# Fixed point results in controlled revised fuzzy metric spaces with an application to the transformation of solar energy to electric power

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*Abstract:*

*Introduction/purpose: This study establishes sufficient conditions for a sequence to be Cauchy within the framework of controlled revised fuzzy metric spaces. It also generalizes the concept of Banach's contraction principle by introducing several new contraction conditions. The aim is to derive various fixed-point results that enhance the understanding of these mathematical structures.*

*Methods: The researchers employ rigorous mathematical techniques to develop their findings. By defining a set of novel contraction mappings and utilizing properties of controlled revised fuzzy metric spaces, they analyze the implications for sequence convergence. The methodology includes constructing specific examples to illustrate the theoretical results.*

*Results: The study presents several fixed-point theorems derived from the generalized contraction conditions. Additionally, it provides a number of non-trivial examples that substantiate the claims and demonstrate the applicability of the results in practical scenarios. A significant application is* *explored regarding the conversion of solar energy into electric power, utilizing differential equations to highlight this connection.*

*Conclusion: The findings deepen the understanding of Cauchy sequences in fuzzy metric spaces and offer a broader perspective on the application of the fixed-point theory in real-world scenarios. The results pave the way for further research in both theoretical mathematics and its practical applications, particularly in the field of renewable energy.*

*Key words: fixed point theorems, revised fuzzy metric space (RFMS), contraction principles(CP), Green's function, differential equation.*

## Introduction and preliminaries

The existence of a unique fixed point (UFP) for self-mappings under suitable contraction conditions over complete metric spaces(CMS) is guaranteed by Banach's fixed point theory (BSP) (also known as "the contraction mapping theorem"), one of the most significant sources of existence and uniqueness theorems in numerous areas of analysis. New extensions and generalizations of fixed point results are important because they increase our understanding of mathematical systems, enable the solution of specific problems, extend current theorems, and lead to the development of new theories and applications. They are an important aspect of mathematical study and have far-reaching ramifications in a variety of fields.

Fuzzy logic (FL) was established by Zadeh (1965). Unlike the theory of traditional logic, some numbers are not contained within the stand. FL afflation of the numbers in the set defines an element within the interval [0,1]. Zadeh has learned theories of fuzzy sets to be art, the problem of indefinity with the aid of uncertainty, the essential part of genuine difficulty.

The theory is seen as a fixed point (FP) in the fuzzy metric space (FMS) for various processes, one of them utilizing fuzzy logic. Later on, following Zadeh's outcomes, Heilpern (1981) established the fuzzy mapping (FM) notion and a theorem on an FP for fuzzy contraction mapping (FCM) in linear MS, expressing a fuzzy general form of BC-theory. In the definition of FMSs provided by Kaleva & Saikkala (1984), the imprecision is introduced if the distance between the elements is not a precise integer. After the first by Kramosil & Michálek (1975) and further work by George & Veeramani (1994), the notation of an FMS was introduced. After that, Klement et al. (2004) presented some problems on trigonomtric terms and releted operators.

Branga & Olaru (2022) proved several fixed point results for self-mappings by utilizing generalized contractive conditions in the context of altered MS. Al-Khaleel et al. (2023) used cyclic contractive mappings of Kannan and Chatterjea type in generalized metric spaces. Czerwik (1993) found the solution of the well-known BFPT in the context of b-metric spaces (b-MS). Mlaiki et al. (2018) defined controlled MS as a generalization of b-MS by utilizing a control function $\alpha: \Xi^2 \to [1, \infty)$ of the otherside of the b-triangular inequality. The relation between b-MS and FMS has been discussed by Hassanzadeh & Sedghi (2018). Li et al. (2022) used Kaleva–Seikkala's type FbMSs and proved several fixed point results by using contraction mappings. Furthermore, Sedghi & Shobe (2012) and Sedghi & Shobkolaei (2014) proved various common FPTs for R-weakly commuting maps in the frame work of FbMSs. Sezen (2021) introduced controlled FMS as a generalization of FMS and FbMS by applying a control function $\alpha : \Xi^2 \to [1, \infty)$ in a triangular inequality of the form:

$$M(b, \omega, t + s) \geq M\left(b, z, \frac{t}{\alpha(b, z)}\right) + M\left(z, \omega, \frac{t}{\alpha(z, \omega)}\right), \text{for all } b, \omega, z \in \Xi \text{ and } s, t > 0 .$$

If we take $\alpha(b, z) = \alpha(z, \omega) = 1$ then its an FMS and for $\alpha(b, z) = \alpha(z, \omega) \geq s$ with $s \geq 1$ it is then an FbMS.

Ishtiaq et al. (2022, 2023) established the theory of double-controlled intuitionistic fuzzy metric-like spaces by "considering the case where the self-distance is not zero"; if the metric's value is 0, afterwards it must be a self-distance and also an established FP theorem for contraction mappings. See (Schweizer & Sklar, 1960) for triangular conorm (TCN), continuous triangular conorm (CTCN) (Kaleva & Seikkala, 1984), and TN of H-type (Hadžić, 1979; Hassanzadeh & Sedghi, 2018). In (Hussain et al, 2022), the authors worked on CFMSs by utilizing orthogonality and pentagonal CFMSs and proved several FPRs for contraction mappings. Rakić et al. (2020) proved a fuzzy version of BFPT by using Ciric-quasi-contraction in the context of FbMSs. Younis et al. (2022) proved several FPRs in the context of dislocated b-metric spaces and solved the turning circuit problem.

In 2018, Šostak (2018) explored the idea of Revised Fuzzy Metric Spaces (shortly, RFM-Space) as a new idea to express a revised fuzzy set. Then Grigorenko et al. (2020) made several proposals based on RFM-Spaces. Building on the work of Šostak (2018), Muraliraj et al. (2023) defined the notion of retinal-type revised fuzzy contraction and proved a fixed point theorem of RFM spaces. Muraliraj & Thangathamizh (Muraliraj & Thangathamizh, 2021ab) partially came up with the idea of fixed point results for RFM in revised fuzzy contraction (shortly, RFM contractions). The following authors (Adabitabar Firozja & Firouzian, 2015; Kider, 2020, 2021; Moussaoui et al, 2022; Muraliraj & Thangathamizh, 2021a, 2023) provided many concepts of RFMS and they proved to be quite useful in this study. Gregori & Miñana (2021) introduced a new version of contraction principles in the context of fuzzy metric spaces. It is defined by the means of t-conorms. After that Muraliraj & Thangathamizh (2022), Thangathamizh et al. (2024) established new revised fuzzy cone contraction type unique coupled fixed point theorems in revised fuzzy cone metric spaces by using the property of triangular.

In this article,

- we prove that a sequence must be Cauchy in the CRFMS under some conditions;
- we prove a fixed point result by using new Ciric-quasi-contraction and generalize the Banach contraction principle by utilizing several new contraction conditions;
- we provide several non-trivial examples to show the validity of the main results;
- we discuss an application concerning the transformation of solar energy to electric power.

Now, we provide several definitions and results that are helpful to understand the main section.

**Definition 1** (Šostak, 2018). A binary operation $\Gamma: [0,1]^2 \to [0,1]$ is a CTCN if it verifies the conditions below:

(C 1) $\Gamma$ is commutative and associative,

(C 2) $\Gamma$ is continuous,

(C 3) $\Gamma(\kappa, 0) = \kappa$ for all $\kappa \in [0,1]$,

(C 4) $\Gamma(\kappa, \rho) \leq \Gamma(c, d)$ for $\kappa, \rho, c, d \in [0,1]$ such that $\kappa \leq c$ and $\rho \leq d$.

The examples of CTCN are $\Gamma_p(a, b) = a + b - a\,.\,b$, $\Gamma_{max}(a, b) = \max\{a, b\}$, and $\Gamma_L(a, b) = \min\{a + b, 1\}$.

**Definition 2** (Hadžić, 1979). Suppose that $\Gamma$ is a TCN and suppose that $\Gamma_\tau: [0,1] \to [0,1], \tau \in \mathbb{N}$ , express the process given below:

$\Gamma_1(b) = \Gamma(b, b), \Gamma_{\tau+1}(b) = \Gamma(\Gamma_\tau(b), b),\ \tau \in \mathbb{N}, b \in [0,1]$.

Then, TCN $\Gamma$ is H-type if the family $\{\Gamma_\tau(b)\}_{\tau\in\mathbb{N}}$ is equi-continuous at $b = 1$.

A TCN of H-type is $\Gamma_{max}$ and see [9, 18] for examples.

Each t-conorm can be generalized in a different way to an n-ary process via associativity, taking $(b_1, \dots, b_n) \in [0,1]^n$ for the values

$\Gamma^1{}_{i=1} b_i = b_1, \Gamma^1{}_{i=1} b_i = \Gamma(\Gamma^1{}_{i=1} b_i, b_\tau) = \Gamma(b_1, \dots, b_\tau)$.

Example 1. An n-ary generalization of the TCN$\Gamma_{min}, \Gamma_L$, and $\Gamma_P$ are:

$\Gamma_{max}(b_1, \dots, b_\tau) = \max\{b_1, \dots, b_\tau\}$

$\Gamma_L(b_1, \dots, b_\tau) = \min\{\sum_{i=1}^{\tau} b_i - (\tau - 1), 1\}$, $\Gamma_P(b_1, \dots, b_\tau) = \prod_{i=1}^{\tau} b_i$

A TCN $\Gamma$ can be extended to accountable infinite operation, for any sequence $(b_\tau)_{\tau\in\mathbb{N}}$ considering from $[0,1]$ the value $\Gamma_{i=1}^{\infty} = \lim_{\tau\to\infty} \Gamma_{i=1}^{\tau} b_i$

The sequence $\{\Gamma_{i=1}{}^{\tau}(b)\}_{\tau\in\mathbb{N}}$ is non-increasing and bounded below and so the limit $\Gamma_{i=1}{}^{\infty}(b_i)$ exists. In the FP theory (Hadžić & Pap, 2001; Hadžić, 1979), it might be interesting to look at the category of TCN$\Gamma$ and sequence $(b_\tau)$ in the range [0,1] such that $\lim_{\tau\to\infty} b_\tau = 0$ and

$$\lim_{\tau\to\infty} \Gamma_{i=1}{}^{\infty}(b_i) = \lim_{\tau\to\infty} \Gamma_{i=1}{}^{\infty}(b_{\tau+i}) \quad (1)$$

Proposition 1. Suppose $(b_\tau)_{\tau\in N}$ is a series of numbers with the range [0,1] such that $\lim_{\tau\to\infty} b_\tau = 0$ and assume $\Gamma$ to be a TCN of H-type. Then, $\lim_{\tau\to\infty} \Gamma_{i=1}{}^{\infty} b_{\tau+i}$. Throughout this study, we utilize $\Xi^2 : \Xi \times \Xi$.

**Definition 3** (Šostak, 2018). A 3-tuple $(\Xi, M, \Gamma)$ is known as an RFMS if $\Xi$ be a some (nonempty) set, $\Gamma$ is a CTCN, and M is an RFM on $\Xi^2 \times (0,\infty)$ and satisfies the following conditions, for all $b, \omega, z \in \Xi$, and $t, s > 0$:

(Rfm 1) $M(b,\omega,t) < 1$,

(Rfm 2) $M(b,\omega,t) = 0 \text{ iff } b = \omega$,

(Rfm 3) $M(b,\omega,t) = M(\omega,b,t)$,

(Rfm 4) $\Gamma(M(b,\omega,t), M(\omega,z,s)) \geq M(b,z,t+s)$,

(Rfm 5) $M(b,\omega,.) : (0,\infty) \to [0,1]$ is continuous.

**Definition 4.** A 3-tuple $(\Xi, M, \Gamma)$ is called an RFbMS if $\Xi$ is a random (non-empty) set, $\Gamma$ is a CTCN, and $M$ is an RFM on $\Xi^2 \times (0,\infty)$ and satisfies the following conditions for all $b, \omega, z \in \Xi$, $t, s > 0$, and $\rho \geq 1$ as a real number:

(Rb 1) $M(b,\omega,t) < 1$,

(Rb 2) $M(b,\omega,t) = 0 \text{ iff } b = \omega$,

(Rb 3) $M(b,\omega,t) = M(\omega,b,t)$,

(Rb 4) $\Gamma\big(M(b,\omega,t), M(\omega,z,s)\big) \geq M(b,z,\rho(t+s))$,

(Rb 5) $M(b,\omega,-): (0,\infty) \to [0,1]$ is continuous.

The following fixed point theorem is using a new Ciric-quasi-contraction in the context of RFbMSs.

**Theorem 1.** Suppose that $(\Xi, M, \Gamma_{\min})$ is a complete RFbMS, assume that $f: \Xi \to \Xi$. If for some $\aleph \in (0,1)$, such that

$$M_\alpha(fb, f\omega, t) \leq \max\left\{\begin{matrix} M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha\left(fb,b,\frac{t}{\aleph}\right), M_\alpha\left(f\omega,\omega,\frac{t}{\aleph}\right), \\ M_\alpha\left(fb,\omega,\frac{2t}{\aleph}\right), M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right) \end{matrix}\right\}, \quad b,\omega \in$$

$\Xi, t > 0$. Then, $f$ has a UFP in $\Xi$.

**Lemma 1.** Let $M(b,\omega,-)$ be an RFbMS. Then, $M(b,\omega,t)$ is b-non-decreasing with respect to for all $b,\omega \in \Xi$.

**Definition 5.** Let $\Xi$ be a non-empty set, $\alpha: \Xi^2 \rightarrow [1, \infty)$, $\Gamma$ is a CTCN, and $M_\alpha$ is an RFM on $\Xi^2 \times [0, \infty)$ and satisfies the following conditions for all $b, \omega, z \in \Xi, s$, and $t > 0$:

$(\text{ERFM}\alpha\ 1)$ $M_\alpha(b, \omega, 0) < 1$,

$(\text{ERFM}_\alpha\ 2)$ $M_\alpha(b, \omega, t) = 0$ iff $b = \omega$,

$(\text{ERFM}_\alpha\ 3)$ $M_\alpha(b, \omega, t) = M_\alpha(\omega, b, t)$,

$(\text{ERFM}_\alpha\ 4)$ $M_\alpha\big(b, z, \alpha(b, z)(t + s)\big) \leq \Gamma(M_\alpha(b, \omega, t), M_\alpha(\omega, z, s))$,

$(\text{ERFM}_\alpha\ 4)$ $M(b, \omega, .): (0, \infty) \rightarrow [0,1]$ is right continuous.

Then, the triple $(\Xi, M_\alpha, \Gamma)$ is said to be an extended revised fuzzy b-metric space and $M_\alpha$ is said to be a controlled RFM on $\Xi$.

**Theorem 2.** Suppose that $(\Xi, M, \Gamma)$ is a complete RFMS with $f: \Xi^2 \rightarrow [1, \infty)$, assume that $\lim_{t \to \infty} M_\alpha(b, \omega, t) = 0$, for all $b, \omega \in \Xi$. Iff $: \Xi \rightarrow \Xi$ satisfies the following for some $\aleph \in (0,1)$, such that

$M_\alpha(fb, f\omega, t) \leq M_\alpha\left(b, \omega, \frac{t}{\aleph}\right)$, for all $b, \omega \in \Xi, t > 0$.

Also suppose that for arbitrary $b_0 \in \Xi$ and $n, q \in N$, there is $\alpha\big(b_n, b_{n+q}\big) \leq \frac{1}{\aleph}$,

where $b_n = f^n b_0$.Then, $f$ has a UFP in $\Xi$.

**Definition 6.** Let $\Xi$ be a non-empty set, $\alpha: \Xi^2 \rightarrow [1, \infty)$, $\Gamma$ is a CTCN, and $M_\alpha$ is an RFM on $\Xi^2 \times [0, \infty)$ and satisfies the following conditions for all $b, \omega, z \in \Xi, s$, and $t > 0$:

$(\text{RFM}_\alpha 1) M_\alpha(b, \omega, 0) < 1$,

$(\text{RFM}_\alpha 2) M_\alpha(b, \omega, t) = 0$ iff $b = \omega$,

$(\text{RFM}_\alpha 3) M_\alpha(b, \omega, t) = M_\alpha(\omega, b, t)$,

$(R\text{FM}_\alpha 4) M_\alpha(b, z, t + s) \leq \Gamma\left(M_\alpha\left(b, \omega, \frac{t}{\alpha(b,\omega)}\right), M_\alpha\left(\omega, z, \frac{s}{\alpha(b,\omega)}\right)\right)$,

$(\text{RFM}_\alpha 5) M(b, \omega, .): (0, \infty) \rightarrow [0,1]$ is right continuous.

Then, the triple $(\Xi, M_\alpha, \Gamma)$ is said to be a CRFMS and $M_\alpha$ is said to be a controlled RFM on $\Xi$.

Muraliraj and Thangathamizh, (2021a) proved the following Banach contraction principle in the context of CRFMS.

**Theorem 3.** Suppose that $(\Xi, M, \Gamma)$ is a complete CRFMS with $\alpha: \Xi^2 \rightarrow [1, \infty)$, assume that $\lim_{t \to \infty} M_\alpha(b, \omega, t) = 0$, for all $b, \omega \in \Xi$.

If $f: \Xi \rightarrow \Xi$ satisfies the following for some $\aleph \in (0, 1)$, such that

$M_\alpha(fb, f\omega, t) \leq M_\alpha\left(b, \omega, \frac{t}{\aleph}\right)$, for all $b, \omega \in \Xi, t > 0$.

Also, suppose that for all $b \in \Xi$, we obtain $\lim_{n \to \infty} M_\alpha(b_n, \omega)$ and $\lim_{n \to \infty} M_\alpha(\omega, b_n)$ which exist and are finite. Then, $f$ has a UFP in $\Xi$.

**Definition 7.** Suppose $M(b, \omega, t)$ is a CRFMS. For $t > 0$, the open ball $B(b, l, t)$ with the center $b \in \Xi$ and the radius $0 < l < 1$ is expressed as a sequence $\{b_\tau\}$:

G-Convergent to b if $M(b_\tau, b, t) \to 0$ as $\tau \to \infty$ or for every $t > 0$.We write $\lim_{\tau\to\infty} b_\tau = b$.

is said to be a Cauchy sequence (CS) if for all $0 < \varepsilon < 1$ and $t > 0$ there exist satisfying $\tau_0 \in N$ such that $M(b_\tau, b_m, t) < \varepsilon$ for all $\tau, m \geq \tau_0$.

The CRFMS$(\Xi, M, \Gamma)$ is a G-complete if every Cauchy sequence is convergent in $\Xi$.

## Main results

In this part, we discuss several new results in the context of CRFMSs.

**Lemma 2.** Suppose $\{b_\tau\}$ is a sequence in a CRFMS$(\Xi, M_\alpha, \Gamma)$. Let $\aleph \in (0,1)$ exist such that

$$M_\alpha(b_\tau, b_{\tau+1}, t) \leq M_\alpha\left(b_{\tau-1}, b_\tau, \frac{t}{\aleph}\right) \quad (2)$$

and $b_\tau \in \Xi$, and $v \in (0,1)$ exist such that

$$\lim_{t\to\infty} M(b, \omega, t) = 0,\ t > 0 \quad (3)$$

Then, $\{b_\tau\}$ is a Cauchy sequence.

**Proof.** Suppose $\kappa \in (\aleph, 1)$ and the total $\sum_{i=1}^{\infty} \kappa^i$ is convergent, $\tau_0 \in N$ exists such that $\sum_{i=1}^{\infty} \kappa^i < 1$ for every $\tau > \tau_0$. Let $\tau > m > \tau_0$. Since $M_\alpha$ b-non-decreasing, by ($RFM_\alpha$ 4), for every $t > 0$, one can obtain

$$M_\alpha(b_\tau, b_{\tau+m}, t) \leq M_\alpha\left(b_\tau, b_{\tau+m}, \frac{\sum_{i=\tau}^{\tau+m-1} \kappa^i}{\alpha(b_\tau, b_{\tau+m})}\right)$$

$$\leq$$

$$\Gamma\left(M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t\kappa^\tau}{\alpha(b_\tau,b_{\tau+1})\alpha(b_\tau,b_{\tau+m})}\right), M_\alpha\left(b_{\tau+1}, b_{\tau+m}, \frac{t\sum_{i=\tau+1}^{\tau+m-1} \kappa^i}{\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_\tau,b_{\tau+m})}\right)\right)$$

$$\leq \Gamma\begin{pmatrix} M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t\kappa^\tau}{\alpha(b_\tau,b_{\tau+1})\alpha(b_\tau,b_{\tau+m})}\right), \\ M_\alpha\left(b_{\tau+1}, b_{\tau+2}, \frac{t\kappa^{\tau+1}}{\alpha(b_{\tau+1},b_{\tau+2})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_\tau,b_{\tau+m})}\right), \\ M_\alpha\left(b_{\tau+2}, b_{\tau+m}, \frac{t\sum_{i=\tau+2}^{\tau+m-1} \kappa^i}{\alpha(b_{\tau+2},b_{\tau+m})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_\tau,b_{\tau+m})}\right) \end{pmatrix}$$

$$
\leq \Gamma\begin{pmatrix}
M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t\kappa^\tau}{\alpha(b_\tau,b_{\tau+1})\alpha(b_\tau,b_{\tau+m})}\right), \\
M_\alpha\left(b_{\tau+1}, b_{\tau+2}, \frac{t\kappa^{\tau+1}}{\alpha(b_{\tau+1},b_{\tau+2})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_\tau,b_{\tau+m})}\right), \\
M_\alpha\left(b_{\tau+2}, b_{\tau+3}, \frac{t\kappa^{\tau+2}}{\alpha(b_{\tau+2},b_{\tau+3})\alpha(b_{\tau+2},b_{\tau+m})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_\tau,b_{\tau+m})}\right) \\
M_\alpha\left(b_{\tau+3}, b_{\tau+4}, \frac{t\kappa^{\tau+3}}{\alpha(b_{\tau+3},b_{\tau+4})\alpha(b_{\tau+2},b_{\tau+3})\alpha(b_{\tau+2},b_{\tau+m})\alpha(b_{\tau+1},b_{\tau+m})\alpha(b_\tau,b_{\tau+m})}\right) \\
\vdots \\
M_\alpha\left(b_{\tau+m-2}, b_{\tau+m-1}, \frac{t\kappa^{\tau+m-2}}{\alpha(b_{\tau+m-2},b_{\tau+m-1})\prod_{i=\tau}^{\tau+m-2}\alpha(b_i,b_{\tau+m})}\right) \\
M_\alpha\left(b_{\tau+m-1}, b_{\tau+m}, \frac{t\kappa^{\tau+m-1}}{\alpha(b_{\tau+m-2},b_{\tau+m-1})\prod_{i=\tau}^{\tau+m-1}\alpha(b_i,b_{\tau+m})}\right)
\end{pmatrix}
$$

From inequality (2), one deduces $M_\alpha(b_\tau, b_{\tau+1}, t) \leq M_a\left(b_0, b_1, \frac{t}{\aleph^\tau}\right)$ and since $\tau > m$ and $\alpha: \Xi^2 \to [1, \infty)$, one can obtain

$$
M_\alpha(b_\tau, b_{\tau+m}, t)
$$

$$
\leq \Gamma\begin{pmatrix}
M_\alpha\left(b_0, b_1, \frac{t\kappa^\tau}{\alpha(b_\tau, b_{\tau+1})\alpha(b_\tau, b_{\tau+m})\aleph^\tau}\right), \\
M_\alpha\left(b_0, b_1, \frac{t\kappa^{\tau+1}}{\alpha(b_{\tau+1}, b_{\tau+2})\alpha(b_{\tau+1}, b_{\tau+m})\alpha(b_\tau, b_{\tau+m})\aleph^{\tau+1}}\right), \\
\vdots \\
M_\alpha\left(b_0, b_1, \frac{t\kappa^{\tau+m-2}}{\alpha(b_{\tau+m-2}, b_{\tau+m-1})\prod_{i=\tau}^{\tau+m-2}\alpha(b_i, b_{\tau+m})\aleph^{\tau+m-1}}\right) \\
M_\alpha\left(b_0, b_1, \frac{t\kappa^{\tau+m-1}}{\prod_{i=\tau}^{\tau+m-1}\alpha(b_i, b_{\tau+m})\aleph^{\tau+m-1}}\right)
\end{pmatrix}
$$

As $\tau \to \infty$ and by utilizing (3),one obtains $M_\alpha(b_\tau, b_{\tau+m}, t) \leq \Gamma(0, 0, \cdots, 0) = 0$. Hence,$\{b_\tau\}$ is a Cauchy sequence in $\Xi$.

**Corollary 1.** Suppose $\{b_\tau\}$ is a sequence in CRFMS $(\Xi, M_\alpha, \Gamma)$ and $\Gamma$ is H-type. If $\aleph \in (0,1)$ exists such that

$$
M_\alpha(b_\tau, b_{\tau+1}, t) \leq M_a\left(b_{\tau-1}, b_\tau, \frac{t}{\aleph}\right), \tau \in \mathbb{N}, t > 0. \quad (4)
$$

Then,$\{b_\tau\}$ is a continuous Sequence.

**Lemma 3.** If for $b, \omega \in \Xi$, and some $\aleph \in (0,1)$,

$$
M_\alpha(b, \omega, t) \leq M_\alpha\left(b, \omega, \frac{t}{\aleph}\right), t > 0. \quad (5)
$$

Then, $b = \omega$.

**Proof.** Inequality (5) implies that

$$
M_\alpha(b, \omega, t) \leq M_\alpha\left(b, \omega, \frac{t}{\aleph^\tau}\right), \tau \in \mathbb{N}, t > 0.
$$

Now,

$$M_\alpha(b,\omega,t) \leq \lim_{\tau\to\infty} M_\alpha\left(b,\omega,\frac{t}{\aleph^\tau}\right) = 0, t > 0.$$

And by $(RFM_\alpha 2)$, it is easy to see that $b = \omega$.

**Theorem 4.** Suppose that $(\Xi, M, \Gamma)$ is a complete CRFMS and suppose that $f\colon \Xi \to \Xi$. Let them exist $\aleph \in (0,1)$ such that

$$M_\alpha(fb, f\omega, t) \leq \min\left\{\begin{matrix} M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha\left(b, f\omega, \frac{t}{\aleph}\right), M_\alpha\left(fb,\omega,\frac{t}{\aleph}\right) \\ \frac{M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right) + M_\alpha\left(fb,\omega,\frac{t}{\aleph}\right)}{2} \\ \frac{M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right) + M_\alpha\left(fb,\omega,\frac{t}{\aleph}\right)}{1+M_\alpha\left(b,\omega,\frac{t}{\aleph}\right)} \end{matrix}\right\}, b,\omega \in$$

$\Xi, t > 0.$ (6)

and $b, \omega \in \Xi$ such that

$\lim_{t\to\infty} M_\alpha(b,\omega,t) = 0, t > 0$ (7)

Then, f has a UFP in $\Xi$.

**Proof.** Suppose $b_0 \in \Xi$ and $b_{\tau+1} = fb_\tau$, $\tau \in \mathbb{N}$. Consider $b = b_\tau$ an d$\omega = b_{\tau-1}$ in (6), then one can obtain

$$M_\alpha(b_\tau, b_{\tau+1}, t) \leq M_a(fb_{\tau-1}, fb_\tau, t),$$

$$\leq$$

$$\min\left\{\begin{matrix} M_\alpha\left(b_{\tau-1}, b_\tau, \frac{t}{\aleph}\right), \Gamma\left(M_\alpha\left(b_{\tau-1}, b_\tau, \frac{t}{\aleph}\right)\right), M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t}{\aleph}\right), \\ M_\alpha\left(b_\tau, b_\tau, \frac{t}{\aleph}\right), \frac{M_\alpha\left(b_{\tau-1}, b_{\tau+1}, \frac{t}{\aleph}\right) + M_\alpha\left(b_\tau, b_\tau, \frac{t}{\aleph}\right)}{2} \\ \frac{M_\alpha\left(b_{\tau-1}, b_{\tau+1}, \frac{t}{\aleph}\right) + M_\alpha\left(b_\tau, b_\tau, \frac{t}{\aleph}\right)}{1+M_\alpha\left(b_{\tau-1}, b_\tau, \frac{t}{\aleph}\right)} \end{matrix}\right\}$$

$$\leq \min\left\{M_\alpha\left(b_{\tau-1}, b_\tau, \frac{t}{\aleph}\right), M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t}{\aleph}\right)\right\}$$

If $M_\alpha(b_\tau, b_{\tau+1}, t) \leq M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t}{\aleph}\right), \tau \in \mathbb{N}, t > 0.$

Then by Lemma 3 such that $b_\tau = b_{\tau+1}, \tau \in \mathbb{N}$, there is

$M_\alpha(b_\tau, b_{\tau+1}, t) \leq M_\alpha\left(b_{\tau-1}, b_\tau, \frac{t}{\aleph}\right), \tau \in \mathbb{N}, M_\alpha(b_\tau, b_{\tau+1}, t)$ and by Lemma 2 it follows that $\{b_\tau\}$ is a CS. Since, $(\Xi, M, \Gamma)$ is complete, $b \in \Xi$ exist such that $\lim_{\tau\to\infty} b_\tau = b$ and

$\lim_{\tau\to\infty} M_\alpha(b, b_\tau, t) = 0, t > 0.$ (8)

By utilizing (6) and $(RFM_\alpha 4)$, it is easy to see that b is a FP for $f$.

Suppose $\kappa_1 \in (\aleph, 1)$ and $\kappa_2 = 1 - \kappa_1$ by (6); one can obtain

$$M_\alpha(fb, b, t) \leq \Gamma\left(M_\alpha\left(fb, b_{\tau-1}, \frac{t\kappa_1}{2\alpha(fb, b_\tau)}\right), M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t\kappa_2}{2\alpha(b_\tau, b)}\right)\right)$$

$$\leq \Gamma\left(\min\left\{\begin{array}{c} M_\alpha\left(b, b_{\tau-1}, \frac{t\kappa_1}{2\alpha(fb,b_\tau)\aleph}\right), M_\alpha\left(b, b_\tau, \frac{t\kappa_1}{2\alpha(fb,b_\tau)\aleph}\right) \\ \Gamma\left(M_\alpha\left(fb, b, \frac{t\kappa_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), M_\alpha\left(b, b_{\tau-1}, \frac{t\kappa_1}{(2)^2\alpha(b,b_{\tau-1})\alpha(fb,b_\tau)\aleph}\right)\right), \\ \frac{M_\alpha\left(b,b_\tau,\frac{t\kappa_1}{2\alpha(fb,b_\tau)\aleph}\right)+\Gamma\left(\begin{array}{c} M_\alpha\left(fb,b,\frac{t\kappa_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), \\ M_\alpha\left(b,b_{\tau-1},\frac{t\kappa_1}{(2)^2\alpha(b,b_{\tau-1})\alpha(fb,b_\tau)\aleph}\right)\end{array}\right)}{2} \\ \frac{M_\alpha\left(b,b_\tau,\frac{t\kappa_1}{2\alpha(fb,b_\tau)\aleph}\right).\Gamma\left(\begin{array}{c} M_\alpha\left(fb,b,\frac{t\kappa_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), \\ M_\alpha\left(b,b_{\tau-1},\frac{t\kappa_1}{(2)^2\alpha(b,b_{\tau-1})\alpha(fb,b_\tau)\aleph}\right)\end{array}\right)}{1+M_\alpha\left(b,b_{\tau-1},\frac{t}{2\alpha(fb,b_\tau)\aleph}\right)} \\ M_\alpha\left(b_\tau, b, \frac{t}{2\alpha(b_\tau,b)}\right) \end{array}\right\}\right)$$

for all $t > 0$. By (8) and as $\tau \to \infty$, one obtains

$$\leq \Gamma\left(\min\left\{\begin{array}{c} 0, 0 \\ \Gamma\left(M_\alpha\left(fb, b, \frac{t\kappa_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), 0\right), \\ \frac{1+\Gamma\left(\begin{array}{c} M_\alpha\left(fb, b, \frac{t\kappa_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), \\ 1 \end{array}\right)}{2} \\ \frac{0.\Gamma\left(\begin{array}{c} M_\alpha\left(fb, b, \frac{t\kappa_1}{(2)^2\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), \\ 1 \end{array}\right)}{1+0} \\ 0 \end{array}\right\}\right)$$

Suppose that $b$ and $\omega$ are two different FP for $f$. Then, by applying (6), one can obtain

$$M_a(b, \omega, t) = M_a(fb, f\omega, t)$$

$$\leq \min\left\{\begin{array}{c} M_\alpha\left(b, \omega, \frac{t}{\aleph}\right), M_\alpha\left(b, f\omega, \frac{t}{\aleph}\right), M_\alpha\left(fb, \omega, \frac{t}{\aleph}\right), \\ \frac{M_\alpha\left(b, f\omega, \frac{t}{\aleph}\right) + M_\alpha\left(fb, \omega, \frac{t}{\aleph}\right)}{2} \\ \frac{M_\alpha\left(b, f\omega, \frac{t}{\aleph}\right).M_\alpha\left(fb, \omega, \frac{t}{\aleph}\right)}{1 + M_\alpha\left(b, \omega, \frac{t}{\aleph}\right)} \end{array}\right\}$$

$$= \min \left\{ \begin{matrix} M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), \\ \frac{M_\alpha\left(b,\omega,\frac{t}{\aleph}\right)+M_\alpha\left(b,\omega,\frac{t}{\aleph}\right)}{2} \\ \frac{M_\alpha\left(b,\omega,\frac{t}{\aleph}\right).M_\alpha\left(b,\omega,\frac{t}{\aleph}\right)}{1+M_\alpha\left(b,\omega,\frac{t}{\aleph}\right)} \end{matrix} \right\} = M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), t > 0.$$

and by Lemma 3, it is easy to see that $b = \omega$.

**Remark 1.** If one takes

$$\min \left\{ \begin{matrix} M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right), M_\alpha\left(fb,\omega,\frac{t}{\aleph}\right), \\ \frac{M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right)+M_\alpha\left(fb,\omega,\frac{t}{\aleph}\right)}{2} \\ \frac{M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right).M_\alpha\left(fb,\omega,\frac{t}{\aleph}\right)}{1+M_\alpha\left(b,\omega,\frac{t}{\aleph}\right)} \end{matrix} \right\} = M_\alpha\left(b,\omega,\frac{t}{\aleph}\right)$$

in the above theorem, one then obtains a revised fuzzy version of the Banach contraction principle.

**Example 2.** Let $\Xi = \{0,1,3\}$, $M_\alpha(b,\omega,t) = e^{-\frac{(b-\omega)^2}{t}}\left(1 - e^{\frac{(b-\omega)^2}{t}}\right)$, and $\Gamma = \Gamma_p$. Then, $(\Xi, M, \Gamma)$ is a complete CRFMS with $\alpha(b,\omega) = b + \omega + 1$. Define the function $f: \Xi \to \Xi$ such that $f(0) = f(1) = 1$ and $f(3) = 0$. Observe that if $b = \omega$ or $\omega \in \{0,1\}$, then $M_a(fb, f\omega, t) = 0$ and $t > 0$ and condition (6) is fulfilled. Suppose b = 1 and $\omega = 3$. Then, $\aleph \in \left(\frac{1}{9},\frac{1}{4}\right)$ and one obtains

$$M_\alpha(fb,f\omega,t) = e^{-\frac{1}{t}}\left(1 - e^{\frac{1}{t}}\right) \le \min\left\{e^{-\frac{4}{\aleph t}}, e^{-\frac{1}{\aleph t}}, e^{-\frac{4}{\aleph t}}, \frac{e^{-\frac{1}{\aleph t}} + e^{-\frac{4}{\aleph t}}}{2}, \frac{e^{-\frac{1}{\aleph t}}.e^{-\frac{4}{\aleph t}}}{1 + e^{-\frac{4}{\aleph t}}}\right\}$$

Now, suppose $b = 1$ and $\omega = 3$, then, choosing $\aleph$ from $\left(\frac{1}{9},\frac{1}{4}\right)$, one deduces

$$M_\alpha(fb,f\omega,t) = e^{-\frac{1}{t}}\left(1 - e^{\frac{1}{t}}\right) \le \min\left\{e^{-\frac{4}{\aleph t}}, e^{-\frac{1}{\aleph t}}, e^{-\frac{4}{\aleph t}}, \frac{e^{-\frac{1}{\aleph t}} + e^{-\frac{4}{\aleph t}}}{2}, \frac{e^{-\frac{1}{\aleph t}}.e^{-\frac{4}{\aleph t}}}{1 + e^{-\frac{4}{\aleph t}}}\right\}$$

Similarly, if $b = 3$ and $\omega = 1$ as well as $b = 3$ and $\omega = 1$, one establishes that for $\aleph \in \left(\frac{1}{9},\frac{1}{4}\right)$ condition (6) is fulfilled for all $b, \omega \in \Xi$, and $t > 0$. Hence, all the conditions of Theorem 4 are satisfied with a UFP $b = 1$.

**Corollary 2.** Supposing that $(\Xi, M, \Gamma)$ is a complete CRFMS with $\alpha: \Xi^2 \to [1, \infty)$, assume that $\lim_{t\to\infty} M_\alpha(b, \omega, t) = 0$. For all $b, \omega \in \Xi$. If $f: \Xi \to \Xi$ satisfies the following, for some $\aleph \in (0, 1)$, such that

$M_\alpha(fb, f\omega, t) \le M_\alpha\left(b, \omega, \frac{t}{\aleph}\right)$, for all $b, \omega \in \Xi, t > 0$. Then, $f$ has a UFP in $\Xi$.

**Example 3.** $\Xi = A \cup B$, where $A = [0,1], B = \frac{\mathbb{N}}{1}$, and $M_\alpha: \Xi^2 \times (0, \infty) \to [0,1]$, is defined by

$$M_\alpha(b, \omega, t) = \begin{cases} 0 \text{ if } b = \omega \\ \dfrac{t}{t + \frac{1}{\omega}} \text{ if } b \in B \text{ and } \omega \in A \\ \dfrac{t}{t + \frac{1}{b}} \text{ if } b \in A \text{ and } \omega \in B \\ \dfrac{t}{t + \max\{b, \omega\}} \text{ otherwise} \end{cases}$$

Then, $(\Xi, M, \Gamma)$ is CRFMS with $\Gamma(b, \omega) = b.\omega$ and a controlled function $\alpha: \Xi^2 \to [0, \infty)$ defined by

$$a(b, \omega) = \begin{cases} 0 \text{ if } b = \omega \\ \min\{b, \omega\} \text{ otherwise} \end{cases}$$

It is easy to see that all the conditions of Corollary 2 are satisfied.

Consider the triangular inequality $(\mathrm{RFEM}_\alpha 4)$ of the revised fuzzy extended b-metric space defined in Definition 5 as

$$M_\alpha(b, z, \alpha(b, z)(t + s)) \le \Gamma(M_\alpha(b, \omega, t), M_\alpha(\omega, z, s))$$

Let $b, z \in A$, and $\omega \in B$, the, $n\alpha(b, z) = 0$. Assume $t = s = 2, b = \frac{1}{2}, z = 1$, and $\omega = 40$. There is

$$\frac{t + s + \frac{1}{\min\{b, z\}}}{t + s + \frac{1}{\max\{b, z\}}} \le \Gamma\left(\frac{t}{t + \frac{1}{\omega}}, \frac{s}{s + \frac{1}{\omega}}\right)$$

One obtains $0.8 > 0.975$, which is a contradiction. Hence, $M_\alpha$ is not an extended revised fuzzy b-metric space. Now, consider the triangular inequality (b 4) of $RF\mathrm{bMS}$ defined in Definition 4 as

$$M_\alpha(b, z, \rho(t + s)) \le \Gamma\big(M_\alpha(b, \omega, t), M_\alpha(\omega, z, s)\big).$$

One obtains

$$\frac{\rho(s + t) + \frac{1}{\min\{b, z\}}}{\rho(s + t) + \frac{1}{\max\{b, z\}}} \le \Gamma\left(\frac{t}{t + \frac{1}{\omega}}, \frac{s}{s + \frac{1}{\omega}}\right)$$

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For $\rho \in [1,9]$, the above inequality is not satisfied.

**Theorem 5.** Supposing that $(\Xi, M_\alpha, \Gamma)$ is a complete CRFMS assuming that $f: \Xi \to \Xi$, then $\aleph \in (0,1)$ exists

$$M_\alpha(fb, f\omega, t) \le \max\left\{M_\alpha\left(b, \omega, \frac{t}{\aleph}\right), M_\alpha\left(fb, \omega, \frac{t}{\aleph}\right), M_\alpha\left(b, f\omega, \frac{t}{\aleph}\right)\right\} \quad (9)$$

For all $b, \omega \in \Xi, t > 0$ such that

$$\lim_{\tau\to\infty} M_\alpha(b, \omega, t) = 0 \quad (10)$$

For all $t > 0$.Then, $f$ has a UFP in $\Xi$.

**Proof.** Suppose $b_0 \in \Xi, b_{\tau+1} = fb_\tau$, and $\tau \in \mathbb{N}$ from (9) with $b = b_\tau$ and $\omega = b_{\tau-1}$, for every $\tau \in \mathbb{N}$ and $t > 0$, one can obtain

$$M_\alpha(b_{\tau+1}, b_\tau, t) \le \max\left\{M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\aleph}\right), M_\alpha\left(b_{\tau+1}, b_\tau, \frac{t}{\aleph}\right), M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\aleph}\right)\right\}$$
$$\le \max\left\{M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\aleph}\right), M_\alpha\left(b_{\tau+1}, b_\tau, \frac{t}{\aleph}\right)\right\}$$

If $M_\alpha(b_{\tau+1}, b_\tau, t) \le M_\alpha\left(b_{\tau+1}, b_\tau, \frac{t}{\aleph}\right)$, $\tau \in \mathbb{N}$, $t > 0$. Then, Lemma 3 implies that $b_\tau, b_{\tau+1}, \tau \in \mathbb{N}$, such that $M_\alpha(b_{\tau+1}, b_\tau, t) \le M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\aleph}\right)$, $\tau \in \mathbb{N}$, $t > 0$.

Moreover, by Lemma 2 $\{b_\tau\}$ is a Continuous Sequence. Hence, $b \in \Xi$ exists such that $\lim_{\tau\to\infty} b_\tau = b$ and

$$\lim_{\tau\to\infty} M_\alpha(b, b_\tau, t) = 0, t > 0. \quad (11)$$

Now, it is shown that b is an FP for$f$. Letting $\kappa_1 \in (\aleph, 1)$ and $\kappa_2 = 1 - \kappa_1$ by (9),one can obtain

$$M_\alpha(fb, b, t) \le \Gamma\left(M_\alpha\left(fb, fb_\tau, \frac{t\kappa_1}{\alpha(fb, fb_\tau)}\right), M_\alpha\left(b_{\tau+1}, fb_\tau, \frac{t\kappa_2}{\alpha(b_{\tau+1}, fb_\tau)}\right)\right)$$
$$\le$$
$$\Gamma\left(\max\left\{\begin{matrix} M_\alpha\left(b, b_\tau, \frac{t\kappa_1}{\alpha(b, b_\tau)}\right), M_\alpha\left(b, fb, \frac{t\kappa_1}{\alpha(b, fb)}\right) \\ M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t\kappa_1}{\alpha(b_\tau, b_{\tau+1})\aleph}\right) \end{matrix}\right\}, M_\alpha\left(b_{\tau+1}, b, \frac{t\kappa_2}{\alpha(b_{\tau+1}, b_\tau)}\right)\right)$$

Taking $\tau \to \infty$ and utilizing (11), one deduces

$$M_\alpha(fb, b, t) \le \Gamma\left(\max\left\{0, M_\alpha\left(b, fb, \frac{t\kappa_1}{\alpha(b, fb)\aleph}\right), 0\right\}\right)$$
$$= \Gamma\left(M_\alpha\left(b, fb, \frac{t\kappa_1}{\alpha(b, fb)\aleph}\right), 0\right) = M_\alpha\left(b, fb, \frac{t}{v}\right), \ t > 0.$$

where $v = \frac{\alpha(b, fb)\aleph}{\kappa_1} \in (0,1)$, one has $M_\alpha(fb, b, t) \le M_\alpha\left(fb, b, \frac{t}{v}\right)$ $t > 0$, and by Lemma 3, one has $fb = b$. Suppose that $b$ and $\omega$ are to different FP for $f$, that is, $fb = b$ and $f\omega = \omega$. By (9), one deduces

$$M_\alpha(fb, f\omega, t) \le \max\left\{M_\alpha\left(b, \omega, \frac{t}{\aleph}\right), M_\alpha\left(b, fb, \frac{t}{\aleph}\right), M_\alpha\left(\omega, f\omega, \frac{t}{\aleph}\right)\right\}$$

$$=\max\left\{M_\alpha\left(b,\omega,\frac{t}{\aleph}\right),0,0\right\} = M_\alpha\left(b,\omega,\frac{t}{\aleph}\right) = M_\alpha\left(fb,f\omega,\frac{t}{\aleph}\right)$$

For $t > 0$, and by utilizing the Lemma 3, one has $fb = f\omega$, which gives $b = \omega$.

**Remark 2.** If one takes

$$\max\left\{M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha\left(fb,b,\frac{t}{\aleph}\right), M_\alpha\left(f\omega,\omega,\frac{t}{\aleph}\right)\right\} = M_\alpha\left(b,\omega,\frac{t}{\aleph}\right)$$

in the above theorem, then one obtains a revised fuzzy version of the Banach contraction principle.

**Example 4.** Suppose $\Xi = (0,2)$, $M_\alpha(b,\omega,t) = e^{-\frac{(b-\omega)^2}{t}}, \left(1 - e^{\frac{(b-\omega)^2}{t}}\right)$, and $\Gamma = \Gamma_p$ Then,$(\Xi, M, \Gamma)$is a complete CFMS with $\alpha(b,\omega) = b + \omega + 2$. Let

$$f(b) = \begin{cases} 2 - b, b \in (0,1) \\ 1, b \in [1,2) \end{cases}$$

**Part 1:** If $b,\omega \in [1,2)$, then $M_\alpha(fb,f\omega,t) = 0, t > 0$ and (9) are trivially verified.

**Part 2:** If $b \in [1,2)$ and $\omega \in (0,1)$, such that $\aleph \in \left(\frac{1}{4},\frac{1}{2}\right)$, one can obtain

$$M_\alpha(fb,f\omega,t) = e^{-\frac{(1-\omega)^2}{t}}\left(1 - e^{\frac{(1-\omega)^2}{t}}\right) = \left(1 - e^{\frac{4\aleph(1-\omega)^2}{t}}\right) =$$

$M_\alpha\left(f\omega,\omega,\frac{t}{\aleph}\right), t > 0$.

**Part 3:** As in the preceding section, for $\aleph \in \left(\frac{1}{4},\frac{1}{2}\right)$, one obtains

$$M_\alpha(fb,f\omega,t) \le M_\alpha\left(fb,b,\frac{t}{\aleph}\right), b \in (0,1), \omega \in [1,2), t > 0.$$

**Part 4:** If $b,\omega \in (0,1)$, then for $\aleph \in \left(\frac{1}{4},\frac{1}{2}\right)$, one has

$$M_\alpha(fb,f\omega,t) = e^{-\frac{(1-\omega)^2}{t}}\left(1 - e^{\frac{(1-\omega)^2}{t}}\right) = e^{-\frac{4\aleph(1-\omega)^2}{t}}\left(1 - e^{\frac{4\aleph(1-\omega)^2}{t}}\right)$$

$= M_\alpha\left(f\omega,\omega,\frac{t}{\aleph}\right), b > \omega, t > 0$ and $M_\alpha(fb,f\omega,t) \le M_\alpha\left(f\omega,\omega,\frac{t}{\aleph}\right), b > \omega, t > 0$

So, condition (9) is fulfilled for $\text{all } b,\omega \in, t > 0$ and by Theorem 5 it follows that $b = 1$ is a UFP for $f$. A new Ciric-quasi-contraction is analyzed in the following theorem.

**Theorem 6.** Supposing that $(\Xi, M, \Gamma_{max})$ is a complete CRFMS, assume that $f: \Xi \to \Xi$. If for some $\aleph \in (0,1)$, such that

$$M_\alpha(fb,f\omega,t) \le \max\left\{\begin{matrix} M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha\left(fb,b,\frac{t}{\aleph}\right), \\ M_\alpha\left(f\omega,\omega,\frac{t}{\aleph}\right), M_\alpha\left(fb,\omega,\frac{2t}{\aleph}\right), M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right) \end{matrix}\right\}, b,\omega \in \Xi, t > 0. \tag{12}$$

Then, $f$ has a UFP in $\Xi$.

**Proof.** Suppose $b_0 \in \Xi$ and $b_{\tau+1} = fb_\tau, \tau \in \mathbb{N}$. By utilizing condition (12) with $b = b_\tau$, $\omega = b_{\tau-1}$, utilizing $(FM_\alpha 4)$, and the assumption that $\Gamma = \Gamma_{max}$, one can obtain

$$M_\alpha(b_{\tau+1}, b_\tau, t) \leq$$

$$\max\left\{\begin{matrix} M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\aleph}\right), M_\alpha\left(b_{\tau+1}, b_\tau, \frac{t}{\aleph}\right), M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\aleph}\right) \\ M_\alpha\left(b_{\tau+1}, b_\tau, \frac{t}{\alpha(b_{\tau+1}, b_\tau)\aleph}\right), M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\alpha(b_\tau, b_{\tau-1})\aleph}\right) \\ M_\alpha\left(b_\tau, b_\tau, \frac{t}{\aleph}\right) \end{matrix}\right\}$$

$$\leq$$

$$\max\left\{M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\alpha(b_\tau, b_{\tau-1})\aleph}\right), M_\alpha\left(b_{\tau+1}, b_{\tau-1}, \frac{t}{\alpha(b_{\tau+1}, b_{\tau-1})\aleph}\right)\right\}, \tau \in \mathbb{N}, t > 0.$$

By Lemma 3 and Corollary 2, it is possible to demonstrate Theorem 5 such that

$M_\alpha(b_{\tau+1}, b_\tau, t) \leq M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\alpha(b_\tau, b_{\tau-1})\aleph}\right), \tau \in \mathbb{N}, t > 0$.and $\{b_\tau\}$ is a CS. So, $b \in \Xi$ exists such that $\lim_{\tau\to\infty} b_\tau = b$,

$$\lim_{\tau\to\infty} M_\alpha(b, b_\tau, t) = 0, t > 0 \tag{13}$$

Suppose $\kappa_1 \in (\aleph, 1)$ and $\kappa_2 = 1 - \kappa_1$. By (12) and $(FM_\alpha 4)$, one deduces

$$M_\alpha(fb, b, t) \leq \max\left\{M_\alpha\left(fb, fb_\tau, \frac{t\kappa_1}{\alpha(fb, fb_\tau)}\right), M_\alpha\left(fb_\tau, fb, \frac{t\kappa_2}{\alpha(fb_\tau, fb)}\right)\right\}$$

$$\leq$$

$$\max\left\{\begin{matrix} \max\left\{\begin{matrix} M_\alpha\left(b, b_\tau, \frac{t\kappa_1}{\alpha(b, b_\tau)}\right), M_\alpha\left(b, fb, \frac{t\kappa_1}{\alpha(b, fb)\aleph}\right), M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t\kappa_1}{\alpha(b_\tau, b_{\tau+1})\aleph}\right), \\ \max\left\{M_\alpha\left(fb, b, \frac{t\kappa_1}{\alpha(b, fb)\alpha(fb, b_\tau)\aleph}\right), M_\alpha\left(b, b_\tau, \frac{t\kappa_1}{\alpha(b, b_\tau)\alpha(fb, b_\tau)\aleph}\right)\right\} \\ M_\alpha\left(b, b_{\tau+1}, \frac{t\kappa_1}{\alpha(b, b_{\tau+1})\aleph}\right) \end{matrix}\right\} \\ M_\alpha\left(b_{\tau+1}, b, \frac{t\kappa_2}{\alpha(b_{\tau+1}, b)}\right) \end{matrix}\right\}$$

For all $\tau \in \mathbb{N}$ and $t > 0$. Taking $\tau \to \infty$ and utilizing (13), one obtains

$$M_\alpha(fb, b, t) \leq \max\left\{\max\left\{\begin{matrix} 0, M_\alpha\left(b, fb, \frac{t\kappa_1}{\alpha(b, fb)\aleph}\right), 0 \\ \max\left\{M_\alpha\left(fb, b, \frac{t\kappa_1}{\alpha(b, fb)\alpha(fb, b_\tau)\aleph}\right), 0\right\}, 0 \end{matrix}\right\}, 0\right\}$$

$$= M_\alpha\left(fb, b, \frac{t\kappa_1}{\alpha(b, fb)\alpha(fb, b_\tau)\aleph}\right), t > 0.$$

and by Lemma 3 with $v = \frac{\alpha(b, fb)\alpha(fb, b_\tau)\aleph}{\kappa_1} \in (0,1)$such that $f\,b = b$. By condition (12), for two different FPs $b = f\,b$and $\omega = f\omega$, one can obtain

$M_\alpha(fb, f\omega, t) \leq$

$$\max\left\{\begin{matrix} M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), M_\alpha\left(fb,b,\frac{t}{\aleph}\right), M_\alpha\left(f\omega,\omega,\frac{t}{\aleph}\right), \\ \max\left\{M_\alpha\left(fb,b,\frac{t}{\alpha(fb,b)\aleph}\right), M_\alpha\left(b,\omega,\frac{t}{\alpha(b,\omega)\aleph}\right)\right\}, M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right)\end{matrix}\right\}$$

$$= \max\left\{\begin{matrix} M_\alpha\left(b,\omega,\frac{t}{\aleph}\right), 1, 1, \\ \max\left\{1, M_\alpha\left(b,\omega,\frac{t}{\alpha(b,\omega)\aleph}\right)\right\}, M_\alpha\left(b,f\omega,\frac{t}{\aleph}\right)\end{matrix}\right\} =$$

$M_\alpha\left(b,\omega,\frac{t}{\alpha(b,\omega)\aleph}\right)$

$= M_\alpha\left(fb,f\omega,\frac{t}{\alpha(b,\omega)\aleph}\right), t > 0$. and by Lemma 3, it follows that $b = \omega$.

The next theorem aims to establish a new contractive condition with the weaker TCN.

**Example 5.** Suppose $\Xi = A \cup B$ where $A = [0,1], B = \mathbb{N}\backslash 1$ and $M_\alpha: \Xi^2 \times (0,\infty) \to [0,1]$ is a revised fuzzy metric defined by

$$M(b,\omega,t) = \begin{cases} 0 & \text{if } b = \omega \\ e^{-\frac{1}{\omega^t}} & \text{if } b \in B \text{ and } \omega \in A \\ e^{-\frac{1}{bt}} & \text{if } b \in A \text{ and } \omega \in B \\ e^{-\frac{\min\{b,\omega\}}{t}}\left(1 - e^{\frac{\min\{b,\omega\}}{t}}\right) & \text{otherwise} \end{cases}$$

Then, $(\Xi, M, \Gamma)$ is a CRFMS with $\Gamma(b,\omega) = b.\omega$ and a controlled function $\alpha: \Xi^2 \to [0,\infty)$ defined by

$$\alpha(b,\omega) = \begin{cases} 0 \text{ if } b = \omega \\ \min\{b+\omega, 1\} \text{ otherwise} \end{cases}$$

It is easy to see that all the conditions of Theorem 6 are satisfied.

Let $b = \frac{1}{2}, \omega = 40, z = 1$, and $t = s = 1$. Then, it does not satisfy the triangle inequality $(EM_\alpha 4)$ of Definition 5. Hence, it is not an extended revised fuzzy b-metric space. Now, show that it is not an RFbMS. Considering the triangle inequality (b4) of Definition 4, there is

$$e^{-\frac{\min\{b,\omega\}}{\rho(t+s)}}\left(1 - e^{\frac{\min\{b,\omega\}}{\rho(t+s)}}\right) \leq \Gamma\left(e^{-\frac{1}{\omega^t}}, e^{-\frac{1}{\omega^s}}\right) \Rightarrow e^{-\frac{1}{2\rho}}\left(1 - e^{\frac{1}{2\rho}}\right)$$

$$\leq e^{-\frac{2}{20}}\left(1 - e^{\frac{2}{20}}\right) = e^{-\frac{1}{10}}\left(1 - e^{\frac{1}{10}}\right)$$

It is clear that the above inequality is not satisfied for $\rho = 2$. Hence, it is not RFbMS.

**Theorem 7.** Supposing that $(\Xi, M, \Gamma), \Gamma \geq \Gamma_p$ is a complete CRFMS, assume that $f: \Xi \to \Xi$. For some $\aleph \in (0,1)$, let

$$M_\alpha(fb, f\omega, t) \le \max\left\{\begin{matrix} M_\alpha\left(b, \omega, \frac{t}{\aleph}\right), M_\alpha\left(fb, b, \frac{t}{\aleph}\right), M_\alpha\left(f\omega, \omega, \frac{t}{\aleph}\right), \\ \sqrt{M_\alpha\left(fb, \omega, \frac{2t}{\aleph}\right), M_\alpha\left(b, f\omega, \frac{t}{\aleph}\right)} \end{matrix}\right\}, b, \omega \in$$

$\Xi, t > 0.$ (14)

And $b, \omega \in \Xi$ exists such that $\lim_{\tau\to\infty} M_\alpha(b, \omega, t) = 0, t > 0.$ (15)

Then, $f$ has a UFP in $\Xi$.

**Proof.** Let $b_0 \in \Xi$ and $b_{\tau+1} = fb_\tau, \tau \in \mathbb{N}$.Taking $b = b_\tau$ and $\omega = b_{\tau-1}$ in (14), by $(RFM_\alpha 4)$ and $\Gamma \ge \Gamma_p$, one can obtain

$$M_\alpha(b_{\tau+1}, b_\tau, t) \le$$

$$\max\left\{\begin{matrix} M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t}{\aleph}\right), M_\alpha\left(fb, b, \frac{t}{\aleph}\right), \\ M_\alpha\left(b_{\tau+1}, b_\tau, \frac{t}{\aleph}\right), M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\aleph}\right) \\ \sqrt{M_\alpha\left(fb, \omega, \frac{2t}{\aleph}\right). M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\alpha(b_\tau, b_{\tau-1})\aleph}\right)}, M_\alpha\left(b_\tau, b_\tau, \frac{t}{\aleph}\right) \end{matrix}\right\}$$

For all $b, \omega \in \Xi, t > 0$. Since $M_\alpha(b, \omega, t)$ is a b-non-decreasing in $t$ and $\sqrt{\kappa. \rho} = \max\{\kappa, \rho\}$, one deduces

$$M_\alpha(b_{\tau+1}, b_\tau, t) \le \max\left\{\begin{matrix} M_\alpha\left(b_{\tau+1}, b_\tau, \frac{t}{\alpha(b_{\tau+1}, b_\tau)\aleph}\right), \\ M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\alpha(b_\tau, b_{\tau-1})\aleph}\right) \end{matrix}\right\} \text{ Or all } \tau \in \mathbb{N},\ t > 0.$$

By Lemmas 2 and 3, one can obtain,

$$M_\alpha(b_{\tau+1}, b_\tau, t) \le M_\alpha\left(b_\tau, b_{\tau-1}, \frac{t}{\alpha(b_\tau, b_{\tau-1})\aleph}\right), \tau \in \mathbb{N},\ t > 0.$$

Hence, $\{b_\tau\}$ is a CS. Since $(\Xi, M, \Gamma)$ is complete, $b \in \Xi$ exist such that $\lim_{\tau\to\infty} b_\tau = b$ and $\lim_{\tau\to\infty} M_\alpha(b, b_\tau, t) = 0, t > 0$ (16)

Supposing $\kappa_1 \in (\aleph, 1)$ and $\kappa_2 = 1 - \kappa_1$, by (14) and $(FM_\alpha 4)$ one can obtain

$$M_\alpha(fb, b, t) \le \Gamma\left(M_\alpha\left(fb, fb_\tau, \frac{t\kappa_1}{\alpha(fb, b_{\tau+1})\aleph}\right), M_\alpha\left(fb_\tau, b, \frac{t\kappa_2}{\alpha(b_{\tau+1}, b)\aleph}\right)\right)$$

$$\le$$

$$\Gamma\left(\max\left\{\begin{matrix} M_\alpha\left(b, b_\tau, \frac{t\kappa_1}{\alpha(b, b_\tau)\aleph}\right), M_\alpha\left(b, fb, \frac{t\kappa_1}{\alpha(b, fb)\aleph}\right), M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t\kappa_1}{\alpha(b_\tau, b_{\tau+1})\aleph}\right) \\ \sqrt{M_\alpha\left(fb, b, \frac{t\kappa_1}{\alpha(fb, b)\alpha(fb, b_\tau)\aleph}\right). M_\alpha\left(b, b_\tau, \frac{t\kappa_1}{\alpha(fb, b_\tau)\alpha(b, b_\tau)\aleph}\right)} \\ M_\alpha\left(b, b_{\tau+1}, \frac{t\kappa_1}{\alpha(b, b_{\tau+1})\aleph}\right) \end{matrix}\right\} \\ M_\alpha\left(b_{\tau+1}, b, \frac{t\kappa_2}{\alpha(b_{\tau+1}, b)}\right)\right)$$

$$\leq \Gamma\left( \max \left\{ \begin{array}{c} M_\alpha\left(b, b_\tau, \frac{t\kappa_1}{\alpha(b,b_\tau)\aleph}\right), M_\alpha\left(b, fb, \frac{t\kappa_1}{\alpha(b,fb)\aleph}\right), M_\alpha\left(b_\tau, b_{\tau+1}, \frac{t\kappa_1}{\alpha(b_\tau,b_{\tau+1})\aleph}\right) \\ \max\left\{M_\alpha\left(fb, b, \frac{t\kappa_1}{\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), M_\alpha\left(b, b_\tau, \frac{t\kappa_1}{\alpha(fb,b_\tau)\alpha(b,b_\tau)\aleph}\right)\right\} \\ M_\alpha\left(b, b_{\tau+1}, \frac{t\kappa_1}{\alpha(b,b_{\tau+1})\aleph}\right) \end{array} \right\} \\ M_\alpha\left(b_{\tau+1}, b, \frac{t\kappa_2}{\alpha(b_{\tau+1},b)}\right) \right)$$

For all $\tau \in \mathbb{N}$ and $t > 0$.Taking $\tau \to \infty$ and utilizing (16), there is

$$M_\alpha(fb, b, t) \leq \Gamma\left( \max \left\{ \begin{array}{c} 0, M_\alpha\left(b, fb, \frac{t\kappa_1}{\alpha(b,fb)\aleph}\right), 0 \\ \max\left\{M_\alpha\left(fb, b, \frac{t\kappa_1}{\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), 0\right\}, 0 \end{array} \right\} \right)$$

$$= M_\alpha\left(fb, b, \frac{t\kappa_1}{\alpha(fb,b)\alpha(fb,b_\tau)\aleph}\right), t > 0.$$

And by Lemma 3 with $v = \frac{\alpha(fb,b)\alpha(fb,b_\tau)\aleph}{\kappa_1} \in (0,1)$ such that $fb = b$.

Let $b$ and $\omega$ are two different FPs for f. By (2.13), one obtains

$$M_\alpha(fb, f\omega, t) \leq \Gamma\left( \begin{array}{c} M_\alpha\left(b, \omega, \frac{t}{\aleph}\right), M_\alpha\left(fb, b, \frac{t}{\aleph}\right), M_\alpha\left(f\omega, \omega, \frac{t}{\aleph}\right) \\ \sqrt{M_\alpha\left(fb, b, \frac{t}{\alpha(fb,b)\aleph}\right) . M_\alpha\left(b, \omega, \frac{t}{\alpha(b,\omega)\aleph}\right)}, M_\alpha\left(b, \omega, \frac{t}{\aleph}\right) \end{array} \right)$$

$$\leq \Gamma\left(M_\alpha\left(b, \omega, \frac{t}{\aleph}\right), 0, 0, \max\left\{0, M_\alpha\left(b, \omega, \frac{t}{\alpha(b,\omega)\aleph}\right)\right\}, M_\alpha\left(b, \omega, \frac{t}{\aleph}\right)\right)$$

$$= M_\alpha\left(b, \omega, \frac{t}{\alpha(b,\omega)\aleph}\right) = M_\alpha\left(fb, f\omega, \frac{t}{\alpha(fb,f\omega)\aleph}\right), t > 0.$$

And thus, by Lemma 3,one obtains $b = \omega$.

**Example 6.** Suppose $\Xi = \{0,1,3\}, M_\alpha(b, \omega, t) = e^{-\frac{(b-\omega)^2}{t}}\left(1 - e^{\frac{(b-\omega)^2}{t}}\right)$ and $\Gamma = \Gamma_p$. Then,$(\Xi, M, \Gamma)$ is a complete CRFMS with $\alpha(b, \omega) = b + \omega + 1$. Define the function $f: \Xi \to \Xi$ such that $f\,0 = f\,1 = 1$ and $f3 = 0$. Observe that if $b = \omega$ or $\omega \in \{0,1\}$, then, $M_\alpha(fb, f\omega, t) = 0, t > 0$ and (14) is fulfilled. Suppose $b = 1$ and $\omega = 3$.Then, $\aleph \in \left(\frac{1}{9}, \frac{1}{4}\right)$, one obtains

$$M_\alpha(fb, f\omega, t) = e^{-\frac{1}{t}}\left(1 - e^{\frac{1}{t}}\right) \leq \max\left\{e^{-\frac{9\aleph}{t}}, e^{-\frac{\aleph}{t}}, e^{-\frac{9\aleph}{t}}, e^{-\frac{\aleph}{t}}, 0\right\}$$

Suppose $b = 1$ and $\omega = 3$.Then, by choosing $\aleph \in \left(\frac{1}{9}, \frac{1}{4}\right)$, one has

$$M_\alpha(fb, f\omega, t) = e^{-\frac{1}{t}}\left(1 - e^{\frac{1}{t}}\right) \leq \max\left\{e^{-\frac{4\aleph}{t}}, 0, e^{-\frac{9\aleph}{t}}, e^{-\frac{\aleph}{t}}, e^{-\frac{\aleph}{t}}\right\}$$

Similarly, if $\mathrm{b}=3$ and $\omega=1$ as well as $\mathrm{b}=3$ and $\omega=1$, then for $\aleph \in \left(\frac{1}{9}, \frac{1}{4}\right)$, condition (14) is met for all $\mathrm{b}, \omega \in \Xi$, and $\mathrm{t}>0$. As a result, Theorem 7 is satisfied with a UFP $\mathrm{b}=1$.

## An application to the transformation of solar energy to electric power

Sun-based boards are currently being distributed and shown widely to reduce people's reliance on petroleum derivatives which are less environmentally friendly. Nearly 19 trillion kilowatts of power were transported internationally in 2007. In comparison, the amount of day light that enters the Earth's surface in a single hour is enough to illuminate the entire planet for a full year. The question is: how do those dazzling and warm beams of light obtain power? A numerical model of the electric flow in an RLC equal circuit, often known as a "tuning" circuit, can be presented with a basic understanding of how light is converted into power. In the fields of radio and communication engineering, this circuit has several uses. The version that is being presented can be used to calculate the production of electric power, provide tools to improve building performance, and can be used as a decision-making tool when designing a hybrid renewable electricity system based on solar power. Every aspect of this system is mathematically expressed as a differential equation in (Younis et al, 2022) using the following equation

$$\begin{cases} \frac{d^2\mathfrak{C}}{d\vartheta^2} = \Omega\left(\vartheta, \mathfrak{C}(\vartheta)\right) - \frac{\mathfrak{R}}{\mathfrak{L}}\frac{d\mathfrak{C}}{d\vartheta} \\ \mathfrak{C}(0) = 0, \mathfrak{C}'(0) = \mathrm{m} \end{cases} \tag{17}$$

where $\Omega: [0,1] \times \mathcal{R}^{+} \rightarrow \mathcal{R}$ is a continuous function that is condition (17) to the integral equation to which it is equivalent.

$$\varrho(\vartheta) = \int_0^{\vartheta} \mathrm{N}(\vartheta, \mathrm{l})\Omega(\mathrm{l}, \mathfrak{C}(\mathrm{l}))\mathrm{dl}, \vartheta \in [0,1] \tag{18}$$

where the Green's function $\mathrm{N}(\vartheta, \mathrm{b})$, it follows:

$$\mathrm{N}(\vartheta, \mathrm{b}) = \begin{cases} (\vartheta - \mathrm{l})e^{\Omega(\mathrm{M}(\mathrm{b},\omega)(\vartheta - \mathrm{l}))} 0 \leq \mathrm{c} \leq \mathrm{l} \leq 1 \\ 0, 0 \leq \vartheta \leq \mathrm{l} \leq 1 \end{cases} \tag{19}$$

where $\Omega(\mathrm{M}(\mathrm{b}, \omega)) > 0$ is a constant, as determined by the values of $\mathfrak{R}$ and $\mathfrak{L}$, mentioned in (3.1).

Let $\Xi = \mathrm{C}([0, \vartheta], \mathcal{R}^{+})$ be the set of all real continuous positive functions that are expressed on the set [0,c]. Let $\Xi$ be endowed with the CRFMS given by the following

$$\mathrm{M}(\mathrm{b}, \omega, \mathrm{t}) = \begin{cases} 0 \text{ if } \mathrm{t} = 0 \\ \sup_{\mathrm{t} \in [0,1]} \frac{\min\{\mathrm{b}, \omega\} + \mathrm{t}}{\max\{\mathrm{b}, \omega\} + \mathrm{t}} \text{ otherwise} \end{cases} \text{for all } \mathrm{b}, \omega \in \Xi \tag{20}$$

One can verify that $(\Xi, \mathrm{M}, \Gamma)$is a complete CRFMS with a controlled function $\alpha: \Xi^2 \to [0,\infty)$,defined by $\alpha(\mathrm{b},\omega) = \mathrm{b} + \omega + 1$.

It is obvious that$\mathrm{b}^*$is a solution of integral Equation (18), and as a result, a solution of differential equation (17) which governs the system of converting solar energy into electric power if and only if $\mathrm{b}^*$ is an FP of $f$. It is installed as a guarantee of the existence of FP of $f$.

**Theorem 8.** Assume the following problem fulfills:

$\mathrm{f}: [0,\vartheta]^2 \to \mathcal{R}^+$ is a continuous function;

there exists a continuous function $\mathrm{N}: [0,\vartheta]^2 \to \mathrm{R}^+$ such that $\sup_{\alpha\in[0,\vartheta]} \int_0^\vartheta \mathrm{N}(\alpha,\mathrm{l}) \geq 1$

$\max\{\mathrm{f}(\alpha,\mathrm{l},\mathrm{b}(\mathrm{l}),\mathrm{f}(\alpha,\mathrm{l},\omega(\mathrm{l}))\} \geq \mathrm{N}(\alpha,\mathrm{b})\max\{\mathrm{D}(\mathrm{b}(\mathrm{l}),\omega(\mathrm{l}))\}$ and

$\min\{\mathrm{f}(\alpha,\mathrm{l},\mathrm{b}(\mathrm{l}),\mathrm{f}(\alpha,\mathrm{l},\omega(\mathrm{l}))\} \geq \mathrm{N}(\alpha,\mathrm{b})\min\{\mathrm{D}(\mathrm{b}(\mathrm{l}),\omega(\mathrm{l}))\}$ for all $\alpha,\mathrm{l},\in [0,1]$, $\mathrm{b},\omega \in \mathcal{R}^+$and $\aleph \in (0,1)$ exists such that

$$\left(\mathrm{b}(\mathrm{l}),\omega(\mathrm{l})\right) = \min\left\{\begin{array}{c} \mathrm{M}_\alpha\left(\mathrm{b}(\mathrm{l}),\omega(\mathrm{l}),\frac{\mathrm{t}}{\aleph}\right), \mathrm{M}_\alpha\left(\mathrm{b}(\mathrm{l}),\mathrm{f}\omega(\mathrm{l}),\frac{\mathrm{t}}{\aleph}\right), \mathrm{M}_\alpha\left(\mathrm{fb}(\mathrm{l}),\omega(\mathrm{l}),\frac{\mathrm{t}}{\aleph}\right) \\ \frac{\mathrm{M}_\alpha\left(\mathrm{b}(\mathrm{l}),\mathrm{f}\omega(\mathrm{l}),\frac{\mathrm{t}}{\aleph}\right) + \mathrm{M}_\alpha\left(\mathrm{fb}(\mathrm{l}),\omega(\mathrm{l}),\frac{\mathrm{t}}{\aleph}\right)}{2} \\ \frac{\mathrm{M}_\alpha\left(\mathrm{b}(\mathrm{l}),\mathrm{f}\omega(\mathrm{l}),\frac{\mathrm{t}}{\aleph}\right) + \mathrm{M}_\alpha\left(\mathrm{fb}(\mathrm{l}),\omega(\mathrm{l}),\frac{\mathrm{t}}{\aleph}\right)}{1 + \mathrm{M}_\alpha\left(\mathrm{b}(\mathrm{l}),\omega(\mathrm{l}),\frac{\mathrm{t}}{\aleph}\right)} \end{array}\right\}$$

Differential equation (17) that represents the solar energy problem has a solution as a result and integral equation (18) also has a solution.

**Proof.** For $\mathrm{b},\omega \in \Xi$, by use of assumptions (I) to (III), one has

$$\mathrm{M}(\mathrm{fb},\mathrm{f}\omega,\mathrm{t}) = \sup_{\mathrm{t}\in[0,1]} \frac{\min\left\{\int_0^\vartheta \mathrm{N}(\vartheta,\mathrm{l})\Omega(\mathrm{l},\mathrm{b}(\mathrm{l}))\mathrm{dl}, \int_0^\vartheta \mathrm{N}(\vartheta,\mathrm{l})\Omega(\mathrm{l},\omega(\mathrm{l}))\mathrm{dl}\right\}+\mathrm{t}}{\max\left\{\int_0^\vartheta \mathrm{N}(\vartheta,\mathrm{l}))\Omega(\mathrm{l},\mathrm{b}(\mathrm{l}))\mathrm{dl}, \int_0^\vartheta \mathrm{N}(\vartheta,\mathrm{l})\Omega(\mathrm{l},\omega(\mathrm{l}))\mathrm{dl}\right\}+\mathrm{t}'}$$

$$= \sup_{\mathrm{t}\in[0,1]} \frac{\int_0^\vartheta \min\{\mathrm{N}(\vartheta,\mathrm{l})\Omega\left(\mathrm{l},\mathrm{b}(\mathrm{l})\right),\ \mathrm{N}(\vartheta,\mathrm{l})\Omega(\mathrm{l},\omega(\mathrm{l}))\}\,\mathrm{dl} + \mathrm{t}}{\int_0^\vartheta \max\left\{\mathrm{N}(\vartheta,\mathrm{l})\right)\Omega\left(\mathrm{l},\mathrm{b}(\mathrm{l})\right), \mathrm{N}(\vartheta,\mathrm{l})\Omega(\mathrm{l},\omega(\mathrm{l}))\} + \mathrm{t}'}$$

$$= \sup_{\mathrm{t}\in[0,1]} \frac{\int_0^\vartheta \mathrm{N}(\vartheta,\mathrm{l})\min\{\Omega(\mathrm{l},\mathrm{b}(\mathrm{l}))\mathrm{dl}\ \Omega(\mathrm{l},\omega(\mathrm{l}))\mathrm{dl}\} + \mathrm{t}}{\int_0^\vartheta \mathrm{N}(\vartheta,\mathrm{l})\max\{\Omega(\mathrm{l},\mathrm{b}(\mathrm{l}))\mathrm{dl}\ \Omega(\mathrm{l},\omega(\mathrm{l}))\mathrm{dl}\} + \mathrm{t}'}$$

$$\leq \sup_{\mathrm{t}\in[0,1]} \frac{\int_0^\vartheta \mathrm{N}(\vartheta,\mathrm{l})\min\left\{\mathrm{D}\left(\mathrm{b}(\mathrm{l}),\omega(\mathrm{l})\right)\right\}\mathrm{dl} + \mathrm{t}}{\int_0^\vartheta \mathrm{N}(\vartheta,\mathrm{l})\max\left\{\mathrm{D}\left(\mathrm{b}(\mathrm{l}),\omega(\mathrm{l})\right)\right\}\mathrm{dl} + \mathrm{t}'} = \mathrm{M}\left(\mathrm{D}(\mathrm{b},\omega,\mathrm{t})\right)$$

Thus, all conditions of Theorem 4 are fulfilled, i.e., the operator $\mathrm{f}$ has an FP which is the solution to differential equation (17) regulating the conversion of solar energy to electrical power.

**Open Problems 1.** The following open problem is provided for further applications of the findings in this article:

Optional appliance renewal is one of the most basic concerns in management science and engineering economics. Corporations periodically purchase new appliances and sell old ones in order to operate the equipment permanently. If $\delta(t, z)$ is the efficiency of the appliance at time period $T$ and $\delta(T)$ is the cost at the purchasing time, then,

$$e^{-\eta t}\delta(T) = \int_{T}^{\lambda^{-1}} e^{-\eta z}[\delta(T, z) - \delta(a(z), z)]du\,, -\infty < T < \infty.$$

where $z$ is the usage time of the machine and $\eta$ is the constant of the industry wide discount rate.

Can the results established in this note or their variants be applied to solve the aforementioned integral equation?

Can the results derived in this article be controlled in graphical revised fuzzy metric spaces? Can one demonstrate the aforementioned findings for multi-valued mappings?

## Conclusions

In the perspective of controlled revised fuzzy metric spaces, this manuscript contains a number of fixed point theorems and a sufficient condition for a sequence to be Cauchy. As a result, the well-known contraction requirements with controlled revised fuzzy metric spaces have been combined to simplify the proofs of several fixed point theorems. Furthermore, an application to transform solar energy to electric power has been discussed. In the future, these results will be enhanced in the framework of tripled controlled revised fuzzy metric spaces and pentagonal controlled revised fuzzy metrics spaces.

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## El punto fijo da como resultado espacios métricos difusos revisados y controlados, con una aplicación a la transformación de energía solar en energía eléctrica

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CAMPO: matemáticas
TIPO DE ARTÍCULO: artículo científico original

*Resumen:*

*Introducción/objetivo: Este estudio establece condiciones suficientes para que una secuencia sea Cauchy dentro del marco de espacios métricos difusos revisados y controlados. También generaliza el concepto del principio de contracción de Banach al introducir varias condiciones nuevas de nuevas. El objetivo es derivar varios resultados de punto fijo que mejoren la comprensión de estas estructuras matemáticas.*

*Métodos: Los investigadores emplean técnicas matemáticas rigurosas para desarrollar sus hallazgos. Al definir un conjunto de asignaciones de contracción novedosas y utilizar propiedades de espacios métricos difusos revisados y controlados, analizan las implicaciones para la convergencia de secuencias. La metodología incluye la construcción de ejemplos específicos para ilustrar los resultados teóricos.*

*Resultados: El estudio presenta varios teoremas de punto fijo derivados de las condiciones de contracción generalizada. Además, proporciona una serie de ejemplos no triviales que fundamentan las afirmaciones y demuestran la aplicabilidad de los resultados en escenarios prácticos. Se explora una aplicación importante con respecto a la conversión de energía solar en energía eléctrica, utilizando ecuaciones diferenciales para resaltar esta conexión.*

*Conclusión: Los hallazgos profundizan la comprensión de las secuencias de Cauchy en espacios métricos difusos y ofrecen una perspectiva más amplia sobre la aplicación de la teoría del punto fijo en escenarios del mundo real. Los resultados allanaron el camino para futuras investigaciones tanto en matemáticas teóricas como en sus aplicaciones prácticas, particularmente en el campo de las energías renovables.*

*Palabras claves: teoremas del punto fijo, espacio métrico difuso revisado (RFMS), principios de contracción (CP), función de Green, ecuación diferencial.*

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## Результаты неподвижной точки в управляемых пересмотренных фазовых метрических пространствах, применяемых для преобразования солнечной энергии

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РУБРИКА ГРНТИ: 27.25.17 Метрическая теория функций,
27.39.15 Линейные пространства, снабженные топологией, порядком и другими структурами
ВИД СТАТЬИ: оригинальная научная статья

*Резюме:*

*Введение/цель: В данном исследовании установлены условия для того, чтобы последовательность Коши находилась в рамках контролируемых пересмотренных нечетких метрических пространств. В статье также обобщается концепция принципа сжатия Банаха, вводя несколько новых условий сжатия. Цель статьи заключается в получении различных результатов с фиксированной точкой, которые улучшат понимание этих математических структур.*

*Методы: В исследовании применялись строгие математические методы для представления открытий. Определяя набор новых сокращающихся отображений и используя свойства контролируемых пересмотренных нечетких метрических пространств, были проанализированы импликации для сходимости последовательностей. Методология включает в себя разработку конкретных примеров, иллюстрирующих теоретические результаты.*

*Результаты: В исследовании представлено несколько теорем о неподвижной точке, полученных из обобщенных условий сжатия. Помимо того, приводится ряд нетривиальных примеров, которые обосновывают утверждения и демонстрируют применимость результатов в практических сценариях. Рассматривается важная сфера применения, связанная с преобразованием солнечной энергии в электрическую с использованием дифференциальных уравнений.*

*Выводы: Полученные результаты углубляют понимание последовательностей Коши в фазовых метрических пространствах и раскрывают более широкую перспективу для применения теории фиксированной точки в реальных сценариях. Результаты прокладывают путь для дальнейших исследований как в области теоретической математики, так и в области ее практического применения, в частности, в области возобновляемых источников энергии.*

*Ключевые слова: теоремы о неподвижной точке, пересмотренное нечеткое метрическое пространство (RFMS), принципы сжатия (CP), функция Грина, дифференциальное уравнение.*

Резултати непомичне тачке у контролисаним ревидираним фази метричким просторима примењени на претварање соларне енергије у електричну

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*Сажетак:*

*Увод/циљ: У студији се успостављају довољни услови да секвенца буде Кошијева у оквиру контролисаних ревидираних фази метричких простора. Такође, генерализује се концепт Банаховог принципа контракције увођењем неколико нових услова контракције. Циљ је да се изведу различити резултати непомичне тачке који доводе до бољег разумевања ове математичке структуре.*

*Методе: Аутори развијају своја открића коришћењем ригорозних математичких техника. Дефинисањем скупа нових пресликавања контракција и коришћењем својства контролисаних ревидираних фази метричких простора анализиранесу импликације за конвергенцију секвенце. Методологија укључује конструисање конкретних примера за илустрацију теоријских резултата.*

*Резултати: Студија представља неколико теорема непомичне тачке изведених из генерализованих услова контракције. Поред тога, наводи бројне нетривијалне примере који поткрепљују тврдње и демонстрирају применљивост резултата у практичним сценаријима. Приказана је важна примена у области претварања соларне енергије у електричну енергију помоћу диференцијалне једначине.*

*Закључак: Налази продубљују разумевање Кошијевих секвенци у фази метричким просторима и нуде ширу перспективу примене теорије непокретне тачке у сценаријима из реалног живота. Резултати отварају пут за даља истраживања, како у теоријској* *математици, тако и у њеним практичним применама, посебно у области обновљиве енергије.*

*Кључне речи: теореме непокретне тачке, прерађени фази метрички простор (RFMS), принципи контракције (CP), Гринова функција, диференцијална једначина.*

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