Collective phenomena

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Abstract:

Introduction/purpose: Quantum field theory techniques are able to describe precisely, inter alia, collective phenomena of statistical and solid state physics.

Method: The path integral method with a Wick rotation shows its complete analogy with the partition function of statistical mechanics.

Results: The Landau–Ginzburg phenomenology successfully describes collective phenomena such as spontaneous magnetization and superconductivity.

Conclusions: Symmetry breaking phenomena could give macroscopic results.

Key words: vacuum energy, symmetry breaking, Landau–Ginzburg model.

Collective phenomena

The techniques we have met so far (Fabiano, 2021a,b, 2022) are not connected exclusively to the field theory. For instance the Anderson–Higgs–Brout–Englert–Guralink–Hagen–Kibble phenomenon (Anderson, 1963; Higgs, 1964a,b; Englert & Brout, 1964; Guralnik et al., 1964) was mutuated essentially from solid state physics. A deeper understanding of physical meaning of the renormalisation group was given by Kadanoff's proposal of block spin renormalisation (Kadanoff, 1966), and Wilson's approach to critical phenomena (Wilson, 1971a,b). Physics does not operate through airlocks.

Path integral versus the partition function

We have already introduced the partition function in (Fabiano, 2022) eq. (28) for a scalar field. Recall that in D dimensions

$$Z = \int \mathcal{D}\phi \; e^{(i/\hbar)\int \mathrm{d}^{\mathrm{D}}x \left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right]} \, . \tag{1}$$

By means of a Wick rotation, we write $d^D x = -i d_E^D x$, where $d_E^D x = dt_E d^{(D-1)} x$, and $(\partial\phi)^2 = (\partial\phi/\partial t)^2 - (\vec{\nabla}\phi)^2$ becomes $(\partial\phi)^2 = (\partial\phi/\partial t_E)^2 + (\vec{\nabla}\phi)^2$, which is the energy operator $E(\phi)$, positive definite, so we obtain the Euclidean version of the path integral

$$Z_E = \int \mathcal{D}\phi \; e^{(i/\hbar)\int \mathrm{d}_E^D x \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]} = \int \mathcal{D}\phi \; e^{-(1/\hbar)E(\phi)} \, . \tag{2}$$

The Euclidean version (2) we have obtained closely resembles the known expression of statistical mechanics. Indeed, making the substitution $\hbar \leftrightarrow kT \equiv 1/\beta$, the probability of a statistical state is proportional to the Boltzmann factor $e^{-\beta E}$, where E is the classical energy of the state. For a system of N particles, classical energy is given by

$$E(p,q) = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + V(q_1, q_2, \ldots, q_N) \, , \tag{3}$$

and the corresponding partition function is proportional to

$$Z = \prod_{i=1}^{N} \int \mathrm{d}p_i \, \mathrm{d}q_i \; e^{-\beta E(p,q)} \, . \tag{4}$$

The kinetic term of energy is known so the integral over p can be explicitly done, and we are left with the (reduced) partition function

$$Z = \prod_{i=1}^{N} \left(\frac{2m_i\pi}{\beta}\right)^{N/2} \int \mathrm{d}q_i \; e^{-\beta V(q_1, q_2, \ldots, q_N)} \, . \tag{5}$$

Going to the continuum limit as we did in (Fabiano, 2022) eq. (24), by promoting $i \to x$ and $q_i \to \phi(x)$ we reobtain the expression found in eq. (2).

We show below a translation table among languages of the field theory in Minkowski space and statistical mechanics, both D dimensional:

Table 1 – Translation table of the field theory and statistical mechanics
Таблица 1 – Таблица перевода теории поля и статистической механики
Табела 1 – Табела превођења теорије поља и статистичке механике

Field theory	$\longleftrightarrow$	Statistical mechanics
x_0	$\longleftrightarrow$	ix_0
$d^D x$	$\longleftrightarrow$	$-id^D x$
$(\partial\phi/\partial t)^2 - (\vec{\nabla}\phi)^2$	$\longleftrightarrow$	$(\partial\phi/\partial t)^2 + (\vec{\nabla}\phi)^2$
$\hbar$	$\longleftrightarrow$	$\beta \equiv 1/(kT)$

Landau–Ginzburg phenomenology

We consider magnetization. Let us briefly review the Goldstone mechanism (Goldstone, 1961) of symmetry breaking, where there is the complex field $\phi \equiv (\phi_1 + i\phi_2)/\sqrt{2}$ in the Lagrangian

$$\mathcal{L} = \partial\phi^\dagger\partial\phi + \mu^2(\phi^\dagger\phi) - \lambda(\phi^\dagger\phi)^2 \ , \tag{6}$$

as the mass term has the wrong sign, we are already in a broken symmetry phase. In Fig. 1, the potential is described by eq. (6). It is clear that it is invariant under rotations, that is invariant under the global $U(1)$ transformation $\phi \to e^{i\alpha}\phi$ because the groups $SO(2)$ and $U(1)$ are isomorphic. We could rewrite eq. (6) in the polar coordinates by $\phi(x) = \rho(x)e^{i\theta(x)}$, so that $\partial_\mu\phi = (\partial_\mu\rho + i\rho\partial_\mu\theta)e^{i\theta(x)}$, finally obtaining

$$\mathcal{L} = \rho^2(\partial\theta)^2 + (\partial\rho)^2 + \mu^2\rho^2 - \lambda\rho^4 \ . \tag{7}$$

In the broken phase, we have

$$v = +\sqrt{\frac{\mu^2}{2\lambda}} \tag{8}$$

and putting $\rho = v + \chi$ we end up with the Lagrangian

$$\mathcal{L} = v^2(\partial\theta)^2 + \left[(\partial\chi)^2 - 2\mu^2\chi^2 - 4\sqrt{\frac{\mu^2\lambda}{2}}\chi^3 - \lambda\chi^4\right] + \left(\sqrt{\frac{2\mu^2}{\lambda}}\chi + \chi^2\right)(\partial\theta)^2 \ , \tag{9}$$

where one recognizes $\theta(x)$ as the massless Goldstone field.

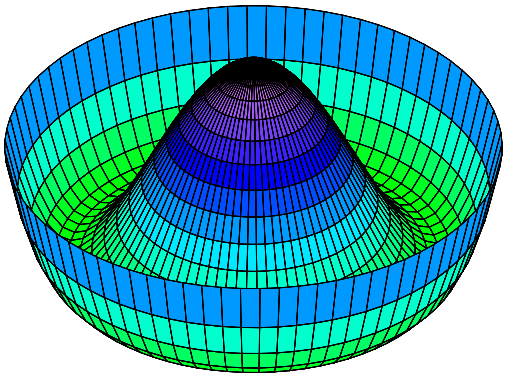

Figure 1 – Complex field potential in the broken symmetry phase
Рис. 1 – Комплексный потенциал поля в фазе нарушенной симметрии
Слика 1 – Потенцијал комплексног поља у фази нарушене симетрије

Consider a ferromagnetic material in the thermal equilibrium at the temperature T. Define the magnetization $\vec{M}(x)$ as the average of atomic magnetic momenta over a region much larger than the relevant microscopic scale.

When the temperature T is high enough, the magnetization vectors should be expected to point out at random, as there is not a preferred direction because of spatial isotropy. The magnetization vector average should add up to zero. However, lowering the temperature under a critical value T_c shows experimentally that the behaviour of magnetism changes in the following manner

$$|\vec{M}| \propto (T_c - T)^{\beta} \; , \qquad (10)$$

where it is experimentally found that β (here a critical exponent, not to be confused with the inverse of the temperature) covers the range of $0.3 - 0.38$ for different materials.

In order to explain this behavior, Landau and Ginzburg (Ginzburg & Landau, 2009) made the hypothesis that the thermodynamic properties of the system should be derivable from a free energy G which is an analytic function of the magnetism $M = |\vec{M}|$ and the temperature T. Then the magnetic field H would be given by

$$H = \frac{\partial}{\partial M} G(M, T) \ . \tag{11}$$

Near T_c where M is small we can write a Taylor series in the powers of M for the free energy G

$$G(M, T) = G_c(T) + a(T)M^2 + b(T)M^4 + \mathcal{O}(M^6) \ , \tag{12}$$

we include only even powers of M because of rotational symmetry and spin up–down symmetry. The magnetic field is therefore given by

$$H = 2a(T)M + 4b(T)M^3 + \mathcal{O}(M^5) \ . \tag{13}$$

At this point, the similarity of the free energy in eq. (12) and the potential of Lagrangian (6) is evident. We are left with two possible phenomena. If $a > 0$, then G will have a single minimum at M_0 (we could assume its value is zero by symmetry) as shown in Fig. 2a, but if $a < 0$ then G shows two symmetric minima around the origin, at the points $\pm M_0$, corresponding to $H = 0$, as in Fig. 2b. The system will choose a state of the minimum free energy which according to Fig. 2b corresponds to a state of non-vanishing magnetization. It is clear that the possibility $a > 0$ corresponds to $T > T_c$, where spontaneous magnetization is zero, while $a < 0$ means $T < T_c$.

Continuing with these analyticity assumptions, we can obtain some of the critical exponents. It is clear that $a(T_c) = 0$ while $b(T_c)$ will have a nonzero value. The next stage is assuming a linear behaviour for a in the vicinity of T_c,

$$a(T) \propto (T_c - T) \ . \tag{14}$$

Then, for $a < 0$ from eqs. (12) and (8), we have

$$M = \sqrt{-\frac{a(T)}{2b(T)}} \propto (T_c - T)^{1/2} \ , \tag{15}$$

for which we identify

$$\beta = \frac{1}{2} \tag{16}$$

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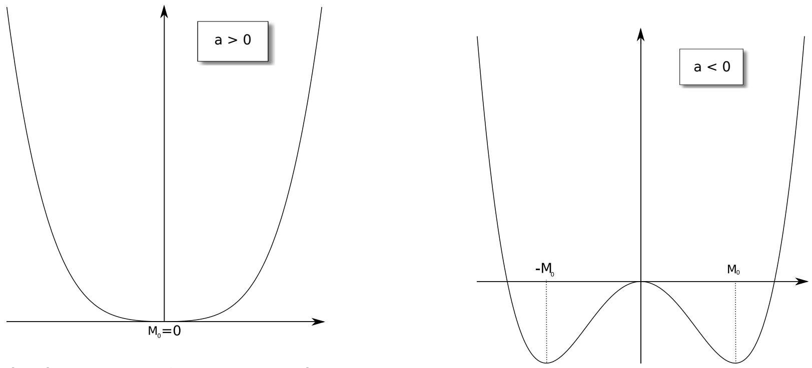

(a) Unbroken symmetry, $a > 0$ and $M_0 = 0$
(а) Непрерывная симметрия, $a > 0$ и $M_0 = 0$
(а) Ненарушена симетрија, $a > 0$ и $M_0 = 0$

(b) Broken symmetry, $a < 0$ and $M_0 > 0$
(б) Нарушенная симметрия, $a < 0$ и $M_0 > 0$
(б) Нарушена симетрија, $a < 0$ и $M_0 > 0$

Figure 2 – Free energy G versus magnetization M
Рис. 2 − Свободная энергия G по сравнению с намагниченностью M
Слика 2 – Слободна енергија G наспрам магнетизације M

as the first critical exponent. If $T > T_c$, then near zero magnetization from (13) we have

$$M = \frac{H}{2a(T)} \,, \tag{17}$$

and defining the magnetic susceptibility as

$$\chi \equiv \left.\frac{\partial M}{\partial H}\right|_{H=0} \tag{18}$$

with the critical exponent γ such that $\chi \propto (T - T_c)^{-\gamma}$, we calculate

$$\chi = \frac{1}{2a(T)} \propto (T - T_c)^{-1} \tag{19}$$

obtaining another critical exponent, $\gamma = 1$.

This approach of the *mean field* does not give very accurate values for critical exponents, but provides a very simple picture of critical behaviour.

Landau and Ginzburg considered the possibility that $\vec{M}$ depends on a position (here the space is three dimensional, we consider only non relativistic effects), then G could be written as

$$G = \int \mathrm{d}^3x\, [\partial_i \vec{M} \partial_i \vec{M} + a\vec{M}^2 + b(\vec{M}^2)^2 + \cdots] \,. \tag{20}$$

Again, this expression is in complete analogy with eq. (6), where a plays the role of a square mass and b is a coupling constant. So the length scale is determined by $1/\sqrt{a}$. For $T > T_c$ add a magnetic field $\vec{H}(x)$ that interacts with the magnetization with the term $-\vec{H} \cdot \vec{M}$ (observe the similarity with the source function in (Fabiano, 2022) eq. (35)). Assuming M is small and minimizing G, we obtain at the first order $(-\partial^2 + a)\vec{M} \propto \vec{H}$ with the solution

$$\begin{aligned}
\vec{M}(x) = \int \mathrm{d}^3y \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot(\vec{x}-\vec{y})}}{\vec{k}^2 + a} \vec{H}(y) = \\
\int \mathrm{d}^3y \iiint \mathrm{d}\phi\, \mathrm{d}\theta\, \frac{\mathrm{d}k}{(2\pi)^3}\, k^2 \sin\theta \frac{e^{ik|\vec{x}-\vec{y}|\cos\theta}}{k^2 + a} \vec{H}(y) = \\
\int \mathrm{d}^3y\, \frac{1}{4\pi|\vec{x} - \vec{y}|} e^{-\sqrt{a}|\vec{x}-\vec{y}|} \vec{H}(y) \,.
\end{aligned} \tag{21}$$

The two point function or the correlation function is defined as

$$\langle \vec{M}(x) \vec{M}(0) \rangle \,, \tag{22}$$

that is, starting from some magnetization $\vec{M}(0)$ at the origin we ask what the magnetization will be at a certain point x. One expects it to be a steeply decreasing function of distance, such as $e^{-|\vec{x}|/\xi}$, where ξ is the correlation length. For $T > T_c$ the critical exponent ν is defined as $\xi \propto (T - T_c)^{-\nu}$. In the Landau–Ginzburg model (Ginzburg & Landau, 2009; Abrikosov, 1957; Gorkov, 1959) the correlation length is given by $\xi = 1/\sqrt{a}$ giving the value $\nu = 1/2$.

Superconductivity

A *superconductor* is a material which below a temperature $T < T_c$ exhibits no resistance to electrical current. The long–standing suspicion that this behaviour was correlated to Bose–Einstein condensation, even if electrons are fermions, was confirmed after the discovery of the Cooper pair (Cooper, 1956), where two electrons bound together at low temperature with some energy lower than the Fermi energy. This condensation is responsible for the effect of superconductivity. Landau and Ginzburg

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had the idea of describing the mechanism by studying the field $\phi(x)$ associated with these condensing bosons without knowing the mechanism of electrons pairing. This description is analogous to the previous one of magnetism. There is a complex field $\phi(x)$ carrying two charge units, the Cooper pair. The Lagrangian is the one already seen in eq. (6) with the variation of $\mathcal{D}_i\phi = (\partial_i - i2eA_i)\phi$ because ϕ is charged[1].

The free energy inclusive of the energy of external magnetic field is given by

$$G = \frac{1}{4}F_{ij}^2 + |\mathcal{D}_i\phi|^2 + a|\phi|^2 + b|\phi|^4 + \cdots \,, \tag{23}$$

clearly invariant by construction under the $U(1)$ transformation $\phi \to e^{2ie\Lambda}\phi$ and $A_i \to A_i + \partial_i\Lambda$.

An important consequence of superconductivity is the Meissner effect: below a critical temperature $T < T_c$ an external magnetic field $\vec{B}$ is expelled from the conductor. This means that a magnetic field inside the material is energetically disfavoured.

If a constant magnetic field is considered, then it costs an energy of the order of $E \propto \vec{B}^2L^3$, where L^3 is the volume of the conductor. In terms of a potential field $\vec{A}$, where $\vec{B} = \vec{\nabla} \times \vec{A}$ there is $\vec{B}^2 \propto \vec{A}^2$. A constant field $\vec{B}$ implies that $\vec{A}$ grows linearly with the distance L, so one obtains $E \propto \vec{A}^2L^3 \propto L^5$. In order to maintain a constant magnetic field inside the superconductor, a huge amount of energy is needed, so such configuration is heavily disfavoured, and as a consequence, $\vec{B}$ is expelled from the superconductor.

As before in (14), we assume a linear behaviour in temperature for the coefficient $a \propto (T - T_c)$ while b remains positive. For $T > T_c$ G has a unique trivial minimum for $|\phi| = 0$, while for $T < T_c$ G has minima for $|\phi| = \sqrt{-a/(2b)} \equiv v$ as found in eq. (8), in complete analogy to Figs. 2a and 2b, respectively, after the exchange of M with $|\phi|$. The free energy (23) becomes below the critical temperature (Nambu, 1960; Hooft, 1971)

$$G = \frac{1}{4}F_{ij}^2 + (2ev)^2A_i^2 + \cdots \,. \tag{24}$$

In order to estimate more precisely the penetration length of a magnetic field inside a superconductor, known as the London penetration length, we

[1] Actually Landau had this intuition much before the discovery of Cooper pairs, but he considered only a single electron instead of a pair.

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need the results of the Landau–Ginzburg model. From (24) one expects the two terms to compete for the same energy, i.e.

$$F_{ij}^2 \propto (ev)^2 A^2 \,. \tag{25}$$

Let l_L be the London penetration length, so the energy of the electromagnetic field is $F_{ij}^2 \propto A^2/l_L^2$, and from the comparison with (25) we obtain

$$l_L \propto \frac{1}{ev} = \frac{1}{e}\sqrt{\frac{2b}{-a}} \,. \tag{26}$$

As already discussed in (20), the characteristic length of the scalar field ϕ is of the order of $l_\phi \propto 1/\sqrt{-a}$, because the coefficient a plays the role of a squared mass.

Comparing the two characteristic lengths, we finally obtain

$$\frac{l_L}{l_\phi} \propto \frac{\sqrt{b}}{e} \,. \tag{27}$$

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Коллективные явления

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РУБРИКА ГРНТИ: 29.19.00 Физика твердых тел
ВИД СТАТЬИ: обзорная статья

Резюме:

Введение/цель: Методы квантовой теории поля способны точно описать также коллективные явления статистической физики и физики твердого тела.

Методы: Метод интеграла по путям с вращением Вика показывает свою полную аналогию со статистической суммой статистической механики.

Результаты: Феноменология Ландау-Гинзбурга успешно описывает такие коллективные явления, как спонтанная намагниченность и сверхпроводимость.

Выводы: Явления нарушения симметрии могут дать макроскопические результаты.

Ключевые слова: энергия вакуума, нарушение симметрии, модель Ландау-Гинзбурга.

Колективни феномени

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ОБЛАСТ: физика чврстог стања, статистичка физика
КАТЕГОРИЈА (ТИП) ЧЛАНКА: прегледни рад

Сажетак:

Увод/циљ: Технике квантне теорије поља такође могу прецизно да опишу колективне појаве статистичке и физике чврстог стања.

Методе: Метода интеграла путање са Виковом ротацијом показује своју потпуну аналогију са партиционом функцијом статистичке механике.

Резултати: Ландау-Гинзбургова феноменологија успешно описује колективне феномене као што су спонтана магнетизација и суперпроводљивост.

Закључак: Феномен нарушавања симетрије могао би дати макроскопске резултате.

Кључне речи: енергија вакуума, нарушавање симетрије, Ландау-Гинзбургов модел.

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