



# APPLICATION OF CHEBYSHEV'S INEQUALITY IN THE PRELIMINARY FEASIBILITY STUDY FOR CONSTRUCTING A SOLAR THERMAL POWER PLANT

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## Abstract:

*Introduction/purpose: This paper examines some of the applications of Chebyshev's inequality. Using Chebyshev's inequality, the analysis of a preliminary feasibility study for constructing a solar thermal power plant in the Banja Luka area has been conducted. The goal of the preliminary analysis is to show, without financial investments, if there is a basis for the climate parameters measurement in the area.*

*Methods: For the known values of the arithmetic means and the standard deviations of the number of cloudy days, the probability of deviation of the number of cloudy days from the mean value was defined by applying Chebyshev's inequality.*

*Results: The diagram shows the values of the upper and lower limits of the number of cloudy days that deviate from the expected value with a probability of 50%.*

*Conclusion: The preliminary assessment of the justification of the realization of a solar thermal power plant justifies the measurements necessary for the analysis and detailed calculation of this type of a plant, because the annual interval of cloudy days is from 94 to 164, or from 26 to 44% in the year.*

*Key words: probability, random variable, dispersion, mean value, cloudy day, solar radiation, solar thermal power plants.*

## Introduction

Many natural phenomena cannot be described in exact ways by applying mathematical rules and schemes. There is not a possibility to unambiguously determine correlations and relations within them. These phenomena are known as random or stochastic and for the probability theory they are the main subject of study. One of the branches of mathematics which deals with the application of the probability theory basic results is mathematical statistics (Dekking et al, 2005; Arandelović et al, 2011). The concept of probability is a completely determined mathematical concept, i.e. probability is a determined function defined by a set of random events. Random events show certain regularity in the frequency of their occurrence if they are observed on a large scale (Elazar, 1972).

All statistic principles are stable if they are related to mass occurrences. Random deviations from the mean value are negligible when observed on a large scale. This stability of mean values of mass occurrences is the essence of the concept of the law of large numbers (Lange, 2010). At a significant number of random occurrences, their mean result ceases to be random and could be predicted with a great precision, which is very useful at analyzing renewable energy sources systems, as they tend to have a random character (Yang et al, 2019; Wang et al, 2017; Hooshmand et al, 2012).

There are certain entry parameters needed when working on a feasibility study of the realization of a certain system. If detailed entry parameters are not available, and their collection demands measuring for a lengthy period of time, it is necessary to determine, according to the existing data, whether conducting the measurement would be justified (Bahmani et al, 2020). A feasibility study related to the construction of a solar thermal plant in the Banja Luka area needs to be based on relevant data related to solar radiation (direct, diffuse and reflected), obtained by long term measurements. Due to the lack of official data on the solar radiation for this area, the available data to be used for this purpose is the data on the number of cloudy days.

Solar thermal plants with parabolic trough collectors, unlike photovoltaic systems, do not use diffuse radiation. Based on the above mentioned, the goal of this paper is to estimate the annual number of cloudy days with a 50% probability, using Chebyshev's inequality. If the preliminary estimation, which does not demand financial investment, leads to a conclusion that the number of cloudy days (with a 50% probability) is under

50% of the year, it is justifiable to invest and conduct measurements on which the feasibility study would be based.

### Inequality in probability theory

It is impossible to predict which of possible values a random variable will have in a specific experiment. It depends on various random circumstances; however, common influence of various random circumstances can lead to the results that almost do not depend on the experiment (Simonović, 1986).

Probability ( $P$ ) is a function that to each event ( $A$ ) assigns a number  $P(A) \in [0, 1]$ , and it is defined by a probability space. A probability space  $(\Omega, F, P)$  consists of the sample space  $\Omega$ , where the set  $F$  represents  $\sigma$  – algebra of the events defined on the space  $\Omega$ . The function  $P: F \rightarrow [0, 1]$  is the probability for  $\Omega$  if it fulfills the following axioms:

- Non-negativity -  $P(A) \geq 0$ ,
- Normalization -  $P(\Omega) = 1$ ,
- Finite additivity - for the family of disjoint sets  $\{A_i : i \in I\} \subseteq F, I \subseteq N$  it is as follows:

$$P\left(\bigcup_{i \in I} A_i\right) = \sum_{i \in I} P(A_i). \quad (1)$$

The conclusion is that the non-negativity and normalization axioms result in the following inequality:  $0 \leq P(A) \leq 1$ , for each  $A \in F$ .

The probability space where  $\Omega$  is a discrete set (finite or countably infinite), and corresponding  $\sigma$ -algebra equals the power set of  $\Omega$ , i.e.  $F = P(\Omega)$ , is known as a discrete probability space (Jaoude, 2016).

In practice, it is important to know the conditions under which the common influence of various random circumstances leads to the results that almost do not depend on the case, as this enables the prediction of the occurrence's possible outcome (Simonović, 1986).

**Definition:** If on the probability space there is a continuous random variable  $X$ , with the density function  $f$ , and if the integral:

$$\int_{-\infty}^{+\infty} |x| f(x) dx, \quad (2)$$

is finite, then the random variable  $X$  has the expectation and the number:

$$EX = \int_{-\infty}^{+\infty} x f(x) dx, \quad (3)$$

and it is called the mathematical expectation of the random variable  $X$  (Maširević & Keglević, 2017).

In the case when  $\Omega$  is a discrete set, and there is a random variable  $X$  on the discrete probability space, the mathematical expectation of the discrete random variable  $X$  can be defined as:

$$EX = \sum_{i \in I \subseteq N} x_i p_i. \quad (4)$$

Understanding the principle of mathematical expectation is only the first step in determining the parameters of a certain distribution. Dispersion (variance) is the next important parameter which represents the extent to which certain distribution deviates from its mean value. The order moment  $k \in N$ , as the mathematical expectation  $E(X^k)$  of the random variable  $X^k$  is defined by the following theorem which represents the basis for analyzing the dispersion concept.

**Theorem:** If for the random variable  $X$  there is a finite (absolute) order moment  $n$ , then there are all order moments  $k < n$  (Arandelović et al, 2011).

If the variance (expressed as  $Var(X), \sigma_x^2, \sigma^2$ ) were defined as  $E[X - E(X)]$ , it would not be a valid choice as this value always equals zero, i.e. it is as follows:

$$E[X - E(X)] = E(X) - E[E(X)] = E(X) - E(X) = 0. \quad (5)$$

In addition, if the variance were defined as  $E(|X - E(X)|)$ , it has been proven that this measure would not have good characteristics important for probability theory. Better characteristics than the above mentioned are present at the random variable  $[X - E(X)]^2$ , i.e. the variance is defined as  $E[X - E(X)]^2$ , provided this expectation exists (Maširević & Keglević, 2017).

Based on the dispersion (2<sup>nd</sup> order moment) of the random variable  $X$ :

$$D(X) = E[X - E(X)]^2, \quad (6)$$

it follows that:

$$D(X) = E(X^2) - 2[E(X)]^2 + [E(X)]^2, \quad (7)$$

which results in the dispersion of random variables more suitable for practical calculation:

$$D(X) = E(X^2) - [E(X)]^2. \quad (8)$$

Dispersion represents the square degree of the deviation of  $X$  from its mean value, hence its root, denoted as a standard deviation of the random variable  $X$  (Arandelović et al, 2011), is often considered, and represented as:

$$\sigma(X) = \sqrt{D(X)} = \sqrt{E(X^2) - [E(X)]^2}. \quad (9)$$

In probability theory, there is a saying that behind every limit theorem there is probability inequality, i.e. that a large number of inequalities has been discovered in very attempts to prove some of the fundamental theorems of probability theory. In this way, Chebyshev discovered his renowned inequality, later named Chebyshev's inequality after him, through which he proved the general form of the Law of large numbers. If we have a series of random variables  $(X_n, n \in N)$  such that for each natural number  $n$  the random variables  $X_1, \dots, X_n$  are mutually independent and their variances are uniformly limited, then the probability that the realization of the random variables'  $X_1, \dots, X_n$  mean value differs from the expected mean in more than a randomly chosen small number has a tendency toward zero with the increasing number of random variables that are taken when calculating the mean. The inequality itself was first presented by French mathematician I.J. Bienaymé in 1853, and 14 years later was proven by Chebyshev (Stellato et al, 2017; Maširević & Keglević, 2017).

If the function of probability distribution is known, then the probability of an event  $\{|X| \geq \varepsilon\}$ ,  $\varepsilon > 0$ , can be determined, where the upper limit of the probability is  $P(|X| \geq \varepsilon)$  (Mitrović, 2007).

**Markov's inequality** (Russian mathematician A.A. Markov, 1856-1922): Let  $X$  be a non-negative random variable, if there is  $E(X^k)$ ,  $k \in N$ , then:

$$P(X \geq \varepsilon) \leq \frac{E(X^k)}{\varepsilon^k} \text{ for each } \varepsilon > 0. \quad (10)$$

**Chebyshev's inequality:** Also according to the author (Chen, 2011), if there is  $Var(X)$ , then:

$$P(|X - E(X)| \geq \varepsilon) \leq \frac{Var(X)}{\varepsilon^2}. \quad (11)$$

### Chebyshev's inequality

Pafnuty Lvovich Chebyshev (1821–1894) was a Russian mathematician. He graduated from the Moscow University where he also started his academic career. Later on he moved to Saint Petersburg where he founded one of the most significant Russian mathematical schools which today is named after him. The subject of his research was probability theory. He demonstrated the weak law of large numbers which eventually was named after him. Some of his students were famous mathematicians Markov, Lyapunov and Korkin (Medić, 2014).

Analyzing the sum of a large number of random variables and their arithmetic means, it can be noticed that partial damping of deviation at summing up causes a decrease in dispersion of the arithmetic mean and enables prediction of its possible outcome at an unlimited increase of the number of addends. Such principles and the conditions under which these principles occur constitute the content of a series of important theorems known under the common name of Law of large numbers, to which Chebyshev's as well as Bernoulli's theorems belong. Chebyshev's theorem is the most general law of large numbers, and Bernoulli's is the simplest. In order for these theorems to be proven, Chebyshev's inequality is needed, as it applies to both discrete and continuous random variables (Jaoude, 2016; Simonović, 1986).

Laws of large numbers are very useful when analyzing renewable energy sources systems, which can be said to have, to a great extent, a random character. An arithmetic mean, i.e. the mean value of many random occurrences ceases to be random and can be defined with a great certainty. These laws consider various forms of convergence of sequence of random variables towards mathematical expectation.

Supposing  $E(X) = \mu$  is an arithmetic mean and  $\sigma$  is a standard deviation of a discrete random variable, Chebyshev formulated the following inequalities (Amidan et al, 2005; Biyya et al, 2017):

$$P(|X - \mu| > k \cdot \sigma = \varepsilon) < \frac{1}{k^2} = \frac{\sigma^2}{\varepsilon^2} \quad \text{and} \quad (12)$$

$$P(|X - \mu| \leq k \cdot \sigma = \varepsilon) \geq 1 - \frac{1}{k^2} = 1 - \frac{\sigma^2}{\varepsilon^2}, \quad (13)$$

where  $k$  and  $\varepsilon$  are positive real numbers defined by the relation  $k \cdot \sigma = \varepsilon$  from which follows:

$$\frac{1}{k^2} = \frac{\sigma^2}{\varepsilon^2}. \quad (14)$$

Based on the given inequalities, it can be said that probability, when the random variable is outside or inside of the following interval (Elazar, 1972):

$$[\mu - \varepsilon = \mu - k \cdot \sigma; \mu + \varepsilon = \mu + k \cdot \sigma], \quad (15)$$

is smaller than the above mentioned relation:

$$\frac{1}{k^2} = \frac{\sigma^2}{\varepsilon^2}, \quad (16)$$

or that it is not smaller than:

$$1 - \frac{1}{k^2} = 1 - \frac{\sigma^2}{\varepsilon^2}. \quad (17)$$

Chebyshev's inequality states that the probability of the random variable  $X$  to deviate from its expectation at its absolute value for greater than or equal to  $k$  standard deviations, is less than or equal to  $1/k^2$  (Maširević & Keglević, 2017).

Chebyshev's inequality gives only the upper limit of the probability of the given deviation. The probability for a random variable to assume the value outside the interval  $(\mu - 3\sigma, \mu + 3\sigma)$ , is in practice generally far smaller than  $1/9$ . If the distribution law is not known, and only  $\mu$  and  $\sigma$  are, the given interval is considered to be an interval of practically possible values of the random variable  $X$ . The statistic rule stating that for normal distribution it is necessary that all the values lie within the three standard deviations of the arithmetic mean is called three sigma or 68-95-99.7 rule. More precisely, 68.27% of the values lie within one standard deviation of the mean, 95.45% of the values lie within two standard deviations of the mean and 99.73% of the values lie within three standard deviations of the mean, which can be presented as:

$$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0,9973. \quad (18)$$

If a product whose unit of measurement is marked as  $J$  is analyzed, the values of the new components are grouped around  $J$  through a normal distribution. Any bigger deviation serves as information on the malfunctioning of the product which should be discarded.

In Figure 1, the values of the random variable  $X_i$  and its arithmetic mean  $\mu$  are applied to the  $X$  axis. Based on the probability addition theorem, for any positive number  $\varepsilon$ , it can be written:

$$P(|X - \mu| > \varepsilon) = \sum_{|X_i - \mu| > \varepsilon} p_i, \quad (19)$$

where  $|X_i - \mu| > \varepsilon$  under the summation sign implies that the sum is spread across all the  $p_i$  values whose corresponding values  $X_i$  lie outside the  $\overline{AB}$  straight line.

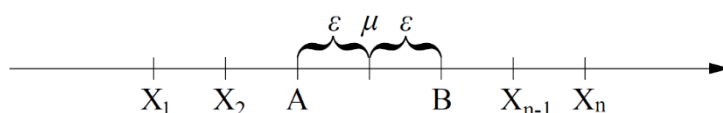


Figure 1 –  $X$  axis with random variables and the arithmetic value  
Рис. 1 – Ось  $X$  со случайными переменными и средним арифметическим  
Слика 1 – Оса  $X$  са случајним промјенљивим и аритметичком средином

Dispersion has the following form:

$$\sigma^2 = E(X - \mu)^2 = \sum_{i=1}^n (X_i - \mu)^2 p_i = \sum_{i=1}^n |X_i - \mu|^2 p_i. \quad (20)$$

Considering that all the elements of the previous sum are positive, the sum can decrease if the values of  $X_i$  that lie outside the  $\overline{AB}$  straight line are used:

$$\sigma^2 > \sum_{|X_i - \mu| > \varepsilon} |X_i - \mu|^2 p_i. \quad (21)$$

If the expression under the summation sign  $|X_i - \mu|$  is replaced with  $\varepsilon$ , the sum will decrease even more:

$$\sigma^2 > \sum_{|X_i - \mu| > \varepsilon} \varepsilon^2 p_i = \varepsilon^2 \sum_{|X_i - \mu| > \varepsilon} p_i, \quad (22)$$

as for all the elements of the sum it is:

$$|X_i - \mu| > \varepsilon. \quad (23)$$

Based on the given relations:

$$P(|X - \mu| > \varepsilon) = \sum_{|X_i - \mu| > \varepsilon} p_i \quad \text{and} \quad \sigma^2 > \sum_{|X_i - \mu| > \varepsilon} \varepsilon^2 p_i = \varepsilon^2 \sum_{|X_i - \mu| > \varepsilon} p_i, \quad (24)$$



a possible result is:

$$\sigma^2 > \varepsilon^2 P(|X - \mu| > \varepsilon), \quad (25)$$

which directly leads to the first Chebyshev's inequality (Elazar, 1972).

Chebyshev's inequality has a great theoretical value. Unfortunately, its practical value is limited as it gives only a rough (sometimes even trivial) probability estimation (Simonović, 1986).

For events that are said to be opposite, the following applies:

$$|X - \mu| \leq \varepsilon \text{ and } |X - \mu| > \varepsilon, \quad (26)$$

and the sum of their probabilities equals one:

$$P(|X - \mu| \leq \varepsilon) + P(|X - \mu| > \varepsilon) = 1. \quad (27)$$

Based on the above demonstrated and the first Chebyshev's inequality, the second Chebishev's inequality results (Elazar, 1972).

## Preliminary feasibility study of solar thermal power plant construction using Chebyshev's inequality

Solar thermal power plants are sources of electrical energy generated by transforming solar energy into thermal energy by the process of heating a fluid or a solid substance and using the product in the circular process for generating electricity.

As all the forms of solar thermal plants need high temperatures to function, they have to have a system to concentrate a large area of sunlight onto a small surface. The oldest and the most commonly used type of plants of this sort are parabolic trough plants (Figure 2). They consist of long rows of parabolic mirrors (curved as a parabola) and a collector placed above them (Pupčević, 2016).



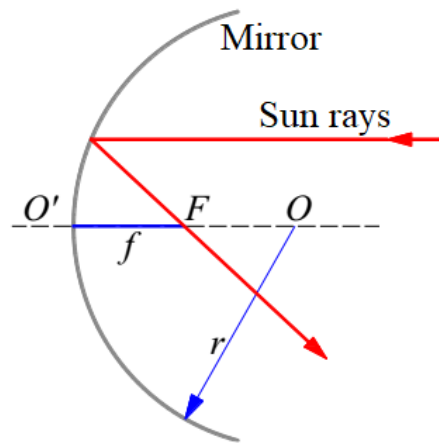


Figure 3 – Spherical mirror and its elements  
 Рус. 3 – Сферическое зеркало и его элементы  
 Слика 3 – Сферно огледало и његови елементи

The elements of spherical mirrors are:

- $O$  - the center of the curvature is the center of the sphere of which the mirror is a part,
- $O'$  - vertex of the mirror is its highest point,
- $r$  - radius of the curvature is the radius of the sphere of which the mirror is a part,
- $OO'$  - principal axis is an imaginary line passing through the vertex, focus and the centre of the mirror,
- $F$  - focus is a point on the optical axis through which rays of light pass, and
- $f$  - focal length is the distance between the vertex and the focus of the mirror.

The focal length equals a half of the curve radius (Han et al, 2021):

$$f = \frac{r}{2} [m]. \quad (29)$$

The efficiency of these plants increases with the installation of energy storing systems, which also contributes to their reliability. These systems rely on the storage of thermal energy into a material of high energy density. The heat storing systems collect the energy during sunny periods (Figure 4), and this energy is spent in the periods of low sun radiation or when there is not radiation at all.

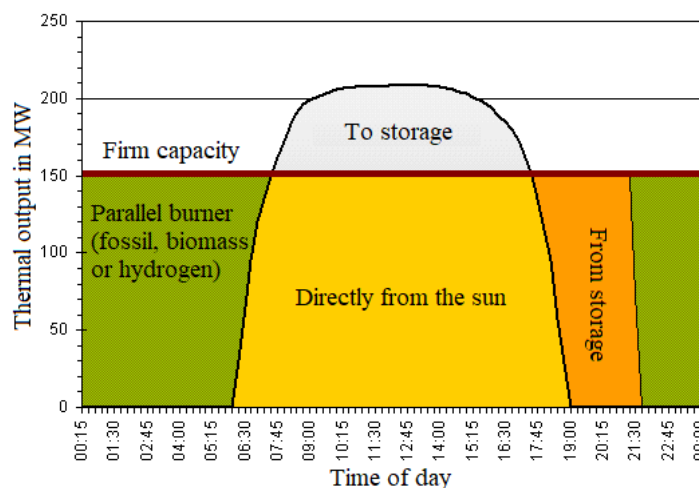


Figure 4 – Energy storing principle (Renewable Energy World, 2003)

Рис. 4 – Принцип накопления энергии (Renewable Energy World, 2003)

Слика 4 – Принцип складиштења енергије (Renewable Energy World, 2003)

Terrestrial radiation consists of:

- $Z_d$  - direct radiation that comes to the Earth,
- $Z_r$  - diffuse radiation, resulted from dispersion in the atmosphere,
- $Z_o$  - reflected radiation from the Earth's surface.

Terrestrial radiation (Figure 5) can be expressed as follows:

$$Z = Z_d + Z_r + Z_o. \quad (30)$$

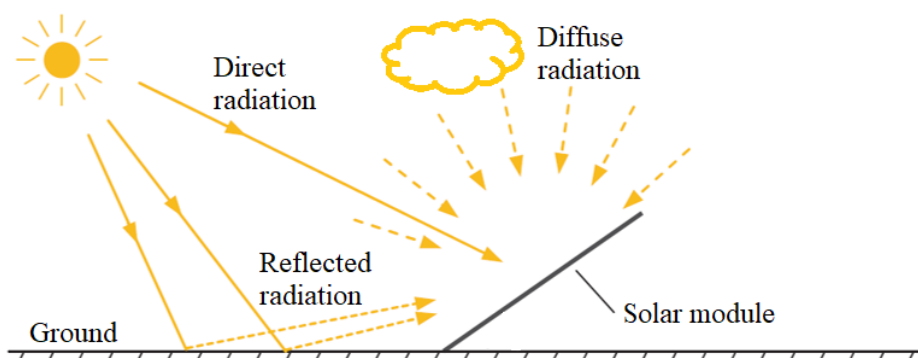


Figure 5 – Direct, diffuse and reflected solar radiation

Рис. 5 – Прямое, рассеянное и отраженное солнечное излучение

Слика 5 – Директно, дифузно и рефлексивно сунчево зрачење

When projecting and planning solar systems, it is necessary to know the exact values of different meteorological elements and parameters (Gomez-Munoz & Porta-Gandara, 2002). Concentrating collectors, in comparison to photovoltaic systems, use only direct solar radiation. Consequently, it can be said that cloudiness, as well as diffuse solar radiation, influence solar thermal plants to a great extent. The climate features of the Bosnia and Herzegovina area result from the influence of a complex climate system – from global, synoptic to mezzo and micro scales. The data necessary for a detailed solar thermal plant study are not available, as measurements have not been done. Measurements relevant for a detailed estimation would have to be taken several years in a row and they demand financial investments. For these reasons, the number of cloudy days in the Banja Luka area has been taken into account for the purposes of a preliminary analysis; more specifically, the data on the number of months a year (with a 50% probability) in which the number of cloudy days does not exceed half of the month.

In the Banja Luka area, it is cloudy or overcast on average for 128.5 days a year (Table 1). The annual number of cloudy days varies slightly less than the number of sunny days. According to the data, it ranges from 71 to 191 days. The winter period, especially January and December, is the period when almost every other day is cloudy.

Table 1 – Number of cloudy days in the Banja Luka area (Pupčević, 2016)  
Таблица 1 – Количество пасмурных дней в Баня-Луке (Pupčević, 2016)  
Табела 1 – Број облачних дана у Бања Луци (Pupčević, 2016)

|            | Jan  | Feb  | Mar | Apr  | May | June | July | Aug | Sep | Oct | Nov | Dec  | Year  |
|------------|------|------|-----|------|-----|------|------|-----|-----|-----|-----|------|-------|
| <i>Mid</i> | 15,8 | 12,8 | 12  | 10,6 | 9,1 | 8,7  | 6,6  | 6,7 | 7,7 | 9,9 | 14  | 14,5 | 128,5 |
| <i>Max</i> | 30   | 23   | 23  | 16   | 20  | 20   | 18   | 16  | 17  | 18  | 25  | 28   | 191   |
| <i>Min</i> | 4    | 0    | 2   | 1    | 3   | 2    | 2    | 1   | 1   | 3   | 6   | 1    | 71    |
| $\sigma$   | 5,8  | 5,2  | 5,5 | 3,3  | 4,1 | 4,2  | 4,3  | 3,6 | 3,5 | 4,3 | 4,7 | 6,1  | 24,8  |

where:

- *Mid* - middle - expected value,
- *Max* - maximum monthly value,
- *Min* - minimum monthly value, and
- $\sigma$  - standard deviation of monthly values.

Let  $X$  be a continuous random variable which denotes the average number of cloudy days per month. The middle, minimum and maximum values of the cloudy days are defined in the table above. According to the experience of the authors (Hajiagha et al, 2015; Crvenković & Rajter, 1999; Jovanović et al, 2008) and using Chebyshev's inequality:

$$P(|X - \mu| < k \cdot \sigma) = 1 - P(|X - \mu| \geq k \cdot \sigma) \geq 1 - \frac{1}{k^2}, \quad (31)$$

as well as based on the known expected values  $\mu$  and the standard deviation  $\sigma$ , the probability of the number of cloudy days deviation from the mean value can be calculated. The minimum value of the number of cloudy days for January is 4, the maximum is 30 and the middle - expected value is  $\mu = 15.8$  days. Taking into account the standard deviation  $\sigma = 5.8$  and the deviation of 10 days, the result for January, as a critical month is as follows:

$$P(|X - 15.8| < 10) = P(|X - \mu| < k \cdot 5.8) \geq 1 - \frac{1}{1.72^2} = 0.664 \text{ implies } 66.4\%. \quad (32)$$

The conclusion that follows is that the probability of the number of cloudy days to deviate for January, for less than two deviations, equals at least 66%. It means that at least 66% of the realization of the random variable  $X$  is within the interval (5.8; 25.8).

Based on the above example, the deviation interval from the standard value at a 50% probability can be expressed in a different way:

$$P(|X - \mu| < k \cdot 5.8) \geq 1 - \frac{1}{k^2} = 0.5. \quad (33)$$

From the above equation,  $k$  can be expressed as:

$$1 - \frac{1}{k^2} = 0.5 \text{ implies } k = \sqrt{2}, \quad (34)$$

and the following correlation is defined:

$$P(|X - 15.8| < \sqrt{2} \cdot 5.8) \geq 0.5 \text{ if and only if } P(7.6 < X < 24) \geq 0.5. \quad (35)$$

The conclusion is that the number of cloudy days in January is under 24 and above 7.6 for the probability more than or equal to 50%.

The upper limit ( $X_g$ ) and the lower limit ( $X_d$ ) of the number of cloudy days per month that varies from the middle - expected value, with a 50% probability, is shown in Table 2.

Table 2 – The number of cloudy days per month with the outcome probability of 50%  
Таблица 2 – Количество пасмурных дней в месяц с вероятностью исхода 50%  
Табела 2 – Број облачних дана мјесечно са вјероватноћом исхода од 50%

|             | Jan   | Feb   | Mar   | Apr   | May   | June  | July  | Aug   | Sep   | Oct   | Nov   | Dec   | Year   |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| $\mu$       | 15.8  | 12.8  | 12    | 10.6  | 9.1   | 8.7   | 6.6   | 6.7   | 7.7   | 9.9   | 14    | 14.5  | 128.5  |
| $\sigma$    | 5.8   | 5.2   | 5.5   | 3.30  | 4.1   | 4.2   | 4.3   | 3.6   | 3.5   | 4.3   | 4.7   | 6.1   | 24.8   |
| $k$         | 1.41  | 1.41  | 1.41  | 1.41  | 1.41  | 1.41  | 1.41  | 1.41  | 1.41  | 1.41  | 1.41  | 1.41  | 1.41   |
| $ X - \mu $ | 8.20  | 7.35  | 7.78  | 4.67  | 5.80  | 5.94  | 6.08  | 5.09  | 4.95  | 6.08  | 6.65  | 8.63  | 35.07  |
| $X_g$       | 24.00 | 20.15 | 19.78 | 15.27 | 14.90 | 14.64 | 12.68 | 11.79 | 12.65 | 15.98 | 20.65 | 23.13 | 163.57 |
| $X_d$       | 7.60  | 5.45  | 4.22  | 5.93  | 3.30  | 2.76  | 2.00  | 1.61  | 2.75  | 3.82  | 7.35  | 5.87  | 93.43  |

Figure 6 shows the minimum (*Min*), maximum (*Max*) and middle (*Mid*) values of the cloudy days number for a period of 30 years. Using Chebyshev's inequality, the upper and the lower limits of the number of cloudy days with the deviation from the expected value with a 50% probability have been calculated.

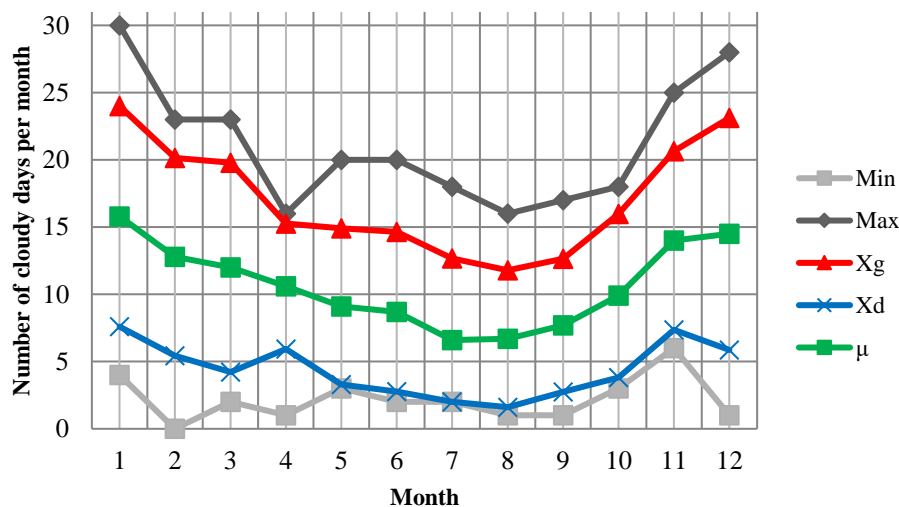


Figure 6 – Number of cloudy days per month (*Min* – *Max*) and the number of cloudy days interval around the average value with a 50% outcome probability.

Рис. 6 – Количество пасмурных дней в месяц (Мин–Макс) и интервал количества пасмурных дней около среднего значения с вероятностью результата 50%.

Слика 6 – Број облачних дана мјесечно (мин-макс) и интервал броја облачних дана око средње вриједности са вјероватноћом исхода од 50%

## Conclusion

Although the estimations obtained from Chebyshev's inequality are generally quite rough, they are often used in practice for their simplicity and the quality of not depending on the values' layout. Chebyshev's inequality has a wide range of applications in cases where probability distribution is not known, while the mean value and the variance are.

The results of the analysis show that, in the worst case scenario, on an annual basis, the Banja Luka area has more than six months with the number of cloudy days that exceeds 15 days a month, (maximum, including April, although its value is higher for only 0.27 days than the upper limit value) with a 50% outcome probability. On the other hand, the measurements show that the minimum number of cloudy days per month

does not exceed eight. The annual interval of cloudy days ranges from 94 to 164, i.e. 26 to 44% of the year. A very important fact is that the middle - expected value of cloudy days per month is less than 50% throughout the year. This lack of sunny days can be compensated by peak energy resources.

The preliminary feasibility study of solar thermal power plant construction in the Banja Luka area using Chebyshev's inequality justifies the measurements necessary for the analysis and detailed estimation of this type of a power plant.

### References

- Amidan, B.G., Ferryman, T.A. & Cooley, S.K. 2005. Data Outlier Detection using the Chebyshev Theorem. In: *IEEE Aerospace Conference*, Big Sky, MT, USA, pp.3814-3819, March 5-12. Available at: <https://doi.org/10.1109/AERO.2005.1559688>.
- Arandelović, I., Mitrović, D.Z. & Stojanović, V. 2011. *Verovatnoća i statistika*. Belgrade, Serbia: Zavod za udžbenike (in Serbian).
- Bahmani, R., Karimi, H. & Jadid, S. 2020. Stochastic electricity market model in networked microgrids considering demand response programs and renewable energy sources. *International Journal of Electrical Power & Energy Systems*, 117 (art.ID: 105606). Available at: <https://doi.org/10.1016/j.ijepes.2019.105606>.
- Biyya, I., Aniba, G. & Maaroufi, M. 2017. Impact of Load and Renewable Energy Uncertainties on Single and Multiple Energy Storage Systems Sizing. In: *IEEE Power & Energy Society Innovative Smart Grid Technologies Conference (ISGT)*, Washington, DC, USA, pp.1-5, April 23-26. Available at: <https://doi.org/10.1109/ISGT.2017.8086031>.
- Chen, X. 2011. A New Generalization of Chebyshev Inequality for Random Vectors. *arXiv preprint arXiv:0707.0805* [online]. Available at: <https://arxiv.org/pdf/0707.0805.pdf> [Accessed: 1 March 2022].
- Crvenković, Z. & Rajter, D. 1999. *Zbirka rešenih zadataka iz verovatnoće i statistike*. Novi Sad, Serbia: University of Novi Sad (in Serbian).
- Dekking, F.M., Kraaikamp, C., Lopuhaä, H.P. & Meester L.E. 2005. *A Modern Introduction to Probability and Statistics*. London: Springer-Verlag. Available at: <https://doi.org/10.1007/1-84628-168-7>.
- Elazar, S. 1972. *Matematička statistika*. Sarajevo, Bosnia and Herzegovina: Zavod za izdavanje udžbenika (in Serbian).
- Gomez-Munoz, V.M. & Porta-Gandara, M.A. 2002. Local wind patterns for modeling renewable energy systems by means of cluster analysis techniques. *Renewable Energy*, 25(2), pp.171-182. Available at: [https://doi.org/10.1016/S0960-1481\(01\)00013-1](https://doi.org/10.1016/S0960-1481(01)00013-1).
- Hajiagha, S.H.R., Hashemi, S.S., Mahdiraji, H.A. & Azaddel, J. 2015. Multi-period data envelopment analysis based on Chebyshev inequality bounds. *Expert*



*Systems with Applications*, 42(21), pp.7759-7767. Available at: <https://doi.org/10.1016/j.eswa.2015.06.008>.

Han, C-Y., Lo, W-T., Chen, K-H., Lee, J-Y., Yeh, C-H., Chen, J-H. 2021. Measurement of Focal Length and Radius of Curvature for Spherical Lenses and Mirrors by Using Digital-Grating Moiré Effect. *Photonics*, 8(7), art.number:252. Available at: <https://doi.org/10.3390/photonics8070252>.

Hooshmand, A., Poursaeidi M.H., Mohammadpour, J., Malki, H.A. & Grigoriads, K. 2012. Stochastic Model Predictive Control Method for Microgrid Management. In: *IEEE PES Innovative Smart Grid Technologies (ISGT)*, Washington, DC, USA, pp.1-7, January 16-20. Available at: <https://doi.org/10.1109/ISGT.2012.6175660>.

Jaoude, A.A. 2016. The paradigm of complex probability and Chebyshev's inequality. *Systems Science & Control Engineering*, 4(1), pp. 99-137. Available at: <https://doi.org/10.1080/21642583.2016.1185044>.

Jovanović, M., Merkle, M. & Mitrović, D.Z. 2008. *Vjerovatnoća i statistika-zbirka riješenih zadataka*. Banja Luka, Republic of Srpska, Bosnia and Herzegovina: Faculty of Electrical Engineering (in Serbian).

Lange, K. 2010. *Applied Probability*. New York, NY, USA: Springer. Available at: <https://doi.org/10.1007/978-1-4419-7165-4>.

Maširević, D.J. & Keglević, N. 2017. Chebyshev's and Markov's inequality in probability theory. *Osječki matematički list*, 17, pp.125-137 [online]. Available at: <https://hrcak.srce.hr/file/285086> (in Serbian) [Accessed: 1 March 2022].

Medić, S. 2014. *Jensen and Chebyshev inequalities for interval-valued functions*. Ph.D. thesis. Novi Sad, Serbia: University of Novi Sad, Faculty of Science (in Serbian) [online]. Available at: <https://nardus.mpn.gov.rs/handle/123456789/1892> [Accessed: 1 March 2022].

Mitrović, D.Z. 2007. *Primijenjena statistika – Teorija i zadaci*. Banja Luka, Republic of Srpska, Bosnia and Herzegovina: University of Banja Luka (in Serbian).

Pupčević, B.M. 2016. *Primjena reverzibilnog ciklusa centrifugalnog rashladnog uređaja za konverziju geotermalne i solarne energije u električnu*. MA thesis. Banja Luka, Republic of Srpska, Bosnia and Herzegovina: University of Banja Luka, Faculty of Mechanical Engineering (in Serbian).

-Renewable Energy World. 2003. Solar thermal power plants - Technology Fundamentals. *Renewable Energy World*, June, pp.109-113.

Simonović, V. 1986. *Uvod u teoriju verovatnoće i matematičku statistiku*. Belgrade, Serbia: Građevinska knjiga (in Serbian).

Stellato, B., Van Parys, B.P.G. & Goulart, P.J. 2017. Multivariate Chebyshev Inequality With Estimated Mean and Variance. *The American Statistician*, 71(2), pp.123-127. Available at: <https://doi.org/10.1080/00031305.2016.1186559>.

Vasquez Padilla, R. 2011. *Simplified Methodology for Designing Parabolic Trough Solar Power Plants*. Ph.D. thesis. University of South Florida, ProQuest Dissertations Publishing [online]. Available at: <https://www.proquest.com/openview/9f7c10b791b1bef807d4fdb59eec0cee/1?pq-origsite=gscholar&cbl=18750> [Accessed: 1 March 2022].



Wang, R., Xiao, G., Wang, P., Cao, Y., Li, G., Hao, J. & Zhu, K. 2017. Energy Generation Scheduling in Microgrids Involving Temporal-Correlated Renewable Energy. In: *GLOBECOM 2017 - 2017 IEEE Global Communications Conference*, Singapore, pp.1-6, December 4-8. Available at: <https://doi.org/10.1109/GLOCOM.2017.8255069>.

Yang, C., Liu, J., Zeng, Y. & Xie, G. 2019. Real-time condition monitoring and fault detection of components based on machine-learning reconstruction model. *Renewable Energy*, 133, pp.433-441. Available at: <https://doi.org/10.1016/j.renene.2018.10.062>.

#### ПРИМЕНЕНИЕ НЕРАВЕНСТВА ЧЕБЫШЕВА В ПРЕДВАРИТЕЛЬНОМ ТЕХНИКО-ЭКОНОМИЧЕСКОМ ОБОСНОВАНИИ СТРОИТЕЛЬСТВА СОЛНЕЧНОЙ ТЕПЛОЙ ЭЛЕКТРОСТАНЦИИ

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РУБРИКА ГРНТИ: 27.43.15 Теория вероятностей и случайные процессы  
ВИД СТАТЬИ: оригинальная научная статья

#### Резюме:

**Введение/цель:** В данной статье описано применение неравенства Чебышева. Используя неравенство Чебышева, был проведен анализ предварительного технико-экономического обоснования строительства солнечной тепловой электростанции на территории Баня-Луки. Цель данного предварительного анализа состоит в том, чтобы без вложений доказать, существует ли основание для измерения климатических параметров в данном регионе.

**Методы:** Путем применения неравенства Чебышева для известных значений средних арифметических и стандартных отклонений в количестве пасмурных дней была определена вероятность отклонения количества пасмурных дней от среднего значения.

**Результаты:** На диаграмме показаны значения верхнего и нижнего пределов количества пасмурных дней, которые с вероятностью 50% не совпадают с ожидаемым значением.

**Выводы:** Предварительная оценка ТЭО по установке солнечной тепловой электростанции оправдывает измерения, необходимые для анализа и детального расчета данного типа установки, так

как годовой интервал пасмурных дней составляет от 94 до 164%, то есть от 26 до 44% в год.

**Ключевые слова:** вероятность, случайная величина, дисперсия, среднее значение, пасмурный день, солнечная радиация, солнечные тепловые электростанции.

## ПРИМЈЕНЕ НЕЈЕДНАКОСТИ ЧЕБИШЕВА У ПРЕЛИМИНАРНОЈ ОЦЈЕНИ ОПРАВДАНОСТИ РЕАЛИЗАЦИЈЕ СОЛАРНЕ ТЕРМАЛНЕ ЕЛЕКТРАНЕ

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ОБЛАСТ: математика

ВРСТА ЧЛАНКА: оригинални научни рад

**Сажетак:**

**Увод/циљ:** У раду су представљене неке примјене неједнакости Чебишева. Помоћу неједнакости Чебишева анализирана је прелиминарна оцјена оправданости реализације соларне термалне електране на простору Бање Луке. Циљ ове прелиминарне анализе јесте да се докаже, без инвестиционих улагања, да ли су оправдана мјерења климатских параметара на том подручју.

**Методе:** За познате вриједности аритметичких средина и стандардних девијација броја облачних дана, примјеном Чебишевљево неједнакости дефинисана је вјероватноћа одступања броја облачних дана од средње вриједности.

**Резултати:** На дијаграму су приказане вриједности горње и доње границе броја облачних дана које одступају од очекиване вриједности са вјероватноћом од 50%.

**Закључак:** Прелиминарна оцјена оправданости реализације соларне термалне електране оправдава мјерења која су неопходна за анализу и детаљни прорачун овакве врсте постројења, јер је годишњи интервал облачних дана од 94 до 164, односно од 26 до 44% периода године.

**Кључне ријечи:** вјероватноћа, случајна промјенљива, дисперзија, средња вриједност, облачан дан, соларно зрачење, соларне термалне електране.

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