Utilizing a hybrid decision-making approach with fuzzy and rough sets on linguistic data for analyzing voting patterns

Sandip Majumder[a], *Samarjit* Kar[b]

National Institute of Technology Durgapur, Department of Mathematics, Durgapur, West Bengal, Republic of India

[a] e-mail: sandip.preseymath.in@gmail.com,
ORCID iD: https://orcid.org/0009-0002-2068-3518

[b] e-mail: skar.maths@nitdgp.ac.in, **corresponding author**,
ORCID iD: https://orcid.org/0000-0002-5503-9338

https://doi.org/10.5937/vojtehg72-48652

FIELD: mathematics, computer sciences
ARTICLE TYPE: original scientific paper

Abstract:

Introduction/purpose: The significance of studying voting behaviour is underscored by its ability to gauge the continuity or divergence of electoral politics from historical trends, elucidating the real impact of the transformative ballot box, and contributing to the examination of democracy as a value among both masses and elites. Additionally, it aids in comprehending the intricate process of political socialization.

Methods: An inherent strength of the rough set lies in its reliance solely on raw data, devoid of external inputs. The decision-theoretic rough set framework, an evolution of the rough set, has garnered widespread application across diverse domains, serving as a proficient tool for acquiring knowledge, particularly in navigating situations marked by vagueness and uncertainty. Despite the proliferation of mathematical models designed to discern people's voting behavior, a decision-based rough set recommendation remains noticeably absent in existing literature. This paper introduces an innovative three-way decision approach grounded in linguistic information for identifying voting behavior. The proposed approach is based on a hybrid probabilistic rough fuzzy model incorporating linguistic information and providing insights into voting patterns.

Results: The three-way decision hybrid models are tested on people and a highly satisfactory result was achieved for identifying their voting behaviours. The justification of results was validated through the mathematical process.

Conclusion: A practical illustration is provided to highlight the importance of this hybrid model and to confirm its usefulness in identifying and forecasting voting behaviour.

Keywords: linguistic term, rough set, fuzzy set, three-way decision, voting behaviour.

Introduction

The fuzzy set and the rough set are two different but complementary approaches for modelling uncertainty and vagueness in knowledge (Dubois & Prade, 1990; Wong & Ziarko, 1987). Zadeh (1965) introduced fuzzy sets to handle uncertainties in datasets. Pawlak (1982) proposed the concept of a rough set. The main advantage is that each one is potentially able to deal with two different types of uncertainty, vagueness for fuzzy sets and indiscernibility for rough sets. Equivalence relation generates the indiscernibility between objects. In rough sets, crisp sets are approximated using equivalence classes which also allow us to eliminate irrelevant attributes. In the fuzzy set theory, indiscernibility between objects is not used, which may be considered as a class with unsharp boundaries. Each of these concepts appears to have its own set of constraints and limitations.

Deviation and extension are two views that provide distinct generalizations of rough sets from the classical set theory. For interpreting rough sets and fuzzy sets, Klir (1994) compared the roles played by non-classical logic which are both extension and deviation of classical logic. In the framework of the set theory, it can be achieved with the combination of a rough set and a fuzzy set. An extended notation of a rough set, namely a rough fuzzy set can be obtained by introducing upper and lower approximations in fuzzy sets (Yao, 1997). Alternatively, by replacing an equivalence relation with a fuzzy similarity relation, one can get a fuzzy rough set. By interpreting the membership function in terms of conditional probability, one can treat a rough set as a special class of fuzzy sets (Wong & Ziarko, 1987). A salient feature of both models is that the theoretic operators, interpretation, and formulation of the membership function are embodied in the theory. Moreover, rough set operators are not truth-functional.

When a rough set and a fuzzy set are put together in a hybrid model, these offer a robust framework to deal with any kind of uncertain, incomplete and complex information. In many contexts, it enhances the quality of the decision-making process. The combination of the rough set and fuzzy set theories has been successfully applied in expert systems, data mining, knowledge acquisition, process control, etc. Using the rough set theory, Sharma et al (2018, 2022) developed various hybrid methods to create decision rules useful for real-world problems. Pal & Kar (2019) represent time series forecasting for stock market prediction through data discretization by fuzzistics and rule generation by the rough set theory. Tufail et al (2022) proposed a novel multiple attributes group decision-making algorithm with the combination of a rough set and a bipolar soft set. Saqlain & Saeed (2024) unravelled the power of similarity measures in multi-polar interval-valued intuitionistic fuzzy soft sets.

Research in multiple attribute group decision-making problems using fuzzy sets, rough sets, and soft sets has increased in recent years (Gurmani et al, 2023). Donbosco & Ganesan (2022) proposed a rough neutrosophic matrix and its application to the MCDM problem for selecting the best building construction site. Narang et al (2023) introduced a fuzzy extension of the MEREC method using a parabolic measure and its applications. Zhan & Wang (2019) defined five forms of the soft covering based rough set and described the relationship between soft rough sets and soft covering based rough sets. Zhan & Wang (2019), Zhan & Sun (2019) and Zhang & Zhan (2019) defined four different fuzzy soft β covering based fuzzy rough sets and contributed fuzzy soft β-coverings. Yang (2022) proposed a fuzzy covering based rough set over a dual universe as an extension of the single universe concept.

Motivation

The examination of voting behaviour typically relies on insights derived from sample surveys, particularly when analysing patterns of election results at the state or country level. However, such aggregated data proves inadequate for scrutinizing individual voter dynamics. A comprehensive understanding of the factors influencing voting necessitates information on individuals, encompassing voting behaviour, attitudes, beliefs, and personal characteristics. Recognizing the impracticality of acquiring such data for every electorate member, conventional practice involves sampling individuals from the population for interviews. Subsequently, survey data undergoes processing and storage conducive to computer-assisted analysis, focusing on elucidating and providing insights into political opinions and patterns of electoral behaviour.

Konlan (2017) introduced a case study in the volta region of Ghana which provides some prediction for identifying voting behaviour. After reviewing the related literature, it revels that some key attention is required. Several scholars have made several approaches to improve the identification of voting behaviour. But their response and effort did not give convincing results. Lees-Marshment (2019) argued that the growing mixed findings are worth scholarly attention as they may lead to poor policy implementation and missed guidance. Bukari et al (2022) introduced the role of social media in the determination of voting behaviour.

The literature review shows that identifying voting behavior is not adequately represented by any existing mathematical framework. This paper delves into the realm of voting behaviour identification, utilizing fuzzy logic, the rough set theory and its expanded applications, including probabilistic rough sets and decision-theoretic rough sets. The objective is to develop a three-way decision model for distinguishing voting behaviour, informed by linguistic data, concerning attribute values. In instances where individuals may not provide precise quantitative descriptions, opinions are expressed using linguistic terms such as "not so good," "good," or "very good."

Neutrosophic logic is also used to deal with incomplete and indeterminate data. The neutrosophic determinacy component (truth, falsehood) and the indeterminacy component exhibit symmetry as the falsehood and the truth look the same and they behave in a symmetrical way with respect to the indeterminacy element which provides a line of symmetry. Sometimes it seems that neutrosophic logic is more generic than fuzzy intuitionistic logic. When it comes to real life scenarios, there are so many cases where intuitionistic fuzzy logic seems more logical than neutrosophic logic. Consider a following situation in neutrosophic logic.

(T,I,F) : (1,1,0) , (1,0,1) , (1,1,1) , (0,0,0) , (0,1,1)

Such a situation is not possible in a real life scenario. But such a scenario will not appear in intuitionistic fuzzy logic.

The main aim of this paper is to contribute some knowledge by proposing a hybrid fuzzy rough empirically testing model which deals with the voting behavior of people in political marketing in India. As a way forward measure, we have introduced a fuzzy logic operator in the model to examine the intention of the voting behaviour of people based on their linguistic information. With the right kind of implementation of our model, political parties can improve their electoral outcome.

Background and research issues

This section provides a succinct overview of the key concepts in the rough set theory, specifically the classical rough set proposed by Pawlak (1982), the probabilistic rough set, the three-way decision based on the probabilistic rough set, and the decision-theoretic rough set model grounded in the Bayesian decision-making process (Hu, 2014; Ziarko, 1993; Yang & Yao, 2012; Yao & Wong, 1992; Yao & Deng, 2014).

Classical rough set

For an information system $\pi = (\tau, B, V, \varphi)$. τ is a finite and non-empty set of objects called the universe. B is a finite set of attributes. V is a set of the values with φ: $\tau \times B \rightarrow V$ as an information function.

Subsequently, for any $F \subseteq B$, the indiscernible relation IND (F) on τ is denoted as

$$IND(F) = \{(x, y) \in \tau \times \tau | f(x) = f(y), \forall f \in F\}$$

Clearly, IND (F) is the equivalence relation on τ. Hence, it leads to a partition of τ.

For any $Y \subseteq \tau$, the lower and upper approximations of Y are shown below.

$\underline{Apr}(Y) = \{x \in \tau | [y] \subseteq Y\}$

$\overline{Apr}\ (Y) = \{x \in \tau | [y] \cap Y \neq \emptyset\}$

It can further be partitioned into three disjoint regions such as:

POSITIVE (Y) = $\underline{Apr}\ (Y)$

BOUNARY (Y) = $\overline{Apr}\ (Y)$ – $\underline{Apr}\ (Y)$

NEGATIVE (Y) = τ −$\overline{Apr}$ (Y).

Hence, if $y \epsilon$ POSITIVE (Y), then y belongs to the concept Y for sure. If $y \epsilon$ NEGATIVE (Y), then y certainly does not belong to Y. If $y \epsilon$ BOUNARY(Y), then y may or may not belong to Y.

Probabilistic rough set theory with a three-way decision framework

While classical rough sets offer distinct decision regions, they struggle with making decisions for the majority of objects. Addressing this limitation, Wong & Ziarko (1987) introduced the probabilistic rough set model, aiming to reduce the boundary region and expand the positive and negative regions. This adjustment relies on the threshold values.

Let $< \tau, E, P >$ be a probabilistic approximation space, where P is the probabilistic measure defined on any subset of the universal set τ. Then for any $y \in Y$, an overlap between the equivalence class $[y]$ and the set Y is defined in the form of conditional probability as

$Pr(Y|[y]) = \frac{|Y \cap [y]|}{|[y]|}$,

where $|.|$denotes the cardinality. (1)

Majumnder, S. et al, Utilizing a hybrid decision-making approach with fuzzy and rough sets on linguistic data for analyzing voting patterns, pp.605-629

Using the threshold pair (α, β) with $0 \leq \delta < \gamma \leq 1$, the lower and upper approximations of the concept Y are defined as follows:

$$\underline{Apr}_{(\gamma,\delta)}(Y) = \{y \in \tau : Pr(Y|[y]) \geq \gamma\}$$
$$= \cup \{[y] \in Y/E : Pr(Y|[y]) \geq \gamma\}$$
$$\overline{Apr}_{(\gamma,\delta)}(Y) = \{y \in \tau : Pr(Y|[y]) > \delta\}$$
$$= \cup \{[y] \in Y/E : Pr(Y|[y]) > \delta\}$$

Based on (γ, δ) - the lower and upper approximation, the probabilistic three-way decision regions are given by

$$POS_{(\gamma,\delta)}(Y) = \underline{Apr}_{(\gamma,\delta)}(Y)$$
$$= \{y \in \tau : Pr(Y|[y]) \geq \gamma\} \quad (2)$$
$$NEG_{(\gamma,\delta)}(Y) = \tau \setminus \overline{Apr}_{(\gamma,\delta)}(Y)$$
$$= \{y \in \tau : Pr(Y|[y]) \leq \delta\} \quad (3)$$
$$BND_{(\gamma,\delta)}(Y) = \overline{Apr}_{(\gamma,\delta)}(Y) \setminus \underline{Apr}_{(\gamma,\delta)}(Y)$$
$$= \{y \in \tau : \delta < Pr(Y|[y]) < \gamma\} \quad (4)$$

The conditional probability of any object may be considered as a confidence level which an object is having the similar description as y belongs to Y. If $\Pr(Y|[y]) \geq \gamma$, i.e, the confidence level is greater than or equal to γ, then we accept y to be in Y, the same is rejected to be in Y if the confidence level is lesser than or equal to δ, i.e., $Pr(Y|[y]) \leq \delta$. If $\delta < Pr(Y|[y]) < \gamma$, then the decision about y to be in Y may be deferred. For $\gamma = 1$ and $\delta = 0$, the probabilistic rough set model coincides with the classical rough set model. For $\gamma = \delta$, one can get the probabilistic two-way decision model.

Decision-theoretic rough set

The probabilistic rough set model faces challenges in determining the threshold pair (γ, δ). To overcome this, the decision-theoretic rough set model emerges as a straightforward application of the Bayesian decision theory. This model provides a systematic procedure for threshold parameter determination, offering a practical and theoretical foundation for probabilistic rough set applications for a given object $y \in$Y.

Let $\pi = \{e_1, e_2, e_3, \ldots\ldots\ldots, e_m\}$ be a finite set of m possible states and $\Sigma = \{b_1, b_2, b_3, \ldots\ldots, b_n\}$ be a set of possible actions. Hence, an $m \times n$, matrix of the loss function can be constructed. If the object is in the state e_r, then $\mu(b_k| e_r)$ represents the loss for selecting the action b_k and $P(e_r|y)$ represents the conditional probability of y belongs to the state

e_r.When the action $b_k$is taken for the object y, the expected risk involved with the action b_k is shown next.

G$(b_k|y)$ = $\sum_{r=1}^{m} \mu(b_k|\, e_r)$ Pr$(e_r|y)$.

Let $Y \subseteq \tau$, then with respect to Y, a set of two states, π = {Y, Y^c} can be constructed and the set of actions is given by $A = \{\, b_P, b_N, b_B\}$ where b_P represent the action for classifying an object $y \in$ POSITIVE (Y), b_N represent the action for classifying an object $y \in$ NEGATIVE (Y) and b_B indicate the action for classifying an object $y \in$ BOUNDARY (Y).Thus, a 3×2 matrix of loss function can be constructed as shown in Table 1 where $\mu(b_P/Y)$ =μ_{PP}, $\mu(b_N/Y)$ = μ_{NP} and μ (b_B/Y) = μ_{BP} represent the loss incurred for taking the action b_P,b_N,b_B, respectively, when an object belongs to Y. Similarly, μ (b_P/Y^c) =μ_{PN}, μ (b_N/Y^c) = μ_{NN}, μ (b_B/Y^c) = μ_{BN} represent the loss incurred for taking the action b_P,b_N,b_B, respectively, when an object belongs to Y^c.

Now, the expected losses for taking different actions when the objects are in [y] can be written as

G $(b_P/[y])$ = μ_{PP} Pr $(Y/[y])$ + μ_{PN}Pr $(Y^c/[y])$

G $(b_B/[y])$ = μ_{BP} Pr $(Y/[y])$ + μ_{BN} Pr $(Y^c/[y])$

G $(b_N/[y])$ = μ_{NP} Pr $(Y/[y])$ + μ_{NN} Pr $(Y^c/[y])$

Now, applying the Bayesian decision procedure, the following rules associated with minimum risk are obtained:

(P): If G $(b_P/[y])$ ≤G $(b_B/[y])$ and G $(b_P/[y])$ ≤G $(b_N/[y])$, then decide $y \in$ POSITIVE(Y).

(N): If G $(b_N/[y])$ ≤G $(b_B/[y])$ and G $(b_N/[y])$ ≤G $(b_P/[y])$, then decide $y \in$ NEGATIVE (Y).

(B): If G $(b_B/[y])$≤G $(b_N/[y])$ and G $(b_B/[y])$ ≤G $(b_P/[y])$, then decide $y \in$ BOUNDARY (Y).

In order for each object to be classified into only one region, the tribreaker rule should be considered. The loss function inequality is considered as

μ_{PP} ≤ μ_{BP}< μ_{NP} and μ_{NN} ≤ μ_{BN}< μ_{PN}, with

$(\mu_{NP} - \mu_{BP})\,(\mu_{PN} - \mu_{BN}) > (\mu_{BP} - \mu_{PP})\,(\mu_{BN} - \mu_{NN})$

Under the special condition of the loss function, the decision rules (P-R), (N-R) and (B-R) can be formulated as follows:

(P-R): If Pr $(Y/[y]) \geq \gamma$, then y belongs to POSITIVE(Y). (5)

(N-R): If Pr $(Y/[y]) \leq \delta$, then y belongs to NEGATIVE (Y). (6)

Majumnder, S. et al, Utilizing a hybrid decision-making approach with fuzzy and rough sets on linguistic data for analyzing voting patterns, pp.605-629

(B-R): If δ< Pr $(Y/[y])$ < γ, then y belongs to BOUNDARY(Y), (7)

where the threshold α and β are given by

$$\gamma = \frac{(\mu_{PN} - \mu_{BN})}{(\mu_{PN} - \mu_{BN}) + (\mu_{BP} - \mu_{PP})} \quad (8)$$

$$\delta = \frac{(\mu_{BN} - \mu_{NN})}{(\mu_{BN} - \mu_{NN}) + (\mu_{NP} - \mu_{BP})} \quad (9)$$

The parameters γ and δ denote the regions and show the associated risk in classification. The parameter γ makes a division between the positive region and the boundary region. The parameter δ similarly creates a division between the boundary region and the negative region.

The parameters of the decision-theoretic rough set model are derived methodically from loss functions, providing a theoretical foundation and practical interpretation for the probabilistic rough set. This approach is also readily applicable to real-world problems involving profit, loss, cost, risk, and other variables.

Fundamentals of the fuzzy set theory

The fuzzy set theory was first presented in 1965 by Professor L.A. Zadeh (1965) as an expansion of the classical set theory. Uncertain boundaries are accommodated by fuzzy sets which use linguistic variables to describe imprecise concepts. Fuzzy sets have thus evolved as a substitute approach to handling uncertainty (Klement & Schwyhla, 1982; Mazandarani et al, 2018; Sugeno, 1985; Wu & Xu, 2016; Pawlak, 1985).

The theory introduces the notion of a membership function, assigning elements a degree of membership within the interval [0, 1]. Let 'Y' denotes the universe of discourse and $m_{\tilde{Q}}$ (y) is the membership function for fuzzy sets $\tilde{Q}$; then, $m_{\tilde{Q}}$ (y) maps each element of Y to the interval [0, 1], i.e.

$m_{\tilde{Q}}$ (y): Y → [0, 1].

Henceforth, the set $\tilde{Q}$ defined on Y is further obtained as $\tilde{Q}$= {(y,$m_{\tilde{Q}}$ (y)) | y∈Y}. In case of an example, let Y = {y_1,y_2,y_3, y_4, y_5 } be the reference set of students and $\widetilde{Q}$ be the reference set of "smart" students, where "smart" is fuzzy term and represented by

$\tilde{Q}$ = {(y_1, 0.6), (y_2, 0.7), (y_3, 1), (y_4, 0.5), (y_5, 0.3)}.

Here, $\widetilde{Q}$ indicates that the smartness of $y_1$is 0.6, y_2 is 0.7, etc. As a result, the membership function gives an indication of how similar an element is to a fuzzy set. Since the membership function is unique to each assessor or group of assessors, it is evidently subjective. In such case, for each y ∈Y, the assessor is able to find an $m_{\tilde{Q}}$ (y).

For a crisp set, the membership function can be obtained as follows:

$m_{\tilde{Q}}\ (y) = \begin{cases} 1, & \text{if } y \in \mathrm{Q} \\ 0, & \text{if } y \notin \mathrm{Q} \end{cases}$

It shows that the crisp set has sharp boundaries, but the fuzzy set contains vague boundaries.

Basic terminology in the fuzzy set

1.μ-cut: A fuzzy set defined on Y and any number $\mu \in [0, 1]$, the μ-cut is the crisp set $\tilde{Q}_{\mu}= \{y|m_{\tilde{Q}}\ (y) \geq \mu\}$ and a strong μ-cut is the set $\tilde{Q}_{\mu*} = \{\ y|m_{\tilde{Q}}\ (y) > \mu\}$

2. Level set of $\tilde{Q}$: All possible levels $\mu \in [0,1]$ that represent distinct μ-cuts of the associated fuzzy set $\tilde{Q}$is denoted as a level set of Q.

$L(\tilde{Q}_{\mu}) = \{|m_{\tilde{Q}}\ (y) = \mu\}$, for some $y \in Y$.

3. Support: For the fuzzy set $\tilde{Q},$ its support is a crisp set defined as s $(\tilde{Q}) = \{\ y|m_{\tilde{Q}}\ (y)\ \geq 0\}$.

4. Normal and subnormal fuzzy sets: The height of the fuzzy set is the maximum value of the membership degree of any fuzzy set. When the height is 1, the fuzzy set $\tilde{Q}$ is normal and subnormal in the case of its height being below 1. The core of a fuzzy set are those y for which $m_{\tilde{Q}}(y) = 1$.

5. Convex fuzzy set: In the case of the fuzzy set, $\tilde{A}$ is convex if $m_{\tilde{A}}$ (r (y_1) + (1 − r) $(y_2)) \geq$ min $\{m_{\tilde{Q}}\ (y_1), m_{\tilde{Q}}(y_2)\}$, $y_1, y_2 \in Y$, r ∈ [0, 1].

6. Cardinality: In the case of a finite fuzzy set $\tilde{Q}$, the cardinality $|\tilde{Q}|$ is defined as $|\tilde{Q}| = \sum_{y \in \mathrm{Y}} m_{\tilde{Q}}\ (y)$and $||\tilde{Q}|| = \frac{|\tilde{Q}|}{|Y|}$ is called relative cardinality of $\tilde{Q}$.

There are numerous definitions available for fuzziness measurement. These facts are discussed in Dubois & Prade (1982), Klement & Schwyhla (1982) and Sugeno (1985).

Elementary operations with the fuzzy set

Let $\tilde{Q}$,$\widetilde{W}$denote distinct fuzzy sets, then they are equivalent if $m_{\tilde{Q}}(y)= m_{\widetilde{W}}\ (y)\ \forall\ y \in$Y and $\tilde{Q} \subseteq \widetilde{W}$ if $m_{\tilde{Q}}(y) \leq m_{\widetilde{W}}\ (y)$,$\forall\ y \in$Y

1. Union:$\tilde{T} = \tilde{Q} \cup \widetilde{W}$ where, $\tilde{T}= \{(\ y, m_{\tilde{T}}(y))\}$ and, $m_{\tilde{T}}(y)=$ max $\{m_{\tilde{Q}}(y), m_{\widetilde{W}}\ (y))\}$.

2. Intersection: $\widetilde{H}= \tilde{Q} \cap \widetilde{W}$where, $\widetilde{H} = \{(y, m_{\widetilde{H}}(y))\}$ and $m_{\widetilde{H}}(y)=$ min $\{m_{\tilde{Q}}(y), m_{\widetilde{W}}\ (y))\}$.

3. Complement: $\tilde{Q}^c= \{(y, m_{\tilde{Q}^c}(y)\}$ where, $m_{\tilde{Q}^c}(y)= 1 - m_{\tilde{Q}}(y)$.

Significance of the fuzzy set

By measuring fuzziness, fuzzy sets provide a useful framework for expressing ambiguous ideas in plain language and addressing uncertainties. Fuzzy variables enable gradual transitions between states, providing a means to express and manage observation and measurement of uncertainties. The Bayesian decision framework may include fuzzy choice objects in the set of states of reality, enabling the creation of decision rules based on a 3×4 matrix of loss functions.

Remark 1: If there are fuzzy decision objects in the set of states of reality, suppose ∇ = {G, H, I, J} where G, H, I, J∈ F(τ) and satisfy G(y) + H(y) + I(y) + J(y) = 1 for any y∈τ . Here, F (τ) is a set of all fuzzy subsets of τ and a set of actions A = $\{b_P, b_N, b_B\}$, then we can formulate a 3×4 matrix for all the values of the loss function. Based on the loss function inequality, one can formulate Bayesian decision rules.

Operation on a linguistic variable

A variable whose values appear as words or phrases in a natural or artificial language is called a linguistic variable. Words and phrases that are obtained through qualitative or quantitative reasoning are included in these values; they are frequently linked to fuzzy or probabilistic systems, see Deng & Yao (2014), Xu (2005), Pawlak (1985), Zadeh (1965), Klir & Yuan (1995) and Chakraborty (2011).

Let M = {k_α| α = 0, 1, ...,r} be a totally ordered discrete term set. In this case, r+1 is the granularity of the set M. As the M is totally ordered, the law of tracheotomy is operated on it, i.e.,$k_\alpha \geq k_\beta$, $k_\alpha \leq k_\beta$, $k_\alpha = k_\beta$iff α ≥ β, α ≤ β, α = β, respectively.

The linguistic term set with the symmetric subscript M = {k | α = −r, ..., −1, 0, 1, ...,r} is also present. Here, 2r + 1 denotes the granularity of M and k_0 represents an assessment of fairness. $k_{-\mathrm{r}}$ and k_r are the lower and upper limits. In the case of the example below:

M = {k_{-3} = very bad, k_{-2}= bad, k_{-1}= slightly bad, k_0= fair, k_1= slightly good k_2= good, k_3= very good}.

The discrete term set M can further be extended to the continuous term set M^* = { k_λ| $k_{-\mathrm{r}} \leq k_{-\lambda} \leq k_\mathrm{r}$, λ ∈ [−r, r]} where k_λ of $M^*$are the same as $k_\alpha$of M for λ = α.

In M^* *,* index of any term denotes the degree of the term. So, we define a real-valued function from M^*as follows:

M^*= {k_λ | $k_{-\mathrm{r}} \leq k_{-\lambda} \leq k_\mathrm{r}$, λ ∈ [−r, r]} be a continuous linguistic term set

J : M^* → [−r,r] be a real-valued function where J(k_λ) = λ for any $k_\lambda \in M^*$.

It deals with decision-making problems under uncertainty. When $k_\lambda \in$ M, then k_λ is the original term, while λ is the original index. In other words, k_λ is the virtual term and λ is the virtual term index. The decision maker consistently employs the initial linguistic terms for assessing alternatives, while the virtual linguistic term is solely applicable during operations.

In case of a continuous term set M^*, for any $k_\lambda, k_\mu \in M^*$ and α, α_1, $\alpha_2 \in$ [0, 1], the following operational laws hold:

(1) $k_\lambda \pm k_\mu = k_{\lambda \pm \mu}$

(2) α $k_\lambda = k_{\alpha\lambda}$

(3) $(\alpha_1+\alpha_2)k_\lambda = \alpha_1 k_\lambda + \alpha_2 k_\lambda$

(4) α $(k_\lambda \pm k_\mu) = \alpha k_\lambda \pm \alpha k_\mu$.

Three-way decision utilizing linguistic information

In this chapter, our main goal is to identify voting patterns using linguistic term-based information pertaining to all attributes. This involves addressing two key challenges:

(i) Computing the conditional probability for each individual concerning the decision object, where the decision object encompasses all individuals within the sample space.

(ii) Determining the threshold values, denoted as γ and δ, is crucial for the lower and upper approximation, respectively, see Greco et al (2008) and Pauker & Kassirer (1980).

In order to address the first challenge, a conceptual framework for probability within a fuzzy event is introduced, specifically tailored to a linguistic-valued attribute set. This framework enables a nuanced analysis of voting behavior, considering the inherent linguistic expressions associated with each attribute.

Definition Let Q = {(y, m_Q (y)) | $y \in R^n$} is a real-valued fuzzy set. The associated crisp probability of a fuzzy event is denoted as P (Q) $=\sum_y m_Q(y)\mathrm{P}(y)$.

Let Q_γ = { y | m_Q (y) ≥ γ}, then the fuzzy probability of the fuzzy event is P (Q) = {(P (Q_γ, γ)) | γ ∈ [0, 1]}.

Linguistic-valued framework for the information system

Linguistic variables provide the attribute values in an information system. Consider a linguistic-valued information system as follows:

$\varphi_1(y_1,a_1) = k_{-3}$, $\varphi_1(y_2,a_1) = k_1$, $\varphi_1(y_3,a_1) = k_0$,

$\varphi_2(y_1, a_2) = k_1$, $\varphi_2(y_2, a_2) = k_{-2}$, $\varphi_2(y_3,a_2) = k_4$,

$\varphi_3(y_1,a_3) = k_0$, $\varphi_3(y_2,a_3) = k_2$, $\varphi_3(y_3,a_3) = k_{-1}$,

$\varphi_4(y_1,a_4) = k_2$, $\varphi_4(y_2,a_4) = k_0$, $\varphi_4(y_3,a_4) = k_{-2}$

In a table, the information system can be represented as follows:

Table 1 – Linguistic valued information system

U/A	a_1	a_2	a_3	a_4
y_1	k_{-3}	k_1	k_0	k_2
y_2	k_1	k_{-2}	k_2	k_0
y_3	k_0	k_4	k_{-1}	k_{-2}

In case of a fuzzy set B having membership valuem_B (y_1) = 0.5, m_B (y_2) = 0.7 and m_B (y_3) = 0.8 and the probabilistic measure P denotes as P(y_1) = 0.2, P(y_2) = 0.3 and P(y_3) = 0.5 Then, $P(B) = \sum_{i=1}^{3} m(y_i)\, P(y_i) = 0.5 \times 0.2 + 0.7 \times 0.3 + 0.8 \times 0.5 = 0.71$.

Next, the associated real-valued function can be defined as below.

Z: M→ [0, 1]

Z (k_λ) = $\frac{J\,(k_\lambda)}{r-1}$ (10)

where r is the total number of terms in M.

For the symmetric subscript linguistic set Z: M^*= → [0, 1] by

Z (k_λ) = $\frac{|\mathrm{J}(k_\lambda) - \mathrm{J}(k_{-\mathrm{r}})|}{2\mathrm{r}}$ (11)

Here, Z (k_λ) is a continuous mapping having transformation between M^*= and [0, 1].

Proposition

Let M^*= {$k_\lambda | k_{-\mathrm{r}}$ ≤ k_λ≤ k_r, λ ∈ [−r, r]} be a set of continuous linguistic terms 'Z' is the transformation between $M^*$and real-valued over [0, 1]; then

1. Z ($k_{-\mathrm{r}}$) = 0, Z (k_0) = 0.5, Z (k_r) = 1
2. Z is an increasing function over M^*

Proof (1): By definition Z (k_λ) = $\frac{|\mathrm{J}(k_\lambda) - \mathrm{J}(k_{-\mathrm{r}})|}{2\mathrm{r}}$

So, Z ($k_{-\mathrm{r}}$) = $\frac{|\mathrm{J}(k_{-\mathrm{r}}) - \mathrm{J}(k_{-\mathrm{r}})|}{2\mathrm{r}}$ = 0

Z (k_0) = $\frac{|\mathrm{J}(k_0) - \mathrm{J}(k_{-\mathrm{r}})|}{2\mathrm{r}}$ = $\frac{|\,0-(-r)|}{2r}$ = 0.5

Z (k_r) = $\frac{|\mathrm{J}(k_\mathrm{r}) - \mathrm{J}(k_{-\mathrm{r}})|}{2\mathrm{r}}$ = $\frac{|\,r-(-r)|}{2r}$ = 1

(2): Let −r ≤ λ_1 ≤ λ_2≤ r, then, k_{λ_2} ≥ k_{λ_1}

Z (k_{λ_2}) = $\frac{|\mathrm{J}(k_{\lambda_2}) - \mathrm{J}(k_{-\mathrm{r}})|}{2\mathrm{r}}$ = $\frac{\lambda_2+r}{2r}$

Z (k_{λ_1}) = $\frac{|\mathrm{J}(k_{\lambda_1}) - \mathrm{J}(k_{-\mathrm{r}})|}{2\mathrm{r}}$ = $\frac{\lambda_1+r}{2r}$, as λ_2 ≥ λ_1 so $\frac{\lambda_2+r}{2r}$ ≥ $\frac{\lambda_1+r}{2r}$

Hence, Z (k_{λ_2}) ≥ Z (k_{λ_1}), so, 'Z' can be considered as an increasing function over M^* . The middle linguistic label denotes an assessment of 'in difference' and the transformation function Z (k_λ) can further be denoted using Z (k_0) as next.

Proposition

Z (k_λ) = $\frac{J(k_\lambda)}{2r}$+ Z (k_0) = 0.5 + $\frac{J(k_\lambda)}{2r}$

Proof:

Z (k_λ) = $\frac{|J(k_\lambda) - J(k_{-r})|}{2r}$

= $\frac{J(k_\lambda) - J(k_{-r})}{2r}$ as λ ∈ [−r, r]

= $\frac{J(k_\lambda)+r}{2r}$ = 0.5 + $\frac{J(k_\lambda)}{2r}$ = Z (k_0) + $\frac{J(k_\lambda)}{2r}$ [as Z (k_0) = 0.5].

Conditional probability in the linguistic valued information system

Let $D \in F(\tau)$ and $y \in \tau$, then the conditional probability of D with respect to y denoted by

$$P\ (D|y) = \frac{\sum_{a_j \in A} \Delta(\ D(y)\ ,Z\ (\varphi_j(y,a_j)))}{\sum_{a_j} Z\ (\varphi_j(y,a_j))}, y \in \tau, \text{ for all attribute j} \quad (12)$$

where Δ: [0, 1] × [0, 1] →[0, 1] is a fuzzy logic operator (Klir & Yaun, 1995; Zadeh, 1965; Wong & Ziarko, 1987).

We define the fuzzy logic operator as Δ (x, y) = min (x, y).

$$\text{Thus, } P\ (D|y) = \frac{\sum_{a_j \in A} D(y)\ \wedge\ Z\ (\varphi_j(y,a_j))}{\sum_{a_j} Z\ (\varphi_j(y,a_j))}, y\ \in\ \tau, \text{ for all attribute j} \quad (13)$$

We demonstrate this with an example from Table 1.

Let M = = {k_λ| k_{-r} ≤ k_λ≤ k_r, λ ∈ [−4, 4]}

z_{a_1}= Z (φ_1, (y_2,a_1)); z_{a_1}= Z (φ_2, (y_2,a_2)); z_{a_3}= Z (φ_3, (y_2,a_3)); z_{a_4}= Z (φ_4, (y_2,a_4))

$$P\ (D|y_2) = \frac{\sum_{a_j \in A} \Delta(\ D(y_2)\ ,Z\ (\varphi_j(y_2,a_j)))}{\sum_{a_j} Z\ (\varphi_j(y_2,a_j))} = \frac{\sum_{a_j \in A} D(y_2)\ \wedge\ Z\ (\varphi_j(y_2,a_j))}{\sum_{a_j} Z\ (\varphi_j(y_2,a_j))}$$

$$= \frac{D(y_2) \wedge z_{a_1} + D(y_2) \wedge\ z_{a_2} + D(y_2) \wedge\ z_{a_3} + D(y_2) \wedge z_{a_4}}{v_{a_1} + v_{a_2} + v_{a_3} + v_{a_{14}}}$$

$$= \frac{0.7 \wedge z(k_1) + 0.7 \wedge z(k_{-2}) + 0.7 \wedge z(k_2) + 0.7 \wedge z(k_0)}{z(k_1) + z(k_{-2}) + z(k_2) + z(k_0)}$$

$$= \frac{0.7 \wedge\ 0.625\ + 0.7 \wedge\ 0.25\ +\ 0.7 \wedge 0.75\ +\ 0.7 \wedge\ 0.5}{0.625+0.25+0.75+0.5} = \frac{2.075}{2.125} = 0.976$$

Clearly, P(D| y) satisfies the axioms of probability. We can now define the lower and higher approximations with the aid of the conditional probability of a fuzzy event with a linguistic description of the attribute.

Let D∈ F (τ) and 0 ≤ δ < γ ≤ 1 and $y \in\tau$, then
$\underline{Apr}_{\gamma}$ (D) = { $y \in \tau$ | P (D| y) ≥ γ}
$\overline{Apr}_{\delta}$ (D) = { $y \in \tau$ | P (D| y)> δ}.

Next, the three-way decision regions are as shown next (Hu, 2014):

$$POS_{(\gamma,\delta)}(D) = \{y \in \tau| \frac{\sum_{a_j \in A} D(y) \wedge Z(\varphi_j(y,a_j))}{\sum_{a_j \in A} Z(\varphi_j(y,a_j))} \geq \gamma\} \quad (14)$$

$$NEG_{(\gamma,\delta)}(D) = \{y \in \tau| \frac{\sum_{a_j \in A} D(y) \wedge Z(\varphi_j(y,a_j))}{\sum_{a_j \in A} Z(\varphi_j(y,a_j))} \leq \delta\} \quad (15)$$

$$BND_{(\gamma,\delta)}(D) = \{y \in \tau | \delta < \frac{\sum_{a_j \in A} D(y) \wedge Z(\varphi_j(y,a_j))}{\sum_{a_j \in A} Z(\varphi_j(y,a_j))} < \gamma\} \quad (16)$$

When the loss function is represented in terms of a linguistic form, the function J is used to determine the threshold values γ and δ, thus resolving the second difficulty. So, inequality in the loss function is:

J (μ_{PP}) ≤ J (μ_{BP}) < J (μ_{NP}) and J (μ_{NN}) ≤ J (μ_{BN}) < J (μ_{PN}) with the condition
{J (μ_{NP}) – J (μ_{BP})} × {J (μ_{PN}) –J (μ_{BN})} > {J (μ_{BP}) – J (μ_{PP})} × {J (μ_{BN}) – J (μ_{NN})},

Then, $$\gamma = \frac{J(\mu_{PN}) - J(\mu_{BN})}{\{J(\mu_{PN}) - J(\mu_{BN})\} + \{J(\mu_{BP}) - J(\mu_{PP})\}} = \{1 + \frac{J(\mu_{BP}) - J(\mu_{PP})}{J(\mu_{PN}) - J(\mu_{BN})}\}^{-1}$$
(17)
$$\delta = \frac{J(\mu_{BN}) - J(\mu_{NN})}{\{J(\mu_{BN}) - J(\mu_{NN})\} + \{J(\mu_{NP}) - J(\mu_{BP})\}} = \{1 + \frac{J(\mu_{NP}) - J(\mu_{BP})}{J(\mu_{BN}) - J(\mu_{NN})}\}^{-1}$$

The parameters γ and δ define the regions and provide associated risk for classifying an object.

Remark: The main focus is to identify the voting behaviour so that political experts can choose the parameter values γ and δ on the basis of their experience (Karni, 2009; Pauker & Kassirer, 1980).

Real life example

This section gives an example which illustrates the main idea of the paper. Here, 26 people from four different age groups are considered and their linguistic information about different attributes related to their voting behaviour is collected. Using this linguistic information, a political party can get a clear idea regarding the share of the vote.

The voting behaviour in the Indian society is highly diversified in nature and composition, influenced by multiple factors. These factors can be classified into three categories: social, economic, and political. There are also some other factors that make an impact on the electoral behaviour in our society: (i) election campaign, (ii) candidate orientation, (iii) habitation (urban or rural), (iv) factionalism (one of the main features in politics from top to bottom levels), (vi) conditions of the economy at the time of elections, such as inflation, food, unemployment etc, and (vii) political events preceding an election like corruption, war, scandals, etc.

Considering all the above factors, here we select seven factors as conditional attributes, namely A_1, A_2,………, A_7, taking into account the past history of a few elections as well as some burning issues related with the coming election. In this example, we divide voters into four groups: Group-I (age from 18 to under 30), Group – II (age from 30 to under 45), Group – III (age from 45 to under 60), and Group – IV (age over 60). The decisional attribute which we denoted by "C" indicates the vote share in favour of the party. We consider different membership values for different groups, which reflects that the electoral behaviour of different age groups is different. For Group-I, the threshold is taken as $\gamma = 0.8$, $\delta = 0.7$, for Group-II, the threshold is taken as $\gamma = 0.7$, $\delta = 0.55$, for Group-III, the threshold is taken as $\gamma = 0.4$, $\delta = 0.25$, and for Group-IV, the threshold is taken as $\gamma = 0.3$, $\delta = 0.2$.

Due to the non-negative and discrete nature of the linguistic term index in this case, we compute the values using equations (10), (11) and (13) and make decisions by using values of $1 - P(\frac{c}{y_i})$ rather than $P(\frac{c}{y_i})$. Let $P^*(\frac{c}{y_i}) = 1 - P(\frac{c}{y_i})$. Therefore, the region of acceptance (positive region) of a three-way choice can be described as follows:

1. Region of acceptance (Positive region) = $\{y_i \mid P^*(\frac{c}{y_i}) \geq \gamma\}$.
2. Region of rejection (Negative region) = $\{y_i | P^*(\frac{c}{y_i}) \leq \delta\}$.
3. Region of non-commitment (Boundary region) = $\{y_i | \delta < P^*(\frac{c}{y_i}) < \gamma\}$.

Here, the positive region indicates a confirmed vote share in favour of the party, the negative region reflects a confirmed vote share against the party, and the non-commitment region indicates that the vote share may or may not be in favour of the party.

Table 2 – Linguistic valued information related to the voting behaviour

Group	People	A_1	A_2	A_3	A_4	A_5	A_6	A_7	C	P $(\frac{c}{y_i})$	1- P $(\frac{c}{y_i})$
(i) (age from 18 to under 30)	y_1	k_0	k_2	k_2	k_1	k_0	k_1	k_2	0.1	0.25	0.75
	y_2	k_1	k_2	k_3	k_2	k_0	k_1	k_3	0.1	0.20	0.80
	y_3	k_2	k_3	k_2	k_3	k_2	k_1	k_3	0.1	0.17	0.82
	y_4	k_4	k_0	k_1	k_0	k_1	k_0	k_1	0.1	0.22	0.77
	y_5	k_3	k_4	k_1	k_3	k_2	k_1	k_3	0.1	0.16	0.83
	y_6	k_1	k_0	k_1	k_0	k_1	k_0	k_1	0.1	0.40	0.60
(ii) (age from 30 to under 45)	y_7	k_0	k_2	k_2	k_1	k_0	k_1	k_2	0.15	0.37	0.62
	y_8	k_1	k_2	k_3	k_2	k_0	k_1	k_3	0.15	0.30	0.70
	y_9	k_3	k_4	k_1	k_3	k_2	k_1	k_3	0.15	0.24	0.75
	y_{10}	k_2	k_3	k_2	k_3	k_2	k_1	k_3	0.15	0.26	0.73
	y_{11}	k_1	k_0	k_1	k_1	k_2	k_0	k_0	0.15	0.48	0.52
	y_{12}	k_4	k_1	k_1	k_2	k_1	k_1	k_3	0.15	0.32	0.67
(iii) (age from 45 to under 60)	y_{13}	k_2	k_0	k_1	k_0	k_1	k_0	k_0	0.35	0.85	0.15
	y_{14}	k_1	k_0	k_2	k_1	k_0	k_1	k_2	0.35	0.82	0.18
	y_{15}	k_3	k_1	k_2	k_3	k_1	k_0	k_3	0.35	0.58	0.41
	y_{16}	k_4	k_0	k_1	k_0	k_1	k_0	k_1	0.35	0.80	0.20
	y_{17}	k_4	k_2	k_1	k_2	k_2	k_1	k_2	0.35	0.64	0.35
	y_{18}	k_3	k_1	k_1	k_3	k_2	k_0	k_3	0.35	0.58	0.41
	y_{19}	k_2	k_0	k_2	k_2	k_1	k_1	k_2	0.35	0.62	0.38
(iv) Age over 60	y_{20}	k_0	k_2	k_2	k_1	k_1	k_0	k_2	0.4	0.85	0.15
	y_{21}	k_1	k_1	k_0	k_2	k_1	k_0	k_0	0.4	0.92	0.08
	y_{22}	k_4	k_0	k_2	k_2	k_1	k_2	k_3	0.4	0.64	0.35
	y_{23}	k_3	k_1	k_1	k_0	k_0	k_0	k_1	0.4	0.76	0.24
	y_{24}	k_3	k_1	k_2	k_3	k_1	k_0	k_2	0.4	0.70	0.30
	y_{25}	k_4	k_0	k_3	k_2	k_1	k_1	k_3	0.4	0.60	0.40
	y_{26}	k_2	k_0	k_1	k_0	k_1	k_1	k_2	0.4	0.88	0.12

For Group G_1: POSITIVE (G_1) = $\{y_2, y_3, y_5\}$, BOUNDARY (G_1) = $\{y_1, y_4\}$ and NEGATIVE (G_1) = $\{y_6\}$.

For Group G_2: POSITIVE (G_2) = $\{y_8, y_9, y_{10}\}$, BOUNDARY (G_2) = $\{y_7, y_{12}\}$ and NEGATIVE (G_2) = $\{y_{11}\}$.

For Group G_3: POSITIVE (G_3) = $\{y_{15}, y_{18}\}$, BOUNDARY (G_3) = $\{y_{17}, y_{19}\}$ and NEGATIVE (G_3) = $\{y_{13}, y_{14}, y_{16}\}$.

For Group G_4: POSITIVE (G_4) = $\{y_{22},\ y_{24},\ y_{25}\}$, BOUNDARY (G_4) = $\{y_{23}\}$ and NEGATIVE (G_4) = $\{y_{20}, y_{21}, y_{26}\}$.

Remark 1: The selection of a threshold is an important issue in a three-way decision-making problem. There are a few methods to calculate the threshold values. One of them is the Bayesian decision process which we discussed briefly in this paper. Most of the time, field experts also select the threshold values using their experience.

Remark 2: Here, we describe the calculation P $(\frac{c}{y_i})$ for the people y_2 as in Table 2**.**

Here, the granularity r = 5, using (13), we get

P (c|y_2) = $\frac{\sum_{a_j \in A} \mathrm{D}(y_2) \wedge \mathrm{Z}(\varphi_j(y_2,a_j))}{\sum_{a_j} \mathrm{Z}(\varphi_j(y_2,a_j))}$, $y \in \tau$, for all attribute j

$$= \frac{\mathrm{D}(y_2) \wedge z_{a_1} + \mathrm{D}(y_2) \wedge z_{a_2} + \mathrm{D}(y_2) \wedge z_{a_3} + \mathrm{D}(y_2) \wedge z_{a_4} + \mathrm{D}(y_2) \wedge z_{a_5} + \mathrm{D}(y_2) \wedge z_{a_6} + \mathrm{D}(y_2) \wedge z_{a_7}}{z_{a_1} + z_{a_2} + z_{a_3} + z_{a_4} + z_{a_5} + z_{a_6} + z_{a_7}}$$

$$= \frac{0.1 \wedge z(k_1) + 0.1 \wedge z(k_2) + 0.1 \wedge z(k_3) + 0.1 \wedge z(k_2) + 0.1 \wedge z(k_0) + 0.1 \wedge z(k_1) + 0.1 \wedge z(k_3)}{z(k_1) + z(k_2) + z(k_3) + z(k_2) + z(k_0) + z(k_1) + z(k_3)}$$

$$= \frac{0.1 \wedge 0.25 + 0.1 \wedge 0.5 + 0.1 \wedge 0.75 + 0.1 \wedge 0.5 + 0.1 \wedge 0 + 0.1 \wedge 0.25 + 0.1 \wedge 0.75}{0.25 + 0.5 + 0.75 + 0.5 + 0 + 0.25 + 0.75}$$

$$= \frac{0.6}{3} = 0.20$$

Here, equation (10), Z (k_λ) = $\frac{J(k_\lambda)}{r-1}$ is used for calculating $z(k_\mathrm{i})$.

Conclusion

In this paper, a three-way decision model based on linguistic information of people is introduced to identify their voting behaviour. Electoral behavior entails an understanding of the factors and reasons that influence voting patterns. The proposed model addresses the research task by seamlessly integrating the rough set theory and the fuzzy set theory into linguistic information, offering a comprehensive approach. It excels in handling complex datasets, providing valuable insights through a computationally intensive yet effective methodology. A comparative analysis based on age groups indicates that this method is more feasible and effective compared to other approaches. When faced with numerous predetermined cases and a large number of boundary regions, the hybrid model proves advantageous by allowing the adjustment of the threshold parameter, reducing the complexity associated with decision making and facilitating a more efficient classification of cases into the specified regions. The significance of the proposed hybrid model has been justified through its application in a real-life scenario.

If political parties are faced with numerous predetermined cases and a large number of boundary regions, the hybrid model provides a chance to get a favourable result in favour of political parties by allowing the adjustment of the threshold parameters γ and δ. It shows that the determination of voting behaviour allows political parties to create an environment for the voters to perform two distinct roles, evaluation of performance and promote the development. It also provides a nice relationship between the determination of voters' behaviour variables like the personality of a political leader, contingent situation, epistemic value, etc., with the voting intention of people.

The adjustment of the threshold values for preferable results indicates that the determination of the voting behaviour model may be considered an important political party resource that can influence the intention of voting behaviour by allowing the political parties to implement programs and policies to meet the requirements and expectations of voters. Current studies show that the effect of a linguistic valued information system hybrid fuzzy rough model has individual preferences with variable specific values. Hence, our model can be considered as an antecedent to voting intention.

Limitation

Like other research, this model is not without its limitations. The limitation of the proposed model is that the integration of the rough set theory and the fuzzy set theory may result in a computationally intensive model, particularly when dealing with large datasets. Analysis methods and alternative data collection such as longitudinal panel data, structural equation modelling, objective performance, and importance-performance matrix are for future studies to test this study's model. For example, the use of longitudinal data did not allow for examining the determination of voting patterns over time. Future researchers should explore cross sectional data to see the pattern of change in voting intentions over time in different political environments. Furthermore, even though this study does not segment the voter market and examine the effect of a specific segment on the model, it would be interesting for future studies to look at the nuances in specific segments and the effects in different contexts. If the data is collected from first-time voters, it may not give proper results. This study focuses only on Indian individuals. Although within a similar context, when India shares some similar characteristics, a few notable differences may appear in applying the hybrid model. Besides this limitation, this hybrid model provides insightful theoretical and practical implications in determining voting patterns.

Further, we may extend our studies beyond Indian politics, which may consider how voters react in the determination of their voting intentions.

For further studies, it would be more interesting to examine the effect of the proposed model on a specific segment of the voter market. Future work should refine the model, considering additional linguistic variables and exploring its application in diverse domains beyond voting behavior analysis. Future research could further focus on refining the hybrid model by exploring advanced machine learning algorithms, enhancing rule-based system interpretability, and developing mechanisms for dynamic rule adaptation.

References

Bukari, Z., Abdul Hamid, A.B., Md. Som, H., Agbemabiese, G.C. & Quansah, F. 2022. Does political issue matter in voting intention in Ghana? A political marketing perspective. *Cogent Business & Management*, 9(1), art.number: 2034227. Available at: https://doi.org/10.1080/23311975.2022.2034227.

Chakraborty, M.K. 2011. On Fuzzy Sets and Rough Sets from the Perspective of Indiscernibility. In: Banerjee, M. & Seth, A. (Eds.) *Logic and Its Applications. ICLA 2011. Lecture Notes in Computer Science*, 6521. Heidelberg: Springer, Berlin. Available at: https://doi.org/10.1007/978-3-642-18026-2_4.

Deng, X. & Yao, Y. 2014. Decision-theoretic three-way approximations of fuzzy sets. *Information Sciences*, 279, pp.702-715. Available at: https://doi.org/10.1016/j.ins.2014.04.022.

Donbosco, J.S.M & Ganesan, D. 2022. The Energy of rough neutrosophic matrix and its application to MCDM problem for selecting the best building construction site. *Decision Making: Applications in Management and Engineering*, 5(2), pp.30-45. Available at: https://doi.org/10.31181/dmame0305102022d.

Dubois, D. & Prade, H. 1982. A CLASS OF FUZZY MEASURES BASED ON TRIANGULAR NORMS A general framework for the combination of uncertain information. *International Journal of General System*, 8(1), pp.43-61. Available at: https://doi.org/10.1080/03081078208934833.

Dubois, D. & Prade, H. 1990. Rough fuzzy sets and fuzzy rough sets. *International Journal of General Systems*, 17(2-3), pp.191-209. Available at: https://doi.org/10.1080/03081079008935107.

Greco, S., Matarazzo, B. & Słowiński, R. 2008. Parameterized rough set model using rough membership and Bayesian confirmation measures. *International Journal of Approximate Reasoning*, 49(2), pp.285-300. Available at: https://doi.org/10.1016/j.ijar.2007.05.018.

Gurmani, S.H., Chen, H. & Bai, Y. 2023. Multi-attribute group decision-making model for selecting the most suitable construction company using the linguistic interval valued T-spherical 150 fuzzy TOPSIS method. *Applied Intelligence*, 53, pp.11768-11785. Available at: https://doi.org/10.1007/s10489-022-04103-0.

Hu, B.Q. 2014. Three-way decisions space and three way decisions. *Information Sciences*, 281, pp.21-52. Available at: https://doi.org/10.1016/j.ins.2014.05.015.

Karni, E. 2009. A theory of medical decision making under uncertainty. *Journal of Risk and Uncertainty*, 39, pp.1-16. Available at: https://doi.org/10.1007/s11166-009-9071-3.

Klement, E.P. & Schwyhla, W. 1982. Correspondence between fuzzy measures and classical measures. *Fuzzy Sets and Systems,* 7(1), pp.57-70. Available at: https://doi.org/10.1016/0165-0114(82)90040-9.

Klir, G.J. 1994. Multivalued logics versus modal logics: alternative frameworks for uncertainty modelling. In: Wang, P.P. (Ed.) *Advances in Fuzzy Theory and Technology*. Durham, North Carolina, USA: Duke University, Department of Electrical Engineering. ISBN: 978-0964345614.

Klir, G.J. & Yaun, B. 1995. FUZZY SETS AND FUZZY LOGIC: Theory and application. Upper Saddle River, NJ, USA: Prentice Hall PTR. ISBN: 0-13-101171-5.

Konlan, B. 2017. *Predictability of Identity Voting Behaviour, Perceived Exclusion and Neglect and the Paradox of Loyalty: A Case Study of a Conflict Involving the Ewe Group in the Volta Region of Ghana and the NDC-led Administrations*. PhD thesis. Ann Arbor, MI, USA: ProQuest LLC & Nova Southeastern University, College of Arts, Humanities, and Social Sciences [online]. Available at: https://www.proquest.com/openview/060098f4656dfc5bc2e04c1edf71d5a3/1?pq-origsite=gscholar&cbl=18750 [Accessed: 10 January 2024].

Lees-Marshment, J. 2019. Marketing Scholars and Political Marketing: the Pragmatic and Principled Reasons for Why Marketing Academics Should Research the Use of Marketing in the Political Arena. *Customer Needs and Solutions*, 6, pp.41-48. Available at: https://doi.org/10.1007/s40547-019-0091-0.

Mazandarani, M., Pariz, N. & Kamyad, A.V. 2018. Granular Differentiability of Fuzzy-Number-Valued Functions. *IEEE Transactions on Fuzzy Systems*, 26(1), pp.310-323. Available at: https://doi.org/10.1109/TFUZZ.2017.2659731.

Narang, M., Kumar, A. & Dhawan, R. 2023. A fuzzy extension of MEREC method using parabolic measure and its applications. *Journal of Decision Analytics and Intelligent Computing*, 3(1), pp.33-46. Available at: https://doi.org/10.31181/jdaic10020042023n.

Pal, S.S. & Kar, S. 2019. Time series forecasting for stock market prediction through data discretization by fuzzistics and rule generation by rough set theory. *Mathematics and Computers in Simulation*, 162, pp.18-30. Available at: https://doi.org/10.1016/j.matcom.2019.01.001.

Pauker, S.G. & Kassirer, J.P. 1980. The Threshold Approach to Clinical Decision Making. *New England Journal of Medicine*, 302(20), pp.1109-1117. Available at: https://doi.org/10.1056/NEJM198005153022003.

Pawlak, Z. 1982. Rough Sets. *International Journal of Computer & Information Sciences*, 11, pp.341-356. Available at: https://doi.org/10.1007/BF01001956.

Pawlak, Z. 1985. Rough sets and fuzzy sets. *Fuzzy Sets and Systems*, 17(1), pp.99-102. Available at: https://doi.org/10.1016/S0165-0114(85)80029-4.

Saqlain, M. & Saeed, M. 2024. From Ambiguity to Clarity: Unraveling the Power of Similarity Measures in Multi-Polar Interval-Valued Intuitionistic Fuzzy Soft Sets. *Decision Making Advances*, 2(1), pp.48-59. Available at: https://doi.org/10.31181/dma21202421.

Sharma, H.K., Kumari, K. & Kar, S. 2018. Air passengers forecasting for Australian airline based on hybrid rough set approach. *Journal of Applied Mathematics, Statistics and Informatics*, 14(1), pp.5-18. Available at: https://doi.org/10.2478/jamsi-2018-0001.

Sharma, H.K., Singh, A., Yadav, D. & Kar, S. 2022. Criteria selection and decision making of hotels using Dominance Based Rough Set Theory. *Operational Research in Engineering Sciences: Theory and Applications*, 5(1), pp.41-55 [online]. Available at: https://oresta.org/menu-script/index.php/oresta/article/view/151 [Accessed: 10 January 2024].

Sugeno, M. 1985. An introductory survey of fuzzy control. *Information Science*, 36(1-2), pp.59-83. Available at: https://doi.org/10.1016/0020-0255(85)90026-X.

Tufail, F., Shabir, M. & Abo-Tabl, E.-S.A. 2022. A Comparison of Promethee and TOPSIS Techniques Based on Bipolar Soft Covering-Based Rough Sets. *IEEE Access*, 10, pp.37586-37602. Available at: https://doi.org/10.1109/ACCESS.2022.3161470.

Wong, S.K.M & Ziarko, W. 1987. Comparison of the probabilistic approximate classification and the fuzzy set model. *Fuzzy Sets and Systems*, 21(3), pp.357-362. Available at: https://doi.org/10.1016/0165-0114(87)90135-7.

Wu, Z. & Xu, J. 2016. Managing consistency and consensus in group decision making with hesitant fuzzy linguistic preference relations. *Omega*, 65, pp.28-40. Available at: https://doi.org/10.1016/j.omega.2015.12.005.

Xu, Z. 2005. Deviation measures of linguistic preference relations in group decision making. *Omega*, 33(3), pp.249-254. Available at: https://doi.org/10.1016/j.omega.2004.04.008.

Yang, B. 2022. Fuzzy covering-based rough set on two different universes and its application. *Artificial Intelligence Review*, 55, pp.4717-4753. Available at: https://doi.org/10.1007/s10462-021-10115-y.

Yang, X. & Yao, J.T. 2012. Modelling Multi-agent Three-way Decisions with Decision-theoretic Rough Sets. *Fundamenta Informaticae*, 115(2-3), pp.157-171. Available at: https://doi.org/10.3233/FI-2012-647.

Yao, Y.Y. 1997. Combination of rough and fuzzy sets based on α-level sets. In: Lin, T.Y. & Cercone, N. (Eds.) *Rough Sets and Data Mining: Analysis for Imprecise Data*. Boston, MA, USA: Springer, pp.301-321. Available at: https://doi.org/10.1007/978-1-4613-1461-5_15.

Yao, Y. & Deng, X. 2014. Quantitative rough sets based on subsethood measures. *Information Sciences*, 267, pp.306-322. Available at: https://doi.org/10.1016/j.ins.2014.01.039.

Yao, Y.Y. & Wong, S.K.M. 1992. A decision-theoretic framework for approximating concepts. *International Journal of Man-Machine Studies*, 37(6), pp.793-809. Available at: https://doi.org/10.1016/0020-7373(92)90069-W.

Zadeh, L.A. 1965. Fuzzy sets. *Information and Control*, 8(3), pp.338-353. Available at: https://doi.org/10.1016/S0019-9958(65)90241-X.

Zhan, J. & Sun, B. 2019. Covering-based soft fuzzy rough theory and its application to multiple criteria decision making. *Computational and Applied Mathematics*, 38, art.number:149. Available at: https://doi.org/10.1007/s40314-019-0931-4.

Zhan, J. & Wang, Q. 2019. Certain types of soft coverings based rough sets with applications. *International Journal of Machine Learning and Cybernetics*, 10(5), pp.1065-1076. Available at: https://doi.org/10.1007/s13042-018-0785-x.

Zhang, L. & Zhan, J. 2019. Fuzzy soft β-covering based fuzzy rough sets and corresponding decision-making applications. *International Journal of Machine Learning and Cybernetics*, 10(6), pp.1487-1502. Available at: https://doi.org/10.1007/s13042-018-0828-3.

Ziarko, W. 1993. Variable precision rough set model. *Journal of Computer and System Sciences*, 46(1), pp.39-59. Available at: https://doi.org/10.1016/0022-0000(93)90048-2.

Utilizar un enfoque híbrido de toma de decisiones con conjuntos difusos y aproximados de datos lingüísticos para analizar patrones de votación

Sandip Majumder, *Samarjit* Kar, **autor de correspondencia**

Instituto Nacional de Tecnología de Durgapur, Departamento de Matemáticas, Durgapur, Bengala Occidental, República de la India

CAMPO: matemáticas, ciencias de computación
TIPO DE ARTÍCULO: artículo científico original

Resumen:

Introducción/objetivo: La importancia de estudiar el comportamiento electoral se ve subrayada por su capacidad para medir la continuidad o divergencia de la política electoral con respecto a las tendencias históricas, dilucidando el impacto real de las urnas transformadoras y contribuyendo al examen de la democracia como un valor entre masas y élites. Además, ayuda a comprender el intrincado proceso de socialización política.

Métodos: Una fortaleza inherente del conjunto aproximado radica en su dependencia únicamente de datos sin procesar, desprovistos de insumos externos. El marco de conjunto aproximado de teoría de decisiones, una evolución del conjunto aproximado, ha obtenido una aplicación generalizada en diversos dominios, sirviendo como una herramienta competente para adquirir conocimientos, particularmente en situaciones marcadas por la vaguedad y la incertidumbre. A pesar de la proliferación de modelos matemáticos diseñados para discernir el comportamiento *electoral de las personas, una recomendación aproximada basada en decisiones sigue notablemente ausente en la bibliografía existente. Este artículo presenta un innovador enfoque de decisión de tres vías basado en información lingüística para identificar el comportamiento electoral. El enfoque propuesto se basa en un modelo probabilístico híbrido aproximado y difuso que incorpora información lingüística y proporciona información sobre los patrones de votación.*

Resultados: Los modelos híbridos de decisión de tres vías se prueban en personas y se obtuvo un resultado altamente satisfactorio para identificar sus comportamientos electorales. La justificación de resultados fue validada mediante el proceso matemático.

Conclusión: Se proporciona una ilustración práctica para resaltar la importancia de este modelo híbrido y confirmar su utilidad para identificar y pronosticar el comportamiento electoral.

Palabras claves: término lingüístico, conjunto aproximado, conjunto difuso, decisión triple, comportamiento electoral.

Гибридный подход принятия решений с нечетким и грубым множествами, примененный к лингвистическим данным для анализа модели голосования

Сандип Маджумдар, *Самарджит* Кар, **корреспондент**

Национальный технологический институт Дургапура, департамент математики, Дургапур, Западная Бенгалия, Республика Индия

РУБРИКА ГРНТИ: 27.47.19 Исследование операций,
28.17.31 Моделирование процессов управления
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Исследование поведения избирателей имеет важное значение, поскольку оно позволяет измерить постоянность избирательной политики, а также отклонение данной политики от исторических тенденций, объясняя реальное влияние преобразующегося избирательного ящика, и вносит вклад в изучение демократической ценности как среди масс, так и среди элит. Кроме того, это способствует пониманию сложного процесса политической социализации.

Методы: Преимущество грубых множеств заключается в том, что они полагаются исключительно на необработанные данные без каких-либо внешних влияний. Структура грубых множеств для теоретического принятия решений как эволюция грубого множества обеспечила широкое применение в различных областях в качестве успешного инструмента приобретения знаний, особенно в ситуациях, когда присутствуют *неуверенность и неопределенность. Несмотря на большое количество математических моделей, разработанных для объяснения поведения избирателей, в существующей литературе явно отсутствует рекомендация об использовании грубых множеств, основанных на принятии решений. В этой статье представлен инновационный трехфакторный подход к принятию решений, основанный на лингвистических данных, с целью выявления поведения избирателей. Предлагаемый подход основан на гибридной вероятностной грубой модели и модели нечеткого множества, которые включают лингвистические данные и дают представление о моделях голосования.*

Результаты: Трехфакторные гибридные модели принятия решений были протестированы на людях и дали весьма удовлетворительные результаты при определении их избирательного поведения. Полученные результаты были подтверждены математическим методом.

Выводы: Пример из практики подчеркивает важность этой гибридной модели и подтверждает ее полезность в выявлении и прогнозировании поведения избирателей.

Ключевые слова: лингвистический термин, грубое множество, нечеткое множество, трехфакторное решение, поведение избирателя.

Хибридни приступ одлучивању са фази и грубим скуповима заснован на лингвистичким подацима ради анализирања образаца гласања

Сандип Мађумдар, *Самарџит* Кар, **аутор за преписку**

Национални институт за технологију Дургапур, Департман за математику, Дургапур, Западни Бенгал, Република Индија

ОБЛАСТ: математика, рачунарске науке
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Проучавање понашања гласача је значајно јер омогућава мерење континуитета изборне политике као и одступања одређене политике од историјских трендова. Оно објашњава стварни утицај трансформативне бирачке кутије и доприноси испитивању демократије као вредности и међу масама и међу елитама. Поред тога, доприноси разумевању сложеног процеса политичке социјализације.

Методе: Снага грубих скупова лежи у њиховом ослањању искључиво на сирове податке, без икаквих спољашњих утицаја. Оквир грубих скупова за теоријско одлучивање, као еволуција грубог скупа, *обезбедио је широку примену у различитим областима као успешна алатка за стицање знања, нарочито у ситуацијама у којима је присутна неодређеност и несигурност. Упркос великом броју математичких модела пројектованих да утврђују понашање гласача, у литератури нема препоруке да се користе груби скупови засновани на одлучивању. Овај рад уводи иновативни трофакторски приступ одлучивању заснован на лингвистичким подацима ради идентификовања понашања гласача. Предложени приступ заснива се на хибридном пробабилистичком моделу грубих и фазних скупова који укључује лингвистичке податке и обезбеђује увид у обрасце гласања.*

Резултати: Трофакторски хибридни модели одлучивања тестирани су на гласачима. У идентификацији њиховог понашања при гласању добијени су веома задовољавајући резултати. који су валидирани путем математичког процеса.

Закључак: Пример из праксе истиче значај овог хибридног модела и потврђује његову корисност за идентификацију и предвиђање понашања гласача.

Кључне речи: лингвистички термин, груби скуп, фази скуп, трофакторска одлука, понашање гласача.

EDITORIAL NOTE: The second author of this article, Samarjit Kar, is a current member of the Editorial Board of the *Military Technical Courier*. Therefore, the Editorial Team has ensured that the double blind reviewing process was even more transparent and more rigorous. The Team made additional effort to maintain the integrity of the review and to minimize any bias by having another associate editor handle the review procedure independently of the editor – author in a completely transparent process. The Editorial Team has taken special care that the referee did not recognize the author’s identity, thus avoiding the conflict of interest.

Paper received on: 11.01.2024.
Manuscript corrections submitted on: 05.06.2024.
Paper accepted for publishing on: 06.06.2024.