, N.V. & Hang, V.T.L., 2016. On relaxions of contraction constants and Caristi's theorem in b-metric spaces. J. Fixed Point Theory Appl., 18(2), pp.267-284. Available at: https://doi.org/10.1007/s11784-015-0273-9.

Harandi, A.A., 2014. *Fixed point theory for quasi-contraction maps in b-metric spaces.* Fixed Point Theory, 15(2), pp.351-358. Available at: http://www.math.ubbcluj.ro/~nodeacj/vol__15(2014)_no_2.php

Huang, H., Radenović, S., Vujaković, J., 2015. On some recent coincidence and immediate consequences in partially ordered b-metric spaces. Fixed Point Theory Appl., 2015:63. Available at: https://doi.org/10.1186/s13663-015-0308-3.

Jeong, G.S. & Rhoades, B.E., 2005, *Maps for which* $F(T)=F(T^n)$, Fixed Point Theory Appl., 6, pp. 71-105, Available at: https://www.novapublishers.com/catalog/product_info.php?products_id=4068.

Jovanović, M., 2016. *Contribution to the theory of abstract metric spaces,* Ph.D. thesis, University of Belgrade, Faculty of Mathematics, Belgrade. Available at: http://nardus.mpn.gov.rs/handle/123456789/7975.

Kaushik, P., Kumar, S., Tas, K., 2017. *A new class of contraction in b-metric spaes and applications.* Abstr. Appl. Anal., Vol.2017. Available at: https://doi.org/10.1155/2017/9718535.

Khamsi, M. A. & Hussain, N., 2010. *KKM mappings in metric type spaces.* Nonlinear Anal., 73(9), pp.3123-3129. Available at: https://doi.org/10.1016/j.na.2010.06.084.

Khan, M. S., Swaleh, M., Sessa, S., 1984. *Fixed point theorems by altering distances between the points.* Bul. Aust. Math. Soc., 30(1), pp.1-9. Available at: https://doi.org/10.1017/S0004972700001659.

Kir, M. & Kiziltunc, H., 2013. On Some Well Known Fixed Point Theorems in *b*-Metric Spaces. Turkish J. Anal. Numb. Theory, 1(1), pp.13-16. Available at: http://dx.doi.org/10.12691/tjant-1-1-4.

Kumam, P., Sintunavarat, W., Sedghi, S., Shobkolaei, N., 2015. *Common fixed point of two R-weakly commuting mappings in b-metric spaces.* J. Funct. Space, Vol.2015. Available at: http://dx.doi.org/10.1155/2015/350840.

Latif, A., Parvaneh, V., Salimi, P., Al-Mazrooei, A.E., 2015. *Various Suzuki type theorems in b-metric spaces.* J. Nonlinear Sci. Appl. 8(4), pp.363-377.

Liu, L. & Gu, F., 2016. Common fixed point theorems for six self-maps in b-metric spaces with nonlinear contractive conditions. J. Nonlinear Sci. Appl., 9(12), pp.5909-5930.

Miculescu, R. & Mihail, A., 2017. *New fixed point theorems for set-valued contractions in b-metric spaces.* J. Fixed Point Theory Appl, pp.1-11. Available at: https://doi.org/10.1007/s11784-016-0400-2.

Ozturk, V. & Ansari, A.H., 2017. Common fixed point theorems for mappings satisfying (E.A)-property via C-class functions in b-metric spaces. Appl. Gen. Topol., 18(1), pp.45-52. Available at: https://doi.org/10.4995/agt.2017.4573.

Parvaneh, V., Roshan, J. R., Radenović, S., 2013. *Existence of tripled coincidence points in ordered b-metric spaces and an application to a system of integral equations.* Fixed Point Theory Appl., 2013:130. Available at: https://doi.org/10.1186/1687-1812-2013-130.

Petrusel, A., Petrucel, G., Yao, J. C., 2017. *Fixed point and coincidence point theorems in b-metric spacs with applications.* Appl. Anal. Discrete Math., 11(1), pp.199-215. Available at: https://doi.org/10.2298/AADM1701199P.

Piri, H. & Kumam, P., 2016. *Fixed point theorems for generalized F-Suzuki-contraction mappings in complete b-metric spaces.* Fixed Point Theory Appl., 2016:90. Available at: https://doi.org/10.1186/s13663-016-0577-5.

Radenović, S., An, T.V., Quan, L.T., 2017a. Some coincidence point results for T-contraction mappings on partially ordered b-metric spaces and applications to integral equations. Nonlinear Analysis: Modelling and Control, 22(4), pp.545-565. Available at: https://doi.org/10.15388/NA.2017.4.9.

Radenović, S., Došenović, T., Ozturk, V., Dolićanin, Č., 2017b. *A note on the paper: "Nonlinear integral equations with new admissibility types in b-metric spaces".* J. Fixed Point Theory Appl. Available at: https://doi.org/10.1007/s11784-017-0416-2.

Rhoades, B. E., 1977. *A comparison of various definitions of contractive mappings.* Transaction of the American Mathematical Society, 226, pp.257-290. Available at: https://doi.org/10.2307/1997954.

Roshan, J.R., Parvaneh, V., Altun, I., 2014a. Some *coincidence point results in ordered b-metric spaces and applications in a system of integral equations.* Appl. Math. Comput., 226, pp.725-737. Available at: https://doi.org/10.1016/j.amc.2013.10.043.

Roshan, J.R., Parvaneh, V., Radenović, S., Rajović, M., 2015. *Some coincidence point results for generalized* (ψ,φ)*-weakly contractions in ordered b-metric spaces.* Fixed Point Theory Appl., 2015: 68. Available at: https://doi.org/10.1186/s13663-015-0313-6.

Roshan, J.R., Parvaneh, V., Shobkolaei, N., Sedghi, S., Shatanawi, W., 2013. *Common fixed points of almost generalized* $(\psi,\varphi)_s-$*contractive mappings in ordered b-metric spaces.* Fixed Point Theory Appl., 2013:159. Available at: https://doi.org/10.1186/1687-1812-2013-159.

Roshan, J.R., Shobkolaei, N., Sedghi, S., Abbas, M., 2014b. *Common fixed point of four maps in b-metric spaces.* Hacett. J. Math. Stat., 43(4), pp.613-624.

Sarwar, M., Jamal, N., Li, Y., 2017. *Coincidence point results via generalized* $(\psi,\phi)-$*weak contractions in partial b-metric spaces with application.* J. Nonlinear Sci. Appl., 10(7), pp.3719-3731. Available at: http://dx.doi.org/10.22436/jnsa.010.07.29.

Sarwar, M. & Rahman, M.U., 2015. Fixed point theorems for Ciric's and generalized contractions in b-metric spaces. Int. J. Anal. Appl., 7(1), pp.70-78.

Sintunavarat, W., 2016. *Nonlinear integral equations with new admissibility types in b-metric spaces.* J. Fixed Point Theory Appl., 18(2), pp.397-416. Available at: https://doi.org/10.1007/s11784-015-0276-6.

Zabihi, F. & Razani, A., 2014. *Fixed point theorems for hybrid rational Geraghty contractive mappings in ordered b-metric spaces.* J. Appl. Math., Vol.2014, Article ID 929821. Available at: http://dx.doi.org/10.1155/2014/929821.

Zhang, C., Li, S., Liu, B., 2017. *Topological structures and the coincidence point of two mappings in cone b-metric spaces.* J. Nonlinear Sci. Appl., 10(4), pp.1334-1344. Available at: http://dx.doi.org/10.22436/jnsa.010.04.05.

УСЛОВИЯ СЖАТИЯ В *b*-МЕТРИЧЕСКИХ ПРОСТРАНСТВАХ

Татьяна М. Дошенович[а], *Мирьяна* В. Павлович[б], *Стоян* Н. Раденович[в]
[а] Университет в г. Нови-Сад, Технологический факультет, г. Нови-Сад, Республика Сербия,
[б] Университет в г. Крагуевац, Естественно-математический факультет, Институт математики и информатики, г. Крагуевац, Республика Сербия,
[в] Белградский университет, Машиностроительный факультет, г. Белград, Республика Сербия

ОБЛАСТЬ: математика (математическая тематическая классификация: первичная 47H10, вторичная 54H25)
ВИД СТАТЬИ: оригинальная научная статья
ЯЗЫК СТАТЬИ, английский

Резюме:

В данной работе представлен анализ различных условий сжатия в b-метрических пространствах, которые недавно были опубликованы. На основании исследований настоящих результатов мы дополнили и откорректировали многие аспекты результатов в данной области. Так, например, исследовав недавние результаты, полученные Р. Микулеску и А. Михаилом (Miculescu & Mihail, 2017, pp.1-11), авторы настоящей статьи доказали, что многие известные результаты в контексте b-метрических пространств могут быть значительно сокращены.

Ключевые слова: метрическое пространство, общая фиксированная точка, функция изменения расстояния, точка совпадения, низкая совместимость.

КОНТРАКТИВНИ УСЛОВИ У *b*-МЕТРИЧКИМ ПРОСТОРИМА

Татјана М. Дошеновић[а], *Мирјана* В. Павловић[б], *Стојан* Н. Раденовић[в]
[а] Универзитет у Новом Саду, Технолошки факултет, Нови Сад, Република Србија,
[б] Универзитет у Крагујевцу, Природно-математички факултет, Институт за математику и информатику, Крагујевац, Република Србија,
[в] Универзитет у Београду, Машински факултет, Београд, Република Србија

ОБЛАСТ: математика (математичка тематска класификација: примарна 47H10, секундарна 54H25)
ВРСТА ЧЛАНКА. оригинални научни чланак
ЈЕЗИК ЧЛАНКА: енглески

Сажетак:

Циљ овог рада јесте да размотри разне контрактивне услове у b-метричким просторима који су недавно објављени. Наши резултати поправљају и допуњују многе недавне резултате из

овог контекста. Користећи недавно добијени резултат Р. Микулескуа и А. Михаила, (Miculescu & Mihail, 2017, pp.1-11) аутори овог чланка показују да докази многих познатих резултата у контексту b-метричких простора могу бити доста скраћени.

Кључне речи: метрички простор, заједничка фиксна тачка, функција промене раздаљине, тачка коинциденције, слаба компатибилност.

Paper received on / Дата получения работы / Датум пријема чланка: 17.08.2017.
Manuscript corrections submitted on / Дата получения исправленной версии работы / Датум достављања исправки рукописа: 20.09.2017.
Paper accepted for publishing on / Дата окончательного согласования работы / Датум коначног прихватања чланка за објављивање: 22.09.2017.