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Critical remarks on “Existence of the solution to second order differential equation through fixed point results for nonlinear F-contractions involving w_0-distance”

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FIELD: mathematics
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Abstract:

Introduction/purpose: In this paper, several critical remarks are presented concerning the paper of Iqbal & Rizwan: Existence of the solution to second order differential equation through fixed point results for nonlinear F-contractions involving w_0-distance from 2020.

Methods: Conventional theoretical methods of functional analysis.

Results: It is shown that their use of the non-decreasing “control” function F instead of a strictly increasing one in Wardowski-type results usually produces contradictions.

Conclusion: It is shown that such results can be obtained in a more general class of metric-like spaces, where strict monotonicity is the only as-

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sumption that has to be imposed on the function F. An example is presented showing that the obtained results are stronger than the classic ones.

Key words: F-contraction, fixed point, metric-like space, strictly increasing function.

Introduction and preliminaries

D. Wardowski's result from (Wardowski, 2012) can be considered as one of the significant generalizations of S. Banach's basic result from 1922. In this generalization, Wardowski used a function $F : (0, +\infty) \to \mathbb{R}$ satisfying the following three properties:

(F1) F is strictly increasing, i.e., $t_1 < t_2$ implies $F(t_1) < F(t_2)$;

(F2) for any sequence $\{a_n\} \subset (0, +\infty)$, $\lim_{n\to+\infty} a_n = 0$ if and only if $\lim_{n\to+\infty} F(a_n) = -\infty$ (i.e., $\lim_{t\to 0+} F(t) = -\infty$);

(F3) there exists $k \in (0, 1)$ such that $\lim_{t\to 0+} t^k F(t) = 0$.

He proved that a self-map T on a complete metric space (X, d) has a unique fixed point if there exists a positive number τ and a function F satisfying the previously mentioned conditions, such that, for all $x, y \in X$ with $Tx \neq Ty$,

$$\tau + F(d(Tx, Ty)) \leq F(d(x, y)) \tag{1}$$

holds.

Later on, in another work (Wardowski, 2018), he generalized the mentioned result, replacing the condition (1) by the following one

$$\varphi(d(x, y)) + F(d(Tx, Ty)) \leq F(d(x, y)), \tag{2}$$

where φ is some function from $(0, +\infty)$ to itself satisfying $\liminf_{s\to t+} \varphi(s) > 0$ for each $t \geq 0$ (he called such mappings (φ, F)-contractions).

In a multitude of subsequent papers, dozens of authors used Wardowski's approach for mappings acting in various spaces (such as b-metric spaces, partial metric spaces, metric-like spaces, cone metric spaces, G-metric spaces, rectangular metric spaces, as well as in spaces endowed with a w-distance) – a review of these results until 2022 can be found in (Fabiano et al, 2022). We recall here the definitions of w-distance p of O. Kada, T. Suzuki and W. Takahashi and metric-like μ of A. Amini-Harandi.

Definition 1. (Kada et al, 1996) Let (X, d) be a metric space and let a mapping $p : X \times X \to [0, +\infty)$ satisfy:

(p1) $p(x,z) \leq p(x,y) + p(y,z)$ for all $x, y, z \in X$;

(p2) for any $x \in X$, the function $p(x, \cdot) : X \to [0, +\infty)$ is d-lower semi-continuous;

(p3) for any $\varepsilon > 0$, there exists $\delta > 0$ such that $p(z, x) < \delta$ and $p(z, y) < \delta$ imply $d(x, y) < \varepsilon$.

Then, p is called a w-distance on X.

Definition 2. (Amini-Harandi, 2012) A metric-like on a nonempty set X is a function $\mu : X \times X \to [0, +\infty)$ if the following conditions hold for all $x, y, z \in X$:

(μ1) $\mu(x,y) = 0$ implies $x = y$;

(μ2) $\mu(x,y) = \mu(y,x)$;

(μ3) $\mu(x,y) \leq \mu(x,z) + \mu(z,y)$.

Then (X, μ) is called a metric-like space.

Note that, see, e.g. (Iqbal & Rizwan, 2020) or (Kadelburg & Radenović, 2024), for a w-distance p on a set X, the mapping

$$\mu(x,y) = \max\{p(x,y), p(y,x)\}$$

is a metric-like on X.

The notions of convergent and Cauchy sequences, and continuous functions, were introduced in metric-like spaces as follows.

Definition 3. (Amini-Harandi, 2012) Let (X, d) be a metric-like space and $\{x_n\}$ be a sequence in X.

1. The sequence $\{x_n\}$ is said to converge to $x \in X$ if $\lim_{n\to+\infty} d(x, x_n) = d(x, x)$.
2. $\{x_n\}$ is a Cauchy sequence if $\lim_{m,n\to+\infty} d(x_m, x_n)$ exists and is finite.
3. The space (X, d) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to some $x \in X$ such that $\lim_{m,n\to+\infty} d(x_m, x_n) = d(x, x) = \lim_{n\to+\infty} d(x_n, x)$.
4. A mapping $T : X \to X$ is continuous at a point $x \in X$ if $\lim_{n\to+\infty} x_n = x$ implies $\lim_{n\to+\infty} Tx_n = Tx$.

On the other hand, some authors considered different conditions on the "control" function F, culminating in O. Popescu's and G. Stan's proof, see

(Popescu & Stan, 2020), Theorem 5, that in fact just condition (F1) is sufficient for obtaining the basic result from (Wardowski, 2012), see also (Fabiano et al, 2022), Theorem 2.3 and Remark 2.4.

It is natural to ask whether the property (F1) of the function F can be replaced by a weaker property that F is non-decreasing, but not strictly increasing. Of course, there are a lot of such functions.

In the recent paper (Iqbal & Rizwan, 2020), the authors tried to generalize the results of papers (Wardowski, 2012) and (Wardowski, 2018) in two ways – firstly, instead of metric d they used w-distance p or metric-like μ. On the other hand, instead of the assumption (F1), the authors of (Iqbal & Rizwan, 2020) used the weaker assumption

(F$1'$) the function F is non-decreasing on $(0, +\infty)$, i.e. $t_1 \leq t_2$ implies $F(t_1) \leq F(t_2)$,

together with the assumptions (F2) and (F3).

Unfortunately, all their results obtained under the assumption (F$1'$) may be incompatible with a Wardowski-type contractive condition. To show that, the following observation can be useful.

Lemma 4. If $F : (0, +\infty) \to \mathbb{R}$ is a non-decreasing, but not strictly increasing function, then there exists an interval $(a, b) \subset (0, +\infty)$ such that the restriction of F to this interval is constant.

Proof. Since F is non-decreasing but not strictly increasing, then there are $a, b \in (0, +\infty)$ such that $a < b$ and $F(a) = F(b)$. But then F is a constant function on (a, b). □

Now, it is easy to construct examples like the following one.

Example. Consider $X = [0, +\infty)$ with the standard metric $d(x, y) = |x - y|$ and the mapping $T : X \to X$ given by $Tx = \frac{3}{4}x$. The function

$$F(t) = \begin{cases} \log t, & 0 < t < 1, \\ 0, & 1 \leq t \leq 2, \\ \log(t-1), & t > 2, \end{cases}$$

satisfies conditions (F$1'$), (F2) and (F3) (but not (F1)!). However, if $\frac{4}{3} \leq |x - y| \leq 2$, then the condition (1) reduces to $\tau + 0 \leq 0$, i.e., $\tau \leq 0$, which is incompatible with the basic assumption $\tau > 0$.

One possible additional assumption when we use the function F satisfying just the condition (F$1'$) could be that

$$\{\, d(x,y) : x, y \in X \,\} \cap \Big(\bigcup_C I_C\Big) = \emptyset,$$

where I_C is the interval (a,b) such that $F(t) = C$ for each $t \in (a,b)$.

In order to improve and generalize the results from (Iqbal & Rizwan, 2020), we first state the following two known lemmas that are of interest in themselves and can be used in proving the Cauchyness of a Picard sequence $\{x_n\} = \{T^n x_0\}$ in both metric and metric-like spaces, see some references on these lemmas in (Fabiano et al, 2022).

LEMMA 5. Let (X,d) be a metric-like space and $\{x_n\}$ be a Picard sequence in it. If

$$d(x_{n+1}, x_n) < d(x_n, x_{n-1}),$$

for all $n \in \mathbb{N}$, then $x_n \neq x_m$ whenever $n \neq m$.

LEMMA 6. Let (X,d) be a metric-like space and $\{x_n\}$ be a sequence in X such that $\{d(x_{n+1}, x_n)\}$ is a non-increasing sequence and that $\lim_{n\to+\infty} d(x_{n+1}, x_n) = 0$. If $\{x_n\}$ is not a Cauchy sequence, then for some $\varepsilon > 0$ there exist two sequences $\{m_k\}$ and $\{n_k\}$ of positive integers with $n_k > m_k > k$, such that the following sequences tend to ε^+ as $k \to +\infty$:

$$d\left(x_{2m_k}, x_{2n_k}\right), \quad d\left(x_{2m_k}, x_{2n_k-1}\right), \quad d\left(x_{2m_k+1}, x_{2n_k}\right),$$
$$d\left(x_{2m_k-1}, x_{2n_k+1}\right), \quad d\left(x_{2m_k+1}, x_{2n_k+1}\right), \quad \ldots$$

REMARK 7. Lemma 6 is true without the hypothesis that the sequence $\{d(x_{n+1}, x_n)\}$ is non-increasing. In that case one can get that the following sequences tend to ε^+ as $k \to +\infty$:

$$d\left(x_{m_k}, x_{n_k}\right), \quad d\left(x_{m_k}, x_{n_k-1}\right), \quad d\left(x_{m_k+1}, x_{n_k}\right),$$
$$d\left(x_{m_k-1}, x_{n_k+1}\right), \quad d\left(x_{m_k+1}, x_{n_k+1}\right), \quad \ldots$$

In this paper, besides already mentioned problems with using non-decreasing functions in Wardowski-type results, we show how generalizations of such results can be derived in the framework of metric-like spaces, using just strict monotonicity of the "control" function F. Modifying the original Wardowski's example, we show that the obtained results are stronger than Banach-type ones.

Results

In this part of the work, φ will be a function that maps $(0, +\infty)$ to itself and for which $\liminf_{s\to t+} \varphi(s) > 0$ is fulfilled for each $t \geq 0$, while F will be a strictly increasing function that maps $(0, +\infty)$ to $\mathbb{R}$. We aim to generalize and improve all three results from (Iqbal & Rizwan, 2020) by replacing the metric d and the w-distance w with a metric-like μ. Concerning the function F, just its strict monotonicity will be assumed. This will also extend D. Wardowski's result from (Wardowski, 2018).

THEOREM 8. Let (X, μ) be a complete metric-like space and $T : X \to X$. If φ and F are functions with the properties stated above, and such that, for all $x, y \in X$,

$$x \neq y \text{ and } \mu(Tx, Ty) > 0 \text{ implies } \varphi(\mu(x, y)) + F(\mu(Tx, Ty)) \leq F(\mu(x, y)), \tag{3}$$

then T has a unique fixed point in X.

Proof. We first prove the uniqueness of a possible fixed point. Indeed, if x and y be two distinct fixed points of the mapping T, then both conditions would be met and still it would hold that $\varphi(\mu(x, y))+F(\mu(x, y)) \leq F(\mu(x, y))$, which is a contradiction with $\varphi(\mu(x, y)) > 0$.

Similar as in the case of metric spaces, the continuity of mapping T follows from the contractive condition (2); however, due to the definition of limits in metric-like spaces (see Definition 3), the proof is a bit different. Namely, we have to prove that, for every sequence $\{x_n\}$ in X, $\mu(x_n, x) \to \mu(x, x) = 0$ as $n \to +\infty$ implies that $\mu(Tx_n, Tx) \to 0$ as $n \to +\infty$.

But it follows from the contractive condition (2) that $\mu(Tx_n, Tx) \leq \mu(x_n, x)$, implying that $\mu(Tx_n, Tx) \to 0$ as $n \to +\infty$. It remains to prove that $\mu(Tx_n, Tx) \to \mu(Tx, Tx)$, which will follow if we show that $\mu(Tx, Tx) = 0$. However, $\mu(Tx, Tx) \leq 2\mu(Tx, Tx_n)$ according to the triangle relation. Thus, we have proved that the continuity follows from the contractive condition.

In order to prove the existence of at least one fixed mapping point of T, starting from an arbitrary point $x_0 \in X$, form the corresponding Picard sequence $\{x_n\}$. If, for some k, $x_k = x_{k-1}$ holds, then according to the first part of the proof, x_{k-1} is a unique fixed point of T. Hence, assume that $x_n \neq x_{n-1}$ for each $n \in \mathbb{N}$. Then, putting $x = x_{n-1}$ and $y = x_n$ into the contractive condition (3), it directly follows that the sequence $\mu(x_n, x_{n+1})$

is non-increasing and so it has a limit as $n \to +\infty$. If we denote it by μ^*, using the property of strict monotonicity of the function F, we get a contradiction with $\varphi(\mu^*) > 0$. Then, according to Lemma 5, assuming that the constructed sequence $\{x_n\}$ is not a Cauchy sequence, the conditions for applying of Lemma 6 are fulfilled. Namely, by putting $x = x_{n_k}$, $y = x_{m_k}$ we get

$$\varphi(\mu(x_{n_k}, x_{m_k})) + F(\mu(x_{n_{k+1}}, x_{n_k+1})) \leq F(\mu(x_{n_k}, x_{m_k})).$$

Passing to the limit as $k \to +\infty$, we get a contradiction with $\liminf_{s \to t+} \varphi(s) > 0$.

Since $\{x_n\}$ is a Cauchy sequence in the complete metric-like space (X, μ), then there exists a (unique) point $x^* \in X$ such that

$$\lim_{m,n \to +\infty} \mu(x_n, x_m) = \lim_{n \to +\infty} \mu(x_n, x^*) = \mu(x^*, x^*) = 0.$$

Since T is continuous, $Tx^* = x^*$ holds. □

We now state several consequences of our Theorem 8.

Putting $\varphi(t) = \tau$ for each $t > 0$ in the contractive condition (3) of Theorem 8, where τ is a positive constant, we get the main result of D. Wardowski from (Wardowski, 2012), but in the framework of metric-like spaces and with a weaker assumption on the function F:

Corollary 9. Let (X, μ) be a complete metric-like space and $T : X \to X$. If $F : (0, +\infty) \to \mathbb{R}$ is a strictly increasing function, such that, for all $x, y \in X$,

$$x \neq y \text{ and } \mu(Tx, Ty) > 0 \text{ implies } \tau + F(\mu(Tx, Ty)) \leq F(\mu(x, y)),$$

then T has a unique fixed point in X.

The following example (which is adapted from (Wardowski, 2012), Example 2.5) shows that our Corollary 9 is stronger than the Banach-type fixed point result in metric-like spaces.

Example. Consider the set $X = \{S_n : n \in \mathbb{N}\}$, where $S_n = \dfrac{n(n+1)}{2}$, and let $\mu(x, y) = \max\{x, y\}$ for $x, y \in X$. Then, (X, μ) is a complete metric-like space (of course, it is not a metric space). Let a mapping $T : X \to X$ be given by $TS_1 = S_1$ and $TS_n = S_{n-1}$ for $n > 1$.

We show first that T is not a Banach-type contraction in (X, μ). Indeed, it is

$$\lim_{n\to+\infty} \frac{\mu(TS_n, TS_1)}{\mu(S_n, S_1)} = \lim_{n\to+\infty} \frac{\max\{S_{n-1}, 1\}}{\max\{S_n, 1\}} = \lim_{n\to+\infty} \frac{S_{n-1}}{S_n}$$
$$= \lim_{n\to+\infty} \frac{\frac{1}{2}(n-1)n}{\frac{1}{2}n(n+1)} = 1,$$

which means that $\mu(TS_n, TS_1) \le \lambda\, \mu(S_n, S_1)$ cannot hold for any $\lambda < 1$ and all $n \in \mathbb{N}$.

Now, we show that the contractive condition of Corollary 9 holds if we take $F(t) = t + \log t$ and $\tau = 1$. In this case, this condition can be rewritten as

$$\frac{\mu(Tx, Ty)}{\mu(x, y)} e^{\mu(Tx,Ty)-\mu(x,y)} \le e^{-1},$$

for all $x, y \in X$ with $x \ne y$ and $\mu(Tx, Ty) > 0$, i.e., in our example, as

$$\frac{\max\{TS_m, TS_n\}}{\max\{S_m, S_n\}} e^{\max\{TS_m,TS_n\}-\max\{S_m,S_n\}} < e^{-1}.$$

Note that $TS_m \ne TS_n$ holds (for $m > n$) if and only if one of the following holds: 1° $m > 2$, $n = 1$ or 2° $m > n > 1$. Consider separately these two cases.

1° In this case we have

$$\frac{\max\{TS_m, TS_1\}}{\max\{S_m, S_1\}} e^{\max\{TS_m,TS_1\}-\max\{S_m,S_1\}} = \frac{S_{m-1}}{S_m} e^{S_{m-1}-S_m} < e^{-m} < e^{-1}.$$

2° Similarly, in this case it is

$$\frac{\max\{TS_m, TS_n\}}{\max\{S_m, S_n\}} e^{\max\{TS_m,TS_n\}-\max\{S_m,S_n\}} = \frac{S_{m-1}}{S_m} e^{S_{m-1}-S_m} < e^{-m} < e^{-1}.$$

Hence, all conditions of Corollary 9 are fulfilled and the conclusion follows.

Remark 10. Since every partial metric space, in the sense of (Matthews, 1994), is also a metric-like space, Theorem 8 and Corollary 9 are also true in the class of partial metric spaces.

Remark 11. Since convergence, Cauchyness, completeness and continuity are defined in the same way for all following classes of spaces: partial, metric-like, partial b-metric (Shukla, 2014) and b-metric-like spaces (Alghamdi et al, 2013), then Theorem 8 can most likely be formulated and proved for all these classes of spaces, including the most general one—b-metric like spaces.

REMARK 12. Just like Theorem 2.1, the other two Theorems 2.2 and 2.3 from (Iqbal & Rizwan, 2020) can be formulated and proved within the class of metric-like spaces. And then only *strict* growth of the function F has to be assumed. One can use some function φ or a given constant τ as in our Theorem 8 or Corollary 9.

REMARK 13. Finally, note that the authors of (Iqbal & Rizwan, 2020), in the examples and applications at the end of the paper, used just functions with *strict* growth, in contrast with the theoretical results in the paper which they claim to hold when a non-decreasing function F is used. Moreover, in all these examples and applications, the only function F that is used is $F(t) = \ln t$, which is trivially known to produce only very well-known results which can be treated in a classical way, without using the ideas from the papers (Wardowski, 2012) and (Wardowski, 2018). That was also one of our motivations to consider and discuss the work (Iqbal & Rizwan, 2020).

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Критические замечания о статье «О существовании решения дифференциального уравнения второго порядка через результаты о неподвижной точке для нелинейных F-сжатий с использованием w_0-дистанцией»

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РУБРИКА ГРНТИ: 27.00.00 МАТЕМАТИКА,
27.25.17 Метрическая теория функций,
27.39.15 Линейные пространства, снабженные топологией, порядком и другими структурами

ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: В этой статье представлено несколько критических замечаний относительно статьи, написанной в 2020 году Iqbal & Rizwan: Existence of the solution to second order differential equation through fixed point results for nonlinear F-contractions involving w_0-distance.

Методы: Общепринятые теоретические методы функционального анализа.

Результаты: Доказано, что использование ими неубывающей "управляющей" функции F вместо строго возрастающей в результатах типа Вардовского обычно приводит к противоречиям.

Выводы: Показано, что такие результаты могут быть получены в более общем классе пространств подобных метрическим, где строгая монотонность является единственным условием, которое необходимо наложить на функцию F. Приведен пример, показывающий, что полученные результаты сильнее классических.

Ключевые слова: F-сжатие, неподвижная точка, метрическое пространство, строго возрастающая функция.

Критичке напомене о чланку „Постојање решења диференцијалне једначине другог реда помоћу резултата о непокретној тачки F-контракција kористењем w_0-дистанцу"

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ОБЛАСТ: математика
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: У овом раду изнето је неколико критичких напомена у вези са радом Iqbal & Rizwan: Existence of the solution to second order differential equation through fixed point results for nonlinear F-contractions involving w_0-distance, из 2020. године.

Методе: Конвенционалне теоријске методе функционалне анализе.

Резултати: Показано је да њихова употреба неопадајуће „контролне" функције F уместо стриктно растуће у резултатима типа Вардовског обично производи контрадикције.

Закључак: Такви резултати могу се добити у општијој класи метричких простора, где је строга монотоност једина претпоставка која се мора наметнути функцији F. Приказан је пример који показује да су добијени резултати јачи од класичних.

Кључне речи: F-контракција, фиксна тачка, простор сличан метрици, строго растућа функција.

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