Modeling of combat operations

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Abstract:

Introduction/purpose: The goal of the research in this paper is to present and evaluate the method of modeling operations by aggregating forces by simulating the battle process with Lanchester's equations. This method is the software basis of a certain number of programs used in NATO, in war simulations, and in the planning and analysis of operations. Its value is in understanding the consequences of decisions made with outcomes and results of combat actions.

Methods: The case study of the well-known Operation Desert Storm gathered the necessary data on operational parameters and the way forces are used in battles. The obtained data were transformed into operational variables of the combat model using the force aggregation method, whose simulation was carried out using the method of differential Lanchester's equations (quadratic law).

Results: By simulating the modeled operation, the parameters of the outcome of the conflict were obtained with numerical indicators of success, consumption of resources, etc. The results were analyzed and a certain correlation with the parameters of the real operation was determined, which enables the validation of the model.

Conclusion: The partial validity of the model describing the conflict on a practical historical example from a case study was confirmed. There are objective limitations in the application of modeling of military operations and optimization of the use of forces. The value of this method is the possibility of a reliable strategic assessment of the adversary's military power at the strategic level.

Key words: air/ground combat operations, attrition, aggregated forces model.

Introduction

The method of modeling and simulation is a scientific tool for visualizing operation plans and predicting the course and the outcome of combat operations. However, in most cases, planners do not know the mathematical background of the program responsible for obtaining results. This can result in subconsciously rejecting the obtained results as unreliable or in giving them too much importance even though there is no basis for either of these.

War and armed conflicts are not a part of the past and will never be. The problem of war is not its occurrence, but wrong decisions made on the assessment of the outcome of the conflict only on the basis of armchair experience and the knowledge of battles from epic history. This often leads to disasters. Examples for this claim are, in addition to Desert Storm which ended disastrously for Iraq, the recent Coalition campaigns in Libya, Iraq and Afghanistan. Finally, the Russian special operation in Ukraine is the last example, but certainly a representative one.

The aim of the research in this paper is to validate the method of modeling the battle process with Lanchester's equations by aggregating heterogeneous forces into homogeneous ones, with the aim of applying it as a scientific tool in the process of planning and analyzing operational-strategic operations. The value of modeling and simulation lies in the simplicity of viewing the consequences of the decisions made in correlation with the essential operational parameters of the results of implementation and the final outcome. This deepens theoretical knowledge about strategy and operational art, which contributes to the verification of the planning process and the predictability of conflict outcomes. The results are noticeable in the preparation and execution of combat operations, their efficiency and effectiveness, assessment of operational capabilities, advantages of new technologies, tactics and purposeful decision making.

The second part of the paper gives the theoretical foundations of the methods used, Lanchester's square law of combat and studies of the equivalence of forces by aggregating heterogeneous forces into homogeneous ones. A brief historical review is given with practical examples of application in solving real combat and practical problems, as well as shortcomings and their evolution into approximate models for software application in computers.

In the third part, experimental modeling was carried out - Operation Desert Storm. The model considered abstracted parameters on the influence of operational factors, combat capabilities, heterogeneity and number of forces, needed for validation, prediction of outcome and course of action, converting them into equivalent values.

The fourth part contains the result analysis and the discussion of their correlation with the actual facts of war, as well as the principles of the quadratic laws of the battle, derived from Lanchester equations. After verification, the validation of the applied methods was made, based on the obtained results and historical facts.

Theoretical background

Lanchester - Osipov's mathematical model, widely known as Lanchester's equations or the law of combat, represents one of the first attempts to scientifically describe armed combat. Lanchester (1916) uses his equations to describe two historical types of combat, which characterizes the process of depletion of forces, influenced by two quantities: the strength of forces and the fighting capability, expressed by the Lanchester attrition-rate coefficient. The first combat type is linear and represents ancient and medieval battles, characterized by the use of cold weapons on foot or on horseback and sometimes by the use of archers, catapults and similar ancient weapons. The analytical expression for this process is (Washburn et al, 2022) where (α) and (β) represent the attrition rates and (X) and (Y) are the numbers of forces engaged in combat:

$$\frac{dX}{dt} = -\beta \qquad \Lambda \qquad \frac{dY}{dt} = -\alpha, \tag{1}$$

for $X > 0 \ \ \Lambda \ \ Y > 0$

The second type of combat is described by Lanchester's square law which characterizes modern combat and warfare with the massive use of firearms, emphasizing the decisive influence of concentration of forces. The general idea (Kress, 2020) in the Lanchester model is to define the variables of the numerical state of the armed forces and the coefficients of the rate of inflicting losses on the adversary, and then solve the resulting equations as a function of time. If there is no change in time for the attrition rates (α) and (β), then the differential equations can be expressed as a system of ordinary differential equations (Washburn, 2000):

$$\frac{dX}{dt} = -\beta * Y \qquad \Lambda \qquad \frac{dY}{dt} = -\alpha * X, \tag{2}$$

for $X > 0 \ \ \Lambda \ \ Y > 0$

Even in the case where both sides have the same attrition rate or one is slightly better, the advantage in force numbers has a decisive influence

(MacKay, 2006). This implies: the winner is the side with better force concentration at the right moment and the right place or maybe has a bigger unit’s army divisions or air squadrons (Lanchester, 1916).

A special form of Lanchester's linear law is area combat. It consists of operations characterized by the law of probability without precision shooting, such as artillery bombardment or air support of an area, which is evenly occupied by opposing armed forces. The side that opens fire inflicts losses on the other side at a certain rate, proportionally to the number of forces located on a certain area, in relation to the total area of the combat layout (Washburn et al, 2022).

Another special form is the logarithmic law of combat, which refers to taking into account other reasons for depleting forces, such as illness, natural disasters, desertion, etc. (Washburn et al, 2022). It is interesting that this particular model proved to be more accurate than the others.

Lanchester's differential equations are the basis for the application of the slightly more complex Deitchman 's (1962) law of mixed combat, which enables the simulation of the combat dynamics of qualitatively different opponents such as the warfare of two adversaries in guerrilla and conventional combat. This problem could be solved by a combination of quadratic and linear laws (Darcom Pamphlet, 1979). A new aspect of the problem of this kind of conflict was given by Kress (2020) by including collateral victims among civilians. Many published works on historical battles partially validated the model which was successfully used to solve certain practical problems. The examples include: Iwo Jima (Engel, 1954), Ardennes campaign (Fricker, 1997) and Kursk (Lucas & Turkes, 2004), artillery and air support; strategy optimization in relation to weapon range, enemy attrition rate and operational costs (Isaacs, 1965); solving air operations problems in terms of combat resources due to the distribution of combat sorties in air support operations, offensive and defensive anti-aircraft operations (Berkovitz and Dresher, 1959), SEAD operations (Barkdoll et al, 2002) and the high level of engagement of the air battle model and expenditure design process (Allen, 1993).

The flaw in the basic model methodology was noticed quite early on. Osipov (Helmbold & Rehm, 1995) immediately pointed out the problem of a constant rate of expenditure of forces, which does not take into account the influence of various parameters such as: maneuver, tactical decisions, logistics, shooting process, operational situation factors (weather, geography, etc.). For these reasons, and in order to improve the initial method, combat modeling by partial differential equations was developed (Protopopescu et al, 1990). Using these methods, even the contribution of intelligence support can be determined (Coulson, 2019). An interesting war model created by Seung-Won Baik (2013) is based on a multi-weapon expansion. Helmbold (1965) noted that the relative rate of attrition of opposing forces depends on the ratio of force sizes. It is also important to mention Bonder (1970), who considered the combat range as a function of time with a constant rate of change of distance.

The fact is that Lanchester's equations and their refinements have been applied with some success in the analysis of historical battles, solving logistical and other operational problems. However, despite the improvements, the basic problem of combat modeling of the heterogeneous structure of forces in battle, in the conditions of changing operational factors, remained. In accordance with the fact that warfare represents a conflict of different types of armed forces, modeling of real warfare implies a heterogeneous combat structure of forces. Given these facts, it is understandable why the application of the basic Lanchester model is not suitable for modeling real war combat.

Another reason is that there is a fundamental difference between modeling the combat of smaller forces versus large, complex forces. The first case is a detailed simulation of each combat entity in the simulation, which is often defined as a high-resolution model approach and can be expressed by several differential equations which describe the combat process. The second case requires many more equations, with more detail. High resolution models involve complex computer programs. Their development and maintenance are complex and expensive. They are usually stochastic, which seems desirable, but actually requires replication to get answers about simulated combat. When trying to model larger forces (divisions, armies, etc.), the number of armed entities makes it impossible to maintain individual resolution.

As Taylor (1980a) said "for small-scale operations it may be possible to reasonably represent force interactions and attendant attrition rates with a few differential equations, but for large-scale operations of conventional armed forces the same approach might well involve hundreds (and possibly even thousands) of differential equations tied together through battlefield operations“. On the basis of these arguments considering methodology complexity for practical solving of this problem, Taylor (1980a) emphasized there were only few developed useful analytical models. Furthermore, he asserts three main approaches in simulating the combat model based on attrition:

- Monte-Carlo simulation,
- Aggregated Force-Fire Power Score approach, and
- Detailed Lanchester’s type model.

For modeling large scale combat operations such as strategic combined operations or campaigns, more suitable are Aggregated Force and Detailed Lanchester's type model. Monte-Carlo simulation is more suitable for small scale combat models (bellow the battalion force level). Disregarding the difference of stochastic and deterministic nature between these methods, a lot of authors consider both models quite similar in sense of results but the deterministic model is more practical for use (Taylor, 1980a).

In general, many experts believe that deterministic models, applicable on computers, give on average similar results to stochastic models, while being more practical. An illustrative description of the problem, by Taylor (1980a), is the consideration of the combat of heterogeneous forces with different types of combat systems with capabilities expressed by the attrition.

In this model of combat, there are a few assumptions which must be considered:

- attrition effects on forces are additive for every specific combat element, without mutual support and synergy effects,
- attrition efficiency of any combat system is proportional to the number of elementary units of that type, and
- each part of forces will attrite all available elements of the opponent according to its own combat capability.

Fire distribution can be considered as special factors (Ψ_{ij}) and (ϕ_{ij}), for both opponents, referring to a part of the forces of one side destroying a part of the forces of the other side, where:

$X_i > 0,\ Y_j > 0 \quad \Lambda \quad 0 < \phi_{\mathrm{ij}},\ \psi_{\mathrm{ij}} < 1.$

According to this and (eq.2), the final model is:

$$\frac{dX_i}{dt} = -\sum_{j=1}^{n} \psi_{ij} * b_{ij} * Y_j \qquad \Lambda \qquad \frac{dY_j}{dt} = -\sum_{i=1}^{m} \phi_{ij} * a_{ij} * X_i \tag{3}$$

This is a combat model where two opponents have heterogenic structures (Taylor, 1980a). The problem seems very simple to solve but that is illusion, because the real solution is very complex, even impossible to resolve. It becomes obvious when someone tries to resolve the combat model of two opponent forces with three and more combat elements (Hsiao & Guu, 2004).

The approximate methods are based on developed procedures for solving the model numerically. Significant contributions to the development of Aggregated Combat Models methodology are the works of: Alan

Washburn, Bill Caldwell, Jim Hartman, Sam Parry and Mark Youngren (Washburn et al, 2022). The numerical approach enables complex problems solving where analytical methods cannot help. They allow solving complex problems for which the solutions satisfy a certain degree of accuracy, which means that there is a certain error with some degree but which is within the limits of tolerance in relation to the analytical solution. Aggregated-force modeling was the basis of various simulation programs of war games, which are still used around the world today, which is why it will be tested as a model base in this work.

Aggregated-force modeling

The basic idea of this model is to aggregate all individual combat elements in the unit into one scalar measure that represents the combat power of the unit. This method combines various weapon systems and forces into one homogeneous force, using two characteristic quantities: the Firepower Index - (FPI) and the Firepower Score - (FPS).

The term firepower score indicates the combat power for each type of a particular weapon system. The firepower index indicates the summarized result, that is, the combat capability of the total, aggregate forces of a unit (Taylor, 1980b). In order to obtain the FPI, a linear model is used to transform all special values of the coefficients of the rate of inflicting losses on the other side, as an aggregate FPS, for the total, combined forces. Also, it is important to emphasize that the conceptual-categorical apparatus is uneven and that different authors use different terms with the same meaning. Since aggregate forces consist of completely different weapon systems, in order to achieve standardization for comparing different systems, the fundamental principle for determining the value is directly proportional to the value of the enemy system it destroys.

Calculating the FPS is relatively complex (Holter,1973), which also complicates the methodological unevenness of this method (Taylor, 1980b). The problem in studies of equivalent forces, as this methodological approach is also called, is to determine the weight or value of all types of weapons of each side in the conflict. Therefore, if we assume that the total value of different, combined weapons systems is a linear function of all those different systems, then it can be expressed by the following Aggregation of Forces (Taylor, 1980b):

$$s_i^x = k_x \cdot \sum_{j=1}^{n} b_{ji} * s_j^y \qquad \Lambda \qquad s_j^y = k_y \cdot \sum_{i=1}^{n} a_{ij} * s_i^x \tag{4}$$

for: $a_{ij} > 0 \;\; \Lambda \;\; b_{ji} > 0$

Where (s_i^x) or (s_j^y) represent the value of one (X_i) or (Y_j) weapon system of the same type on one side which is directly proportional to the total value of the opposing forces destroyed by those weapons per unit of time. This means that aside from constants of proportionality (k_x) and (k_y), the kill rate matrix (b_{ji}) denotes the attrition rate at which one (Y_j) system kills or destroys (X_i) systems in a certain combat situation and vice versa.

In relation to the initial analytical form of the battle of heterogeneous forces (eq.3), we consider the total value of the opposing forces (X) and (Y), as the value or the FPI of (V_x) and (V_y). Then the FPI represents the combat potential or the value of a military unit, where the score or the sum of that is a weapon system and indicates the number of combat elements in the unit:

$$V_x = \sum_{i=1}^{m} s_i^x * x_i \qquad \Lambda \qquad V_y = \sum_{j=1}^{n} s_j^y * y_i \qquad (5)$$

The values of the constant of proportionality (k_x) and (k_y) from (eq.4) are more convenient to be expressed as (Taylor, 1980b):

$$\left(\frac{1}{K_x}\right) = c_x \qquad \Lambda \qquad \left(\frac{1}{K_y}\right) = c_y \qquad (6)$$

In that case, the intensity of combat losses of aggregate forces (X) and (Y) and the values (c_x) and (c_y) can be interpreted as the Lanchester coefficient of attrition rate of loss of composite forces in the process where aggregate forces are consumed with time. The meaning of these constants is a direct consequence of the premise that there are positive values $(\mathrm{c_x})$ and $(\mathrm{c_y})$ which can determine the relationships between the values of different weapons or the FPS (s_i^x) and (s_j^y).

Finally, according to Taylor (1980b), it follows that the ratio expresses the equality of the average infliction of losses in time of (X) or (Y) sides as $(\frac{d\bar{Y}}{dt})$ or as $(\frac{d\bar{X}}{dt})$ and the product of the negative constant ($-c_y$) or ($-c_x$) and the average "weight" of the other and represents a unique value for all types of weapons. This can be written, in terms of Lanchester's square law, as:

$$\frac{dV_x}{dt} = -c_y * V_y \qquad \Lambda \qquad \frac{dV_y}{dt} = -c_x V_x \qquad (7)$$

This also means that if it is possible to determine the values (c_x) and (c_y) and the FPS vectors $[s^x]$ and $[s^y]$ of the total aggregated forces FPI in the time (V_x) and (V_y), by transformation of a heterogeneous conflict model into a homogeneous one, the mathematical model can be expressed as the classic Lanchester's quadratic law of combat (Darcom Pamphlet, 1979):

$$V_x(t) = V_x^0 cosh\sqrt{c_y * c_x * t} - V_y^0 \sqrt{\frac{c_y}{c_x}} \cdot sinh\sqrt{c_y * c_x * t} \qquad (8)$$

According to Taylor (1980b), this calculation is repeated for all parts of the forces if they are geographically separated and the losses actually represent a daily (temporal) decrease in combat power caused by combat operations. Individual losses, of special parts of power, are obtained through the process of disaggregation (Taylor, 1980b).

Algorithm for obtaining the FPS and the FPI[1]

The starting point of the procedure is based on the value of the equivalent forces by the equation:

$$C^2 * [a_{ij}] * [b_{ji}] * [S_i^x]^k = [W_i]^k \qquad (9)$$

Where $[S_i]^k$ is a new vector defined as a relative value – the FPS of the (*i*) weapon type (Darcom Pamphlet, 1979) and (*C*) is the single scaling factor for convenience, which brings the arms of real values into relation (Holter, 1973). Sizes indicate the relative value of individual weapons. For example, in relation to the value - a tank, so it can be concluded that some Blue type (*A*) weapons are effectively similar to Red type (*B*) weapons and each worth as two tanks (M60A3).

At the beginning, all components of the FPS vector $[s_i^x]^k$ are determined to have a value one, where the exponent *(k=1)* denotes the start of the iterative process. According to Holter (1973), this yields a fast convergent algorithm, leading to a unique value(Λ) and the FPS- $[S_i^x]^k$. By calculating (eq.9), a new vector – a relative FPS $[W_i]^k$is obtained, in which the weakest component (infantry) $[S_{inf}]^k$, is determined as the equivalent force value in relation to which other elements are determined.

[1] Considering the methodological complexity of the procedure for obtaining the rating and the index of firepower, it is not suitable and possible to give a detailed description; however, the essence of the method is shown. For more detailed information, see the works of Taylor (1980), Holter (1973) and a group of authors in the Handbook (Darcom Pamphlet, 1979), where the method is fully and thoroughly presented, with appropriate examples.

Then a new FPS is calculated according to the following relation:

$$[S_i]^{k+n} = \Lambda^k * [W_i]^k , \tag{10}$$

$$where: \quad \Lambda^k = {}^{1}\!/_{(S_{inf})^k} \tag{11}$$

where $[S_i]^{k+n}$ is the next vector of the relative FPS and where e.g.$(S_i)^k$ - infantry is the weakest weapon component.

The previous step (eq.9) is repeated, increasing (k) by a unit at each iteration, until the value: $\Lambda^{k+n+1} \approx \Lambda^{k+n}$, at some stage or iteration (k) is within a certain degree of accuracy. The iterations converge to a unique value (Λ) and the vector $[s_j^y]$ under the assumption that the matrix with $[a_{ij}]$ * $[b_{ji}]$ is irreducible (Darcom Pamphlet, 1979).

After the last iteration, the final value for (Λ) and the vector $[s_i^x]$ is reached:

$$[s_i^x]^{k+n+1} = \Lambda^{k+n} * [W_i]^{k+n} \tag{12}$$

Finally, the FPS $[s_j^y]$ is calculated:

$$C * [b] * [s_i^x] = \left[s_j^y\right] \tag{13}$$

$$\text{where} \qquad C = \sqrt{\Lambda} \tag{14}$$

The final vectors $[s_i^x]$ and $[s_j^y]$, represent the FPS for both opponent's weapon types or classes. The total value or the Fire Power Index – FPI $V_0(X)$ and $V_0(Y)$, of both opponents, is given by the relation:

$$V_0(X) = \left[s_i^x\right]^T * [X_i] \qquad \Lambda \qquad V_0(Y) = \left[s_j^Y\right]^T * [Y_j] \tag{15}$$

This represents only the basic structure of the model, which according to Taylor (1980b) forms the basis for the software tool in various war game simulations for the operational level, such as: ATLAS, TAGS, CEM, IDAGAM and TACWAR or the more recent FATHM (Washburn & Kress 2009). This type of model is also used in this paper.

Although this method is determined by the rate of fire (product of fired projectiles and carriers) in a certain time, it is nevertheless based on a certain subjectivism in the development of the FPI and is therefore subject to certain objections. It has been criticized by several experts, due to the method of calculation, where the FPI depends on the circumstances of the way of use, which affect the effectiveness of each particular element of the forces of one of the opponents. At the same time, the quantification of the combat capabilities of each special element represents a number that indicates its value in special combat conditions, in relation to other elements.

Also, it is important to emphasize that the conceptual-categorical apparatus is uneven and that different authors use different terms with the same meaning. Since aggregate forces consist of completely different weapon systems, the fundamental principle for determining the value is directly proportional to the value of the enemy system it destroys. In order to achieve standardization for comparing different systems, this maxim is developed into the view that the value of a weapon system is directly proportional to the rate at which the value of an enemy weapon system is destroyed. According to Taylor (1980b), this has continued to be the basis for large force conflict analyses in the US Armed Forces and NATO countries during 70s and 80s and even today, due to the simple fact that it is by far the most suitable for software application. The fact is that these methods are still in use through software tools which are applied for simulations of the conflict of forces of strategic groups on the battlefield. However, it has been criticized by a lot of authors.

Due to the nature of war as a phenomenon and the limited availability of relevant facts, modeling was done followed by the evaluation of the method based on the results of a case study, a representative historical example of a strategic air operation (campaign) Operation Desert Storm (Keaney & Cohen, 1993).

Experiment – combat simulation

The essential question is both complex and difficult to answer: whether the created combat model behaves consistently in a way that corresponds to reality? The key is the assessment of the parameters that are an integral part of the model. By practical verification, on the example of a combat situation, a comparison can be made and the real applicability of the approximate method can be verified. The validation of the model was carried out by simulating the combat operation Desert Storm, due to fortunate circumstances that a large statistical material is publicly available, with a wealth of data such as: data on planning and formation of forces (Gulf War Air Power Survey, 1993a), the number of flights performed, the consumption of ammunition and fuel, the number and type of targeted objects, tactics and training and combat capabilities (Gulf War Air Power Survey, 1993c), expected effects of actions, etc. For the sake of simplicity of application and data processing, a certain approximation was made, which refers to the generalization of the forces and the determination of their combat capabilities.

Blue (Coalition Force) has m = 3 types of combat forces, which are then grouped according to their type and purpose, and deployed in the appropriate order of battle: air force, air defense and army force.

Red (Iraq Force) has n = 4 types of combat forces, which are then also grouped according to their type and purpose, and deployed in the appropriate order of battle: air force, air defense, army force and tactical ballistic missiles.[2]

The reviewed forces, according to their numerical strength status are given in the following Tables from 1 to 3 (Gulf War Air Power Surwey, 1993d) and the combat capabilities of the opponents are given in Tables 4 and 6 (Gulf War Air Power Surwey, 1993b). The ground forces are shown as a collection of elementary parts, which together form wholes of special types of combat units of mechanized and armored divisions and brigades. The Iraqi army represents: 8 divisions of the Republican Guard and 36 divisions of the Regular Army on the Kuwaiti battlefield, while armored brigades form the composition of 22 divisions of the Iraqi army in Iraq. Actual numbers of Ground combat force elements are shown in Table 3.

Table 1 – Comparative strenght of Coalition and Iraqi Forces by types
Таблица 1 – Сравнительная численность коалиционных и иракских сил по видам
Табела 1 – Упоредна снага коалиционих и ирачких снага по врстама

X_i	X_F	X_B	X_{STH}	X_{EW}	X_{FA}	X_{SEAD}	X_{AH}
	205	420	40	59	2150	450	681
Y_j	Y_F	Y_{FA}	Y_A	Y_{EW}	Y_{SEAD}	Y_{recon}	Y_{AH}
	56	164	908	20	12	32	442

Table 2 – Comparative strenght of Coalition and Iraqi Forces by types
Таблица 2 – Сравнительная численность коалиционных и иракских сил по видам
Табела 2 – Упоредна снага коалиционих и ирачких снага по врстама

X_i	X_{ADL}	X_{ADM}	X_{ADS}	X_{WMD}	X_{MD}	X_{ABr}
	96	44	-	-	24600	21000
Y_j	Y_{ADL}	Y_{ADM}	Y_{ADS}	Y_{WMD}	Y_{ArmD}	Y_{MD}
	18	270	558	110	33000	22000

[2] Meanings of abbreviations are: F-fighter, B-bomber, FB- fighter bomber, STH-stealth, EW-electronic warfare, FA-fighter attack, SEAD-suppression of enemy air defense, AH-attack helicopter, ADF-air defense (L-long, M-medium, S-short range), A-artillery, E-infantry, ARM-armored; T-tanks, AFV-armored fighting vehicles, WMD-weapons of mass destruction, MD-Mechanized divisions, ABr-Armored Coalition' s brigades, ArmD-Iraqi' s armored divisions, MD-Mechanized divisions.

Table 3 – Comparative formation composition of ground forces by types
Таблица 3 – Сравнительный структурный состав сухопутных войск по видам
Табела 3 – Упоредни формацијски састав копнених снага по врстама

	T	AFV	Artillery	Infantry
X_i	7716	13160	4556	486400
Y_j	6490	4620	4151	330000

The method implies that through the process of aggregation of forces, the values of the equivalent forces FPS and FPI of both opponents are defined, considering the rate of attrition through operational capabilities Bulger (1997). After that, the combat model is programmed with a set of analytical equations which describe the "attrition" or combat losses of each opponent's forces, according to the Lanchester quadratic law of combat (Eq.8). Each separate element of the aggregate forces is recalculated by the reverse process, according to a given time step in the operation or campaign.

When modeling with this method, the following assumptions were made:

- the impact of the force maneuver is related to the speed of expenditure of forces and has no other influence,
- there is no change in the rate of attrition of force, during the execution of a special stage or sequence of the operation,
- there is no operational pause during combat engagement,
- all combat forces of both opponents are simultaneously engaged in combat until the desired end state is achieved: neutralization, defeat or retreat, and
- air operations on strategic targets were not considered, such as air strikes on logistics bases, warehouses, energy plants, etc.

The combat capabilities of the forces in this case mean the speed of inflicting losses by a certain combat system of one party to a certain combat system of the other party. They are given in Tables from 4 to 6.

In the mentioned simulations, which were used or are still used by NATO member armies, it is possible to program different operational situation conditions and types of combat: such as attack or defense, maneuver combat, winter or summer, mountainous terrain, surprise, etc. This is important to note because in these cases the composition and the number of forces changes, as well as the combat capabilities of special elements of the forces, which affects the aggregation of forces and the Firepower Index or Value of the forces.

Table 4 – Comparative combat capabilities of the Coalition forces by targets
Таблица 4 – Сравнительные боевые возможности коалиционных сил по целям
Табела 4 – Упоредне борбене способности коалиционих снага према циљевима

α	Y (F,FA,A,AH)	Y (EW,SEAD)	Y (ADL,ADM,ADS)	Y_{WMD}	Y (AmD,MD)
X_{F}	0.257	0.427	0.427	0.860	0.860
X_{B}	0.012	0.012	0.186	0.727	0.727
X_{STH}	-	-	0,800	0,600	0,600
X_{EW}	0.156	0.574	0.574	0.439	0.439
X_{FA}	0.156	0.574	0.574	0.439	0.439
X_{SEAD}	-	-	0,357	0,357	0,400
X_{AH}	0.001	0.001	0.270	0.900	0.600
X_{ADL}	0.480	0.480	-	0.480	-
X_{ADM}	0.455	0.455	-	0.455	-
X_{ADS}	0.052	-	-	-	-
X_{MD}	-	-	-	0.001	0.030
X_{ABr}	-	-	-	0.010	0.500

Table 5 – Comparative combat capability of Iraqi forces by targets I
Таблица 5 – Сравнительная боеспособность иракских сил по целям I
Табела 5 – Упоредна борбена способност ирачких снага према циљевима I

β	X (F,B,FA,SEAD,AH)	X_{STH}	X_{EW}	X (ADL,ADM)	X_{ADS}
Y_{F}	0.131	0.004	0.01	0.116	-
Y_{FA}	0.12	0.001	0.02	0.136	0.119
Y_{A}	0.07	-	0.052	0.472	0.702
Y_{EW}	0.07	0.07	0.052	0.2	-
Y_{SEAD}	0.07	0.07	0.052	0.2	-
Recon	0.038	-	0.038	0.05	-
Y_{AH}	0.0005	-	0.0005	0.211	0.2
Y_{ADL}	0.091	0.091	0.091	-	-
Y_{ADM}	0.327	0.027	0.327	-	-
Y_{ADS}	0.057	0.057	0.057	-	-
Y_{WMD}	-	-	-	0.0499	0.0499
Y_{ArmD}	-	-	-	-	-
Y_{MD}	-	-	-	-	-

Table 6 – Comparative combat capability of Iraqi forces by targets II
Таблица 6 – Сравнительная боеспособность иракских сил по целям II
Табела 6 – Упоредна борбена способност ирачких снага према циљевима II

β	X_{WMD}	X_{MD}	X_{ABr}
Y_F	-	-	-
Y_{FA}	0.119	0.119	0.119
Y_A	0.702	0.702	0.702
Y_{EW}	-	-	-
Y_{SEAD}	-	-	-
Y_{recon}	-	-	-
Y_{AH}	0.9	0.6	0.9
Y_{ADL}	0.091	-	-
Y_{ADM}	0.327	-	-
Y_{ADS}	-	-	-
Y_{WMD}	0.5	0.2	0.1
Y_{ArmD}	0.001	0.03	0.05
Y_{MD}	0.01	0.5	0.3

The essence of the force aggregation method (Darcom Pamphlet, 1979) is reflected in the iterative procedure by which all special elemental forces of each opponent with special combat capabilities of destroying each special element of the opponent's forces are expressed as a total measure or value of the relative strength of the forces of one and the other opponent.

Results and analysis

Finally, the validation of the model and the evaluation of the representativeness of the output results of the simulation was performed by comparing the parameters of the Operation Desert Storm (data from the real world) with the results obtained by the simulation and the operational assessment method. A computer testing of the operation model was performed, according to the available data, where certain discrepancies (errors) were taken into account. The Summary Report of Desert Storm, based on an exceptional database from the Gulf War Survey (Keaney & Cohen, 1993), served to validate the model. This was a necessary condition, by which it was possible to arrive at a relatively reliable structure and functioning of the operation, as well as relatively reliable data.

The overall estimated strength of Iraqi forces is given by characteristic periods and reflects losses throughout the campaign. The situation in January 1990 marks the period of the Operation Desert Shield, and the situation in February-March 1991 includes the situation before and after the Operation Desert Storm. For more details it is useful to consult the Survey, Chapter VII (Elliot, 1994), with the list of tasks for different combat missions, with the number of flights performed and the percentage of the total sorties performed. The total number of flights during the counterair and strategic attack as a part of the campaign was about 95,000, and during the air support and air interdiction phase of operation was about 15,000 (Engelhard, 1991). The real losses of the air forces for the both opponents were as follows (Gulf War Air Power Survey, 1993d).

The comparison of actual and modeled number of flights in different missions and the consumption of ammunition were given according to Keaney & Cohen (1993) in Tables 7 and 8. At first glance, the planned combat distribution of forces by the process of targeting and the duration of the three-day cycle allows a simple calculation in simulation. In practice, a whole series of factors in real world affect execution of tasks, from weather conditions, through the correctness of the aircraft, to the specifics mission terms, target characteristics, topography and local tactical conditions in the area of operation, etc. In the case of applying the deterministic mathematical model of force aggregation, these situations can only be expressed by a probability of execution or attrition loss coefficients. This implies that we cannot use simple calculations for the precise, daily number of combat flights and ammunition consumption because the number of possible or required actions is not symmetrical with the actually performed ones, but only probable. [3]

Some facts relevant for the objectivity of modeling should be noted:

- The Iraqi Air Force initially attempted somewhat larger air defense and fighter air support operations, then only sporadically, resulting in defections to Iran, and eventually ceased operations;
- About 140 Iraqi aircraft defected to Iran, which would probably have been destroyed if they had participated in the battle. These aircraft were never recovered by Iraq;
- In the operation model, air operations were considered by available Iraqi aircraft that could be detected on the ground or in the air. Due to methodological limitations, the model, in this case,

[3] Ammunition consumption and the number of dedicated flights performed in the model were calculated, based on an assessment in relation to the required and probable number of hits of a certain type of ammunition to destroy/neutralize the target.

simultaneously calculates the probable average expenditure of precision-guided air-to-air and air-to-ground missiles;

- The model implies no possibility that part of the forces in the conflict will be out of combat and that part of the force cannot be acted upon, e.g. ammunition and equipment in shelters;
- Some of the support missions, such as air transport, aerial refueling, reconnaissance, etc., are not shown, as they are not supported by the model.

Table 7 – Comparison of real and modeled Coalition Forces air missions
Таблица 7 – Сравнение реальных и смоделированных миссий Коалиционных сил
Табела 7 – Поређење стварних и моделованих мисија коалиционих снага

Executed missions	Real World	Model
Strategic attack and interdiction	38277	33469
Air support	6128	9943
Offensive/Defensive counterair operations	19419	18228
Suppression of enemy air defense	4326	6547
Electronic warfare	2918	
Reconnaissance	3236	-
Overall Operational support	45267	-
Overall combat	68150	68188

Table 8 – Comparison of real and modeled consumption of the Coalition air weapons
Таблица 8 – Сравнение реального и смоделированного расхода оружия Коалиции
Табела 8 – Поређење реалне и моделоване потрошње наоружања Коалиције

Type of weapons	Real world	Model
Overall munitions	228182	228908
Air to Air missiles	174	738
Air defense missiles	360	316
Unguided air bombs	210004	211067
Guided Air to Ground missiles and bombs	15372	14605
Cruise missiles	333	
Anti-radiation missiles	2039	2182
Targeting phases	14	15

The final results of the Desert Storm simulation modeling are given in the overview of the state of forces for Blue (Coalition) and Red (Iraq) and in diagrams in Figures from 1 to 4.

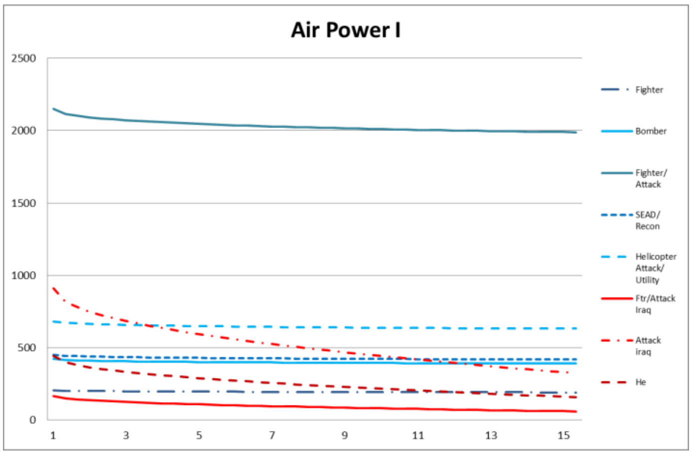

Figure 1 – Depiction of the air forces attrition process in the Operation, part 1[4]
Рис. 1 – Изображение процесса истощения ВВС в операции, часть 1
Слика 1 – Приказ процеса трошења ваздухопловних снага у операцији, 1. део

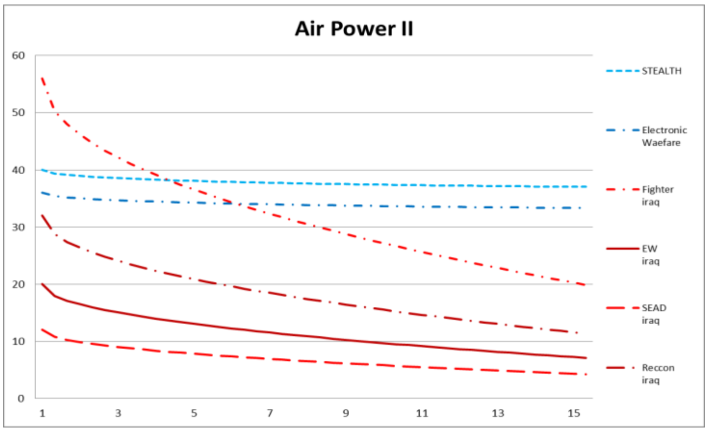

Figure 2 – Depiction of the air forces attrition process in the Operation, part 2
Рис. 2 – Изображение процесса истощения ВВС в ходе операции , часть 2.
Слика 2 – Приказ процеса трошења ваздухопловних снага у операцији, 2. део

[4] The values on the abscissa indicate the number of cycles in the targeting process, where one cycle represents 3 days. The values on the ordinate represent the numbers of elements of a combat system (the number of aircraft or elements of the tactical formation of ground units).

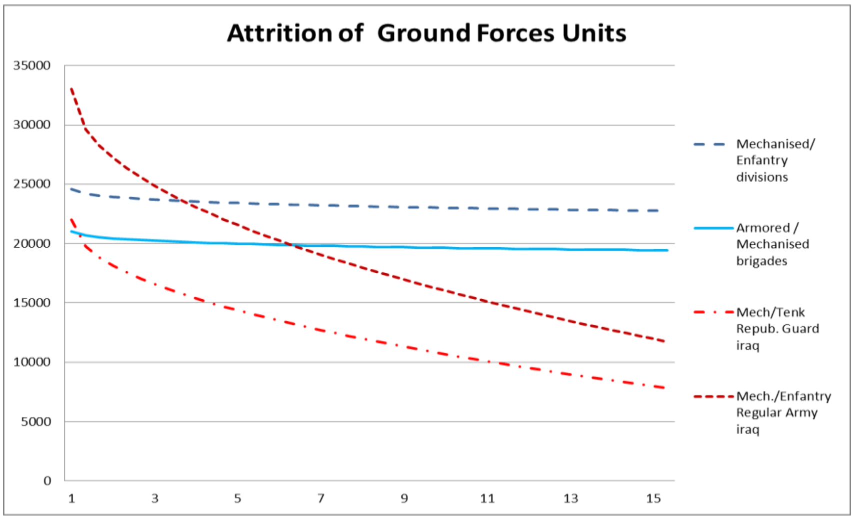

Figure 3 – Attrition process of the aggregated ground forces
Рис. 3 – Процесс истощения объединенных сухопутных войск
Слика 3 – Процес трошења агрегираних копнених снага

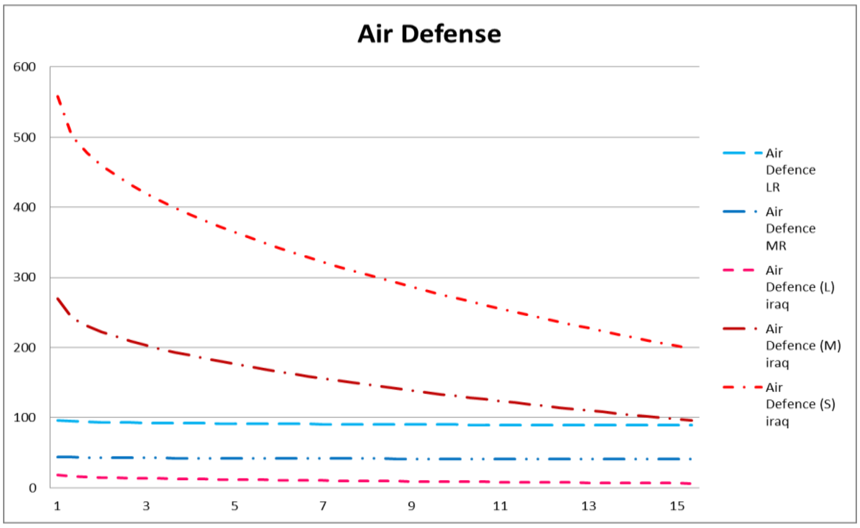

Figure 4 – Air defense attrition process
Рисунок 4 – Процесс истощения ПВО
Слика 4 – Процес исцрпљивања снага противваздухопловне одбране

An overview of the state of forces by type at the beginning and at the end of the modeled duration of the Operation Desert Storm, in a period of about 45 days, is shown in Tables 9 and 10.

Table 9 – Comparison of the Coalition and Iraqi air forces by type during the Operation
Таблица 9 – Сравнение коалиционных и иракских военно-воздушных сил по типам в ходе операции
Табела 9 – Поређење коалиционих и ирачких снага по врстама током операције

Xi	X_F	X_B	X_{STH}	X_{EW}	X_{FA}	X_{SEAD}	X_{AH}
	205	420	40	59	2150	450	681
	190	389	37	33	1989	416	630
Yj	Y_F	Y_{FA}	Y_A	Y_{EW}	Y_{SEAD}	Y_{recon}	Y_{AH}
	56	164	908	20	12	32	442
	20	58	322	7	4	11	157

Table 10 – Comparison of the Ccoalition and Iraqi air forces by type during the Operation
Таблица 10 – Сравнение коалиционных и иракских военно-воздушных сил по видам в ходе операции
Табела 10 – Поређење коалиционих и ирачких снага по врстама током операције

Xi	X_{ADL}	X_{ADM}	X_{ADS}	X_{WMD}	X_T	X_{AFV}	X_A	X_E
	96	44	-	-	7716	13160	4556	486400
	89	41	0	0	7143	12182	4217	450257
Yj	Y_{ADL}	Y_{ADM}	Y_{ADS}	Y_{WMD}	Y_T	Y_{AFV}	Y_A	Y_E
	18	270	558	110	6490	4620	4151	330000
	6	96	198	39	1480	1480	7808	11712

An analysis with a comparison of real statistical data, based on the Gulf War review (Gulf War Air Power Survey, 1993d), was performed and the modeling results were presented. Certain deviations were observed, and the results are presented comparatively as available statistical data / data obtained by the simulation process:

- operation lasted about 42 days (14 targeting cycles)/ 45 days (15 targeting cycles) in the model;
- 75 aircraft (airplanes and helicopters) of Coalition forces were shot down, and 141 were damaged / 298 in the model;
- actually destroyed planes and helicopters of Iraq were about 259, including 122 lost in air-combat, 121 defected to Iran later confiscated and about 81 destroyed on the ground/ in the model 769 planes and 285 helicopters;
- surface-to-air missile batteries lost about 115-35 / 546 in the model;
- destroyed armored forces of Iraq: 4,550 tanks and 2,840 AFV 4,139/ 2,947 in the model;

- destroyed armored forces of the Coalition 664/ 1,551 in the model;
- destroyed artillery pieces of Iraq: about 2,917/ 2,647 in the model;
- between 20,000 and 26,000 Iraqi military personnel were killed and 75,000 others were wounded/210,468 in the model; and
- Coalition forces suffered about 984 deaths / 36,143 in the model.

According to the attrition of forces diagrams, during the execution of the operation, a disproportionately higher number of losses of Iraqi forces can be clearly observed. It is also clear that the military power of the Coalition was overwhelming, resulting in a massive victory. This is a significant feature of the Operation Desert Storm. However, considering the comparison of numerical indicators (combat exhaustion), it is obvious that there are deviations, which is why the model is not fully valid and is only relatively reliable, in terms of the required precision, in the process of operational planning. It is easy to see that the losses of Iraqi air and ground forces, the losses of the Coalition forces and the number of combat sorties are not identical. The data in the model were obtained by estimating the rate of losses based on the data from the actual operation and were numerically calculated. The operational duration of the operation is only conditional because it is based on a time estimate according to the conditionality of applying Lanchester's equations (the time step must be appropriately small due to the consistency of the model).

The data differ somewhat in ammunition consumption, where there are smaller discrepancies for unguided and precision-guided weapons on surface targets. Somewhat larger deviations are observed in anti-aircraft operations and ammunition consumption. A large difference was observed in close air support (attrition and weapon consumption) and infantry casualties of both opponents.

There is an interesting observation by American experts that the assessment of the expenditure of forces in modeled combat operations from the Vietnam War to the Operation Desert Storm is constantly exaggerated and relatively wrong in relation to reality. Also, it should be noted that when checking the ATLAS model by SHAPE Headquarters, based on the data on the numerical superiority of the Allies in the war in Europe in 1940, a conclusion was reached about the very quick defeat of the Germans (Dupuy, 1997). The general conclusion is that models lose their fidelity when trying to simulate large campaigns because they cannot faithfully replicate their enormous complexity, a correlation already emphasized by Taylor (1980a). This is an essential issue in the application of computer simulations, where most military-political experts do not know the mathematical basis of the program. They cannot explain countermeasures, execution kinematics, deception, decisions by fighters and commanders in real time, changes in tactics as the campaign progresses, moral, etc. When considering the application of this method, objections to methodological inconsistency need to be emphasized, as seen in the Handbook (Darcom Pamphlet, 1979) where Howes and Thrall discuss several different methods for determining the relative weights or values, and give examples of their recommended ideal weights (Howes & Thrall, 1973). However, a bigger problem is that weights or values should be cross-structured so that the total representative strengths or equivalent combat powers can be determined on the same homogeneous scale and in terms of the same weapon (Holter, 1973). Many models only extrapolate individual force engagements in combat from scenarios versus complex ones (Berenson, 1997) which is a gross methodological error.

In this case, it is important to note that the mathematical model is deterministic and discrete, with calibration performed for certain deviations that have appeared in relation to reality but can be considered acceptable for several reasons.

The first reason is that the model processes operational actions on the battlefield and in the operational depth, according to the doctrinal principles of use, but also taking into account the specific situation in this conflict. This means that it was practically difficult to project a real combat sortie and the availability of Iraqi aircraft, air defense and other types of weapons to act as targets, due to the atypical use, because the Iraqis decided to preserve their aviation and army forces by masking them, expecting a ground operation. On the other hand, the Coalition forces avoided air-ground combat until the last 100 hours of the operation;

Secondly, actions on strategic objects, such as communications, energy, industrial and economic, or political infrastructure of Iraq, were partially taken into account, where a part of guided aerial bombs, missiles and cruise missiles were probably used;

Thirdly, given the stochastic nature of the actual process of armed struggle, certain interruptions and changes in the planned actions, caused by various causes, were sure to occur, which affected the change of action plans, increased the consumption of ammunition in reality and caused atypical use of the methodology; and

Lastly and most importantly, the force aggregation method requires a recalculation for each special phase or stage of the operation, due to the change in the operational situation, which is reflected in the operational capabilities and combat order or the strength of the forces in battle (firepower index and force value - Firepower Score).

According to formal criteria, the observed Operation Desert Storm can be viewed as a realistic system described at a higher level, while the created deterministic model is at a lower level of description and has been formally verified, in terms of the accuracy of the calculation of the given parameters. The partial validity of the model, which describes the conflict on a practical historical example from a case study, was confirmed, given that the creator of the model is methodologically allowed to determine the maximum degree of deviation. As stated, the combat was not conducted according to doctrinal principles, which would have meant an air-ground battle and the engagement of the full combat potential of both sides. In this sense, the entire campaign can be generally divided into the first part, which includes a strategic air operation: "crushing the military power of Iraq" and the second part: "an offensive air-ground operation," which expelled the Iraqi forces from Kuwait and then destroyed them. According to the formal objectives of the real operation and the results obtained, it can be said that the model is approximately satisfactory, considering the final numerical results, in terms of the large disproportion of Iraqi losses in relation to the Coalition forces and the duration of the operation.

Larger discrepancy is observed in the Coalition ground troop losses, which is a problem of force aggregation combat modeling, where it is assumed that all forces participate in operations simultaneously. It is interesting that the Coalition planners also assumed higher losses around 45,000 (Correll et al, 2021), which resulted in a change in the way of using forces and abandoning the then valid doctrinal principles of an air-ground battle. The result is the strategic use of air power in crushing Iraq's military power. When the last phase of the operation began, there was almost no ground combat, with a few exceptions.

An unsolved part of the problem of applying this model as a means of support in the process of operational planning is the possibility of optimizing the use of forces in combat - the course of action, due to the limitations of the application of the multi-criteria optimization method.

However, the real problem of the model's reality arises during the duration of the process, when operational conditions are applied and power losses lead to absurd situations. As an example, we can cite the situation of fighting forces that do not have the possibility of fighting each other, which can happen due to the percentage decrease in the power of joint units. It would be an example of a battle between naval and land forces (ships at sea, tanks in plains and infantry against modern aviation). The model would still recalculate losses even though the possibility of interaction between combat entities does not exist.

Conclusion

The created deterministic, discrete mathematical model of a strategic campaign can be used during further experimentation and consideration as a strategic planning tool, to obtain certain data, which deepens and expands knowledge, with certain limitations on reliability.

The complexity of applying the model is precisely the problem of the power aggregation method. Modeling requires iteratively repeating the aggregation process for each distinct phase of the operation. The reason for this is that, due to a change in intermediate objectives and/or methods of execution of action, there is a change in combat capabilities and the size of the forces fighting in certain regions, directions and in a certain operational environment. These changes affect the operational capabilities of the force and the coefficient of force attrition, which implies changes in the FPS and the FPI in the model. This, consequently, requires phase modeling, for each specific phase or area of the battlefield, which implies recalculation and the use of far more complex software, in order to obtain the results necessary for the planning process in real time.

For these reasons, there are certain objective limitations for the application of modeling of military operations, and especially for the optimization of the use of forces at the tactical and operational level of the battlefield. However, the model provides a relatively reliable assessment of the outcome of the operation, with conditionally adequate assessment of numerical indicators, with the above assumptions.

A special problem for the optimization of the force use model (optimal course of action) is the methodological basis of the method itself, which does not ensure the use of any of the multi-objective programming methods. This prevents practical application in the targeting process, which implies optimal planning by grouping forces with the arrangement of objects of action and the required targeted effects, which is the core of the operational planning process. The problem could eventually be solved by applying multi-attribute optimization methods, which would require the development of several scenarios with the complete process of building a combat model and simulation. However, this again would not provide a real solution - optimization and is not practical for use in operational command conditions.

The essential model is usable at the operational-strategic level, where the fight of joint units and strategic formations of the armed forces is considered. The existing model offers a highly probable assessment of the outcome of a conflict or as a means of comparing the military power of two adversaries, which is its proven value. Also, it can be useful in a rough estimation of the required funds and possible losses, but these results, especially the losses and the duration of the operation, should be taken with caution.

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Моделирование боевых действий

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РУБРИКА ГРНТИ: 27.47.00 Математическая кибернетика,
27.47.19 Расследование операций,
28.17.31 Моделирование процессов управления
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Целью данного *исследования являются проверка и оценка приближенного метода модели стратегических боевых действий объединенных сил, основанной на уравнениях Ланчестера.*

Методы: На примере известной операции «Буря в пустыне» были собраны необходимые данные о боевых возможностях и численности противника, оперативной обстановке, доктринальных принципах и способах применения сил в боевых задачах. Полученные данные обрабатывались методом агрегирования сил, преобразуя силы разнородного состава в однородные. Моделирование боевых действий проводилось с помощью метода дифференциальных уравнений Ланчестера.

Результаты: В ходе исследования была п*одтверждена частичная валидность модели, описывающей вооруженный конфликт, на практическом историческом примере* анализированного случая *с учетом влияния параметров исхода конфликта, соотношения потерь, расхода боеприпасов и количества выполненных воздушных операций.*

Выводы: В результате исследования были выявлены объективные ограничения в применении моделирования боевых действий, оптимизации применения сил на тактическом и оперативном уровнях. Ценность данного метода заключается в возможности надежной стратегической оценки военной мощи противника на стратегическом уровне.

Ключевые слова: боевые действия воздух-земля, истощение, модель объединенных сил.

Моделовање борбених операција

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КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Циљ истраживања је провера и процена апроксимативне методе модела стратегијских борбених дејстава агрегатних снага, базираним на Ланчестеровим једначинама.

Методе: Студијом случаја познате операције „Пустињска олуја" прикупљени су потребни подаци о борбеним способностима и снази непријатеља, оперативним факторима, доктринарним принципима и начину употребе снага у борби. Добијени подаци су обрађени методом агрегације сила, трансформишући снаге хетерогеног састава у хомогене. Моделовање битке је спроведено методом диференцијалних Ланчестерових једначина.

Резултати: Потврђена је делимична валидност модела који описује сукоб на практичном историјском примеру из студије случаја, узимајући у обзир утицај параметара исхода сукоба, односа губитака, потрошње убојних средстава и броја изведених ваздухопловних мисија.

Закључак: Постоје објективна ограничења у примени моделовања војних операција, и оптимизацији употребе снага на тактичком и оперативном нивоу. Вредност овог метода јесте могућност поуздане стратешке процене војне моћи противника на стратешком нивоу.

Кључне речи: ваздухопловно-копнене борбене операције, исцрпљивање, модел агрегатних снага.

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