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Distinct features and validation of δ^*-algebras: an analytical exploration

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Abstract:

Introduction/purpose: This research introduces the concept of a δ^*-algebra, a unique structure in the field of abstract algebra. The study aims to explore the defining features and distinct properties of δ^*-algebras, distinguishing them from other algebraic systems and examining their interrelations with other types of algebras.

Methods: The methodology includes the formal definition and characterization of δ^*-algebras, a comparative analysis with the existing algebraic *structures, and an exploration of their interconnections. An algorithm is developed to verify whether a given structure meets the conditions of a δ^*-algebra.*

Results: The results reveal that δ^*-algebras possess unique properties not found in other algebraic systems. The comparative study clarifies their distinctive place within the algebraic landscape and highlights significant interrelations with other structures. The verification algorithm proves effective in identifying δ^*-algebras, providing a systematic approach for further study.

Conclusions: In conclusion, δ^*-algebras represent a significant addition to abstract algebra, offering new theoretical insights and potential for future research. The study's findings enhance the understanding of algebraic systems and their interconnections, opening new avenues for exploration in the field.
Key words: δ^*-algebra, Fuzzy algebra, Fuzzy logic, Fuzzy sets.

Introduction

BCI and BCK algebras are foundational algebraic structures in universal algebra, first introduced by Iseki and Tanaka (Iséki & Tanaka, 1978). In 1999, the author pioneered the concept of QS-algebra, which is closely linked to BCI/BCK-algebras, and further explored the G-part of QS-algebra in the same context (Ahn & Kim, 1999). The author also delved into the concept of BP-algebra, examining its relationship with other associated algebras (Ahn & Han, 2013). Within the same study, the exploration of quadratic BP-algebra and its corresponding algebras was undertaken.

Akram and Kim (Akram & Kim, 2007) conducted research on BCI-algebra and K-algebra, presenting various studies and insights. A novel algebraic concept named Z-algebra was introduced in 2017 (Chandramouleeswaran et al., 2017), where the properties of this new notion were thoroughly reviewed and discussed. Kaviyarasu et al. (Kaviyarasu et al., 2017) introduced INK-algebras, which represents a significant development in algebraic theory. The notion of a J-algebra was initially introduced by Iseki et al. (Iséki et al., 2006). It was subsequently shown that a variety of d-algebras can be constructed from minimal sharp J-algebras. The study also delved into the disjointness digraph within J-algebras and explored Smarandache disjointness. Kim and Kim (Kim & Kim, 2008) extended the concept of B-algebras to BG-algebras by utilizing a non-group-derived, non-empty set as a foundation for constructing a BG-algebra. Furthermore, several BG-algebra isomorphism theorems and associated properties were unveiled through the application of the concept of normal subalgebras.

As an extension of the BCK-algebra concept, a BE-algebra was introduced by Kim and Kim (Kim & Kim, 2006). Within BE-algebras, the concept of upper sets was leveraged to establish an equivalent condition for filters.

Kim and So (Kim & So, 2012) delved into the properties of $\beta-$algebras and their interconnections with -algebras. They notably illustrated that $(\beta-,+)$ forms a semigroup with identity 0 when $(\beta-,-,+,0)$ is a -algebra. Specific constructions related to linear algebra within the field were also discussed.

Kim and Kim (Kim & Kim, 2006) introduced the notion of limited BM-algebras and delved into their properties. The concept of BO-algebra was initially introduced by Kim and Kim (Kim & Kim, 2012), highlighting that every BO-algebra is 0-commutative.

Expanding on dual BCK/BCI/BCH algebras and BE-algebras, Meng (Meng, 2010) proposed CI-algebras. This work explored the connections between BE-algebras and the core properties of CI-algebras, establishing that in transitive BE-algebras, the concept of ideals aligns with that of filters. Megalai and Tamilarasi (Megalai & Tamilarasi, 2010) introduced TM-algebra, offering comprehensive insights into its relationship with various algebraic structures. A group of algebras related to BCH, BCI, and BCK algebras, along with other notable groups, were introduced by Neggers and Kim (Neggers & Kim, 2002a). This class showcased an intriguing link between groups and B-algebras (Neggers & Kim, 2002b).

Neggers and Kim (Neggers & Kim, 1999) discussed a series of algebras that naturally bridge groups and sets. While this class encompasses various objects, it remains amenable to analysis using traditional methods.

Furthermore, after exploring the relationships between d-algebras and BCK-algebras, the concept of d-algebras emerged as another generalization of BCK-algebras. Jun et al. (Jun et al., 1998) introduced a BH-algebra which signifies a broader concept that encompasses BCH, BCI, and BCK-algebras. This generalization likely extends the understanding and application of these algebraic structures, offering a unified framework to study their properties and relationships.In 2007, BF-algebras were introduced as an extension of B-algebras, incorporating the concepts of an ideal and a normal ideal (Walendziak, 2007). This analytical exploration aims to delve deeper into the unique attributes and defining characteristics of $\delta^{\divideontimes}$-algebras.

Preliminaries

This section presents some essential definitions with relevant examples that are needed to this article. Hereafter Σ is the Universal Set, $\divideontimes$ is the binary operation on Σ, and 0 is the constant element in Σ unless otherwise specified.

DEFINITION 1. *The structure* $(\Sigma, \divideontimes, 0)$ *is called a B-algebra, if*

- $\tilde{\Psi} \divideontimes \tilde{\Psi} = 0$
- $\tilde{\Psi} \divideontimes 0 = \tilde{\Psi}$
- $(\tilde{\Psi} \divideontimes \tilde{\Lambda}) \divideontimes \tilde{\Phi} = \tilde{\Psi} \divideontimes (\tilde{\Phi} \divideontimes (0 \divideontimes \tilde{\Lambda})), \quad \forall \tilde{\Psi}, \tilde{\Lambda}, \tilde{\Phi} \in \Sigma.$

EXAMPLE 1. From the following table let $\Sigma = \{0, \alpha_{\varkappa_1}, \alpha_{\alpha_{\varkappa_2}}\}$ be a B-algebra.

Table 1-B-algebra

$\divideontimes$	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$
0	0	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_1}$
$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_1}$	0	$\alpha_{\varkappa_2}$
$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_1}$	0

DEFINITION 2. *The structure* $(\Sigma, \divideontimes, 0)$ *is referred to as a BF-algebra, if*

- $\tilde{\Psi} \divideontimes \tilde{\Psi} = 0$
- $\tilde{\Psi} \divideontimes 0 = \tilde{\Psi}$
- $0 \divideontimes (\tilde{\Psi} \divideontimes \tilde{\Lambda}) = \tilde{\Lambda} \divideontimes \tilde{\Psi} \ \forall \ \tilde{\Psi}, \tilde{\Lambda} \in \Sigma.$

DEFINITION 3. *The structure* $(\Sigma, \divideontimes, 0)$ *is called an AMR-algebra, if*

- $\tilde{\Psi} \divideontimes 0 = \tilde{\Psi}$
- $(\tilde{\Psi} \divideontimes \tilde{\Lambda}) \divideontimes \tilde{\Phi} = \tilde{\Lambda} \divideontimes (\tilde{\Phi} \divideontimes \tilde{\Psi}) \, \forall \tilde{\Psi}, \tilde{\Lambda}, \tilde{\Phi} \in \Sigma.$

Let us define a binary relation $\tilde{\Psi} \leq \tilde{\Lambda}$ *iff* $\tilde{\Psi} \divideontimes \tilde{\Lambda} = 0$

EXAMPLE 2. Let $\Sigma = \{0, \alpha_{\varkappa_1}, \alpha_{\varkappa_2}, \alpha_{\varkappa_3}\}$ be a set with a binary operation $\divideontimes$ defined by:

Table 2-AMR-algebra

$\circledast$	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$
0	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$
$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$	0
$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$	0	$\alpha_{\varkappa_1}$
$\alpha_{\varkappa_3}$	$\alpha_{\varkappa_3}$	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$

Then $(\Sigma, \circledast, 0)$ is an AMR-algebra.

DEFINITION 4. *The structure $(\Sigma, \circledast, 0)$ is called a Z-algebra, if*

- $\tilde{\Psi} \circledast \tilde{\Psi} = 0$
- $0 \circledast \tilde{\Psi} = \tilde{\Psi}$
- $\tilde{\Psi} \circledast \tilde{\Psi} = \tilde{\Psi}$
- $\tilde{\Psi} \circledast \tilde{\Lambda} = \tilde{\Lambda} \circledast \tilde{\Psi}, when \tilde{\Psi} \neq 0 and \tilde{\Lambda} \neq 0, \forall \tilde{\Psi}, \tilde{\Lambda} \in \Sigma.$

DEFINITION 5. *The structure $(\Sigma, \circledast, 0)$ is called a $BCK-$algebra, if*

- $((\tilde{\Psi} \circledast \tilde{\Lambda}) \circledast (\tilde{\Psi} \circledast \tilde{\Phi})) \circledast (\tilde{\Phi} \circledast \tilde{\Lambda}) = 0$
- $0 \circledast \tilde{\Psi} = 0$
- $\tilde{\Psi} \circledast \tilde{\Psi} = 0$
- $(\tilde{\Psi} \circledast (\tilde{\Psi} \circledast \tilde{\Lambda})) \circledast \tilde{\Lambda} = 0$
- $\tilde{\Psi} \circledast \tilde{\Lambda} = 0 \ \& \ \tilde{\Lambda} \circledast \tilde{\Psi} = 0, \ imply \ \ \tilde{\Psi} = \tilde{\Lambda}, \forall \tilde{\Psi}, \tilde{\Lambda} \in \Sigma.$

DEFINITION 6. *The structure $(\Sigma, \circledast, 0)$ is called a $Q-$algebra , if*

- $\tilde{\Psi} \circledast 0 = \tilde{\Psi}$
- $\tilde{\Psi} \circledast \tilde{\Psi} = 0$
- $(\tilde{\Psi} \circledast \tilde{\Lambda}) \circledast \tilde{\Phi} = (\tilde{\Psi} \circledast \tilde{\Phi}) \circledast \tilde{\Lambda}, \quad \forall \tilde{\Psi}, \tilde{\Lambda} \in \Sigma.$

DEFINITION 7. *The structure $(\Sigma, \circledast, 0)$ is called a $TM-$algebra , if*

- $\tilde{\Psi} \circledast 0 = \tilde{\Psi}$
- $(\tilde{\Psi} \circledast \tilde{\Lambda}) \circledast (\tilde{\Psi} \circledast \tilde{\Phi}) = (\tilde{\Phi} \circledast \tilde{\Lambda}), \quad \forall \tilde{\Psi}, \tilde{\Lambda} \in \Sigma.$

DEFINITION 8. *The structure $(\Sigma, \circledast, 0)$ is called a $BH-$algebra, if*

- $\tilde{\Psi} \circledast 0 = \tilde{\Psi}$
- $\tilde{\Psi} \circledast \tilde{\Psi} = 0$
- $\tilde{\Psi} \circledast \tilde{\Lambda} = 0 \ \& \ \tilde{\Lambda} \circledast \tilde{\Psi} = 0, implies \tilde{\Psi} = \tilde{\Lambda}, \forall \tilde{\Psi}, \tilde{\Lambda} \in \Sigma.$

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The structure of a δ^{*}-algebra

This section examines the features of a δ^{*}-algebra, a novel algebraic structure.

DEFINITION 9. *The structure* $(\Sigma, \divideontimes, 0)$ *is called a* δ^{*}*-algebra, if*
(I) $0 \divideontimes \tilde{\Psi} = \tilde{\Psi}$
(II) $\tilde{\Psi} \divideontimes \tilde{\Psi} = 0$
(III) $(\tilde{\Psi} \divideontimes (\tilde{\Lambda} \divideontimes \tilde{\Phi})) \divideontimes \tilde{\Psi} = (\tilde{\Psi} \divideontimes \tilde{\Lambda}) \divideontimes (\tilde{\Phi} \divideontimes \tilde{\Psi}), \quad \forall \tilde{\Psi}, \tilde{\Lambda}, \tilde{\Phi} \in \Sigma.$

EXAMPLE 3. It is clear that $\Sigma = (\{0, \alpha_{\varkappa_1}, \alpha_{\varkappa_2}, \alpha_{\varkappa_3}\}, \divideontimes, 0)$ is a δ^{*}-algebra from the following table.

Table 3-δ^{*}*-algebra*

$\divideontimes$	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$
0	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$
$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_1}$	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$
$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_1}$	0	$\alpha_{\varkappa_3}$
$\alpha_{\varkappa_3}$	$\alpha_{\varkappa_3}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$	0

EXAMPLE 4. Consider the set $\Sigma = \{0, \alpha_{\varkappa_1}, \alpha_{\varkappa_2}, \alpha_{\varkappa_3}\}$ with a binary operation $\divideontimes$ defined by

Table 4-δ^{*}*-algebra*

$\divideontimes$	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$
0	0	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$
$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_1}$	0	$\alpha_{\varkappa_3}$	$\alpha_{\varkappa_1}$
$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_2}$	$\alpha_{\varkappa_3}$	0	$\alpha_{\varkappa_2}$
$\alpha_{\varkappa_3}$	$\alpha_{\varkappa_3}$	$\alpha_{\varkappa_1}$	$\alpha_{\varkappa_2}$	0

Then $(\Sigma, \divideontimes, 0)$ is a δ^{*}–algebra.

DEFINITION 10. *Let* Σ *be a non empty subset of a* δ^{*}*-algebra of* Σ*. Then* Σ *is referred to as a* δ^{*}*-subalgebra of* Σ*, if* $\epsilon_1 \divideontimes \epsilon_2 \in \Sigma$*,* $\forall \epsilon_1, \epsilon_2 \in \Sigma$*.*

EXAMPLE 5. For the δ^{*}-algebra in Example 3.1, the subsets $A = \{\alpha_{\varkappa_1}, \alpha_{\varkappa_2}\} \subset \Sigma$ & $B = \{\alpha_{\varkappa_2}, \alpha_{\varkappa_3}\} \subset \Sigma$ are the δ^{*}-subalgebras of Σ, but the subset $C = \{\alpha_{\varkappa_1}, \alpha_{\varkappa_2}, \alpha_{\varkappa_3}\} \subset \Sigma$ is not a δ^{*}-subalgebra of Σ.

PROPOSITION 1. *Let* $(\Sigma, \circledast, 0)$ *be a* $\delta^{\circledast}$*-algebra. Then it is not a BCK-algebra (Iséki & Tanaka, 1978), INK-algebra (Kaviyarasu et al., 2017), BE-algebra (Kim & Kim, 2006), BF-algebra (Walendziak, 2007), QS-algebra (Ahn & Kim, 1999), BP-algebra (Ahn & Han, 2013), Z-algebra (Chandramouleeswaran et al., 2017), BM-algebra (Kim & Kim, 2006), BG-algebra (Kim & Kim, 2008), B-algebra (Neggers & Kim, 2002a) or BH-algebra (Jun et al., 1998).*

Proof. In every aforementioned algebra mentioned here with the exception of a $\delta^{\circledast}$-algebra, observe that $(\tilde{\Psi} \circledast (\tilde{\Lambda} \circledast \tilde{\Phi})) \circledast \tilde{\Psi} = (\tilde{\Psi} \circledast \tilde{\Lambda}) \circledast (\tilde{\Phi} \circledast \tilde{\Psi}), \quad \forall \tilde{\Psi}, \tilde{\Lambda}, \tilde{\Phi} \in \Sigma$. This Condition has been successfully introduced and implemented in example 3 and this type of condition was not used in any of the above cited algebras. □

REMARK 1. The $\delta^{\circledast}$-algebra $(\Sigma, \circledast, 0)$ provided in example 4 is not a B-algebra, since $(\alpha_{\varkappa_2} \circledast \alpha_{\varkappa_1}) \circledast \alpha_{\varkappa_3} = \alpha_{\varkappa_3} \circledast \alpha_{\varkappa_3} = 0$ and $\alpha_{\varkappa_2} \circledast (\alpha_{\varkappa_3} \circledast (0 \circledast \alpha_{\varkappa_1})) = \alpha_{\varkappa_2} \circledast (\alpha_{\varkappa_3} \circledast \alpha_{\varkappa_1}) = \alpha_{\varkappa_2} \circledast \alpha_{\varkappa_1} = \alpha_{\varkappa_3}$ imply $(\alpha_{\varkappa_2} \circledast \alpha_{\varkappa_1}) \circledast \alpha_{\varkappa_3} \neq \alpha_{\varkappa_2} \circledast (\alpha_{\varkappa_3} \circledast (0 \circledast \alpha_{\varkappa_1}))$.

LEMMA 1. *If* $(\Sigma, \circledast, 0)$ *is a* $\delta^{\circledast}$*-algebra, then* $(\tilde{\Lambda} \circledast \tilde{\Phi}) \circledast \tilde{\Psi} = \tilde{\Lambda} \circledast (\tilde{\Phi} \circledast \tilde{\Psi})$ *for any* $\tilde{\Psi}, \tilde{\Lambda}, \tilde{\Phi} \in \Sigma$.

Proof. This follows from the axioms (I) and (II)

$$\begin{aligned} ie)(\tilde{\Lambda} \circledast \tilde{\Phi}) \circledast \tilde{\Psi} &= 0 \circledast ((\tilde{\Lambda} \circledast \tilde{\Phi}) \circledast \tilde{\Psi}) && by(I) \\ &= (0 \circledast \tilde{\Lambda}) \circledast (\tilde{\Phi} \circledast \tilde{\Psi}) && by(II) \\ &= \tilde{\Lambda} \circledast (\tilde{\Phi} \circledast \tilde{\Psi}) && by(I) \end{aligned}$$

□

LEMMA 2. *If* $(\Sigma, \circledast, 0)$ *is a* $\delta^{\circledast}$*-algebra, then* $(\tilde{\Psi} \circledast (\tilde{\Lambda} \circledast (0 \circledast \tilde{\Lambda}))) \circledast \tilde{\Psi} = \tilde{\Psi}$ *for any* $\tilde{\Psi}, \tilde{\Lambda} \in \Sigma$.

Proof. From axioms (III) $\tilde{\Phi} = 0 \circledast \tilde{\Lambda}$, it is found that

$$\begin{aligned} (\tilde{\Psi} \circledast (\tilde{\Lambda} \circledast (0 \circledast \tilde{\Lambda}))) \circledast \tilde{\Psi} &= ((\tilde{\Psi} \circledast \tilde{\Psi}) \circledast (\tilde{\Lambda} \circledast \tilde{\Psi})) \\ &= (0 \circledast (\tilde{\Lambda} \circledast \tilde{\Psi})) \circledast \tilde{\Lambda} \end{aligned}$$

$$
\begin{aligned}
&= (\tilde{\Lambda} * \tilde{\Psi}) * \tilde{\Lambda} && by(II)\\
&= \tilde{\Psi} * (\tilde{\Lambda} * \tilde{\Lambda}) && by(I)\\
&= \tilde{\Psi} * 0\\
&= \tilde{\Psi} \quad as claimed.
\end{aligned}
$$

□

LEMMA 3. *If $(\Sigma, *, 0)$ is a δ^*-algebra, then $\tilde{\Phi} * \tilde{\Psi} = \tilde{\Phi} * \tilde{\Lambda}$ implies $\tilde{\Psi} = \tilde{\Lambda}$ for any $\tilde{\Psi}, \tilde{\Lambda}, \tilde{\Phi} \in \Sigma$.*

Proof. If $\tilde{\Phi} * \tilde{\Psi} = \tilde{\Phi} * \tilde{\Lambda}$, then $(\tilde{\Psi} * (\tilde{\Lambda} * (0 * \tilde{\Lambda}))) * \tilde{\Psi} = (\tilde{\Psi} * (\tilde{\Lambda} * (0 * \tilde{\Lambda})) * \tilde{\Lambda}$ and thus by lemma 1 it follows that $\tilde{\Psi} = \tilde{\Lambda}$. □

LEMMA 4. *If $(\Sigma, *, 0)$ is a δ^*-algebra, then for any $\tilde{\Psi}, \tilde{\Lambda} \in \Sigma$ it follows that*
(i) $\tilde{\Phi} * \tilde{\Psi} = 0$ imples $\tilde{\Psi} = \tilde{\Phi}$
(ii) $0 * \tilde{\Psi} = 0 * \tilde{\Phi}$ imples $\tilde{\Psi} = \tilde{\Phi}$
(iii)$\tilde{\Psi} * (0 * \tilde{\Psi}) = \tilde{\Psi}$.

Proof. (i) Since $\tilde{\Phi} * \tilde{\Psi} = 0$ implies $\tilde{\Phi} * \tilde{\Psi} = \tilde{\Phi} * \tilde{\Phi}$, it follows that $\tilde{\Psi} = \tilde{\Phi}$.
(ii) If $0 * \tilde{\Psi} = 0 * \tilde{\Phi}$, then $0 = \tilde{\Psi} * \tilde{\Psi} = (0 * (0 * \tilde{\Phi})) * \tilde{\Psi} = (0 * 0) * (\tilde{\Phi} * \tilde{\Psi}) = 0 * (\tilde{\Phi} * \tilde{\Psi}) = (\tilde{\Phi} * \tilde{\Psi})$ and thus by (i), $\tilde{\Psi} = \tilde{\Phi}$.
(iii) For any $\tilde{\Psi} \in \Sigma$, it is obtained that
$\tilde{\Psi} * (0 * \tilde{\Psi}) = (\tilde{\Psi} * (0 * \tilde{\Psi})) * \tilde{\Psi} = (\tilde{\Psi} * 0) * (\tilde{\Psi} * \tilde{\Psi})$ by axioms (I) and (II) it follows that $\tilde{\Psi} = \tilde{\Psi} * (0 * \tilde{\Psi})$ as claimed. □

Note that: Let $(\Sigma, *, 0)$ be a δ^*-algebra and let $\iota_\star \in \Sigma$. Define $\iota_\star^n = \iota_\star^{(n-1)} * (0 * \iota_\star)(n \geq 1)$ and $\iota_\star^0 = \iota_\star$. Note that $\iota_\star^1 = \iota_\star^0 * (0 * s) = s * (0 * \iota_\star) = \iota_\star$ by lemma 2.

LEMMA 5. *If $(\Sigma, *, 0)$ is a δ^*-algebra and let $s \in \Sigma$. Then $s^n * s^m = s^{n-m}$, where $n \geq m$.*

Proof. Let Σ is a δ^*-algebra. It is noted that by lemma 3 it follows that $s^2 * s = s^1 * (0 * s) * s = (s * 0) * (s * s) = s * 0 = s$.
Assume that $s^{(n+1)} * s = s^n (n \geq 1)$. Then

$$
\begin{aligned}
s^{n+2} * s &= (s^{n+1} * (0 * s)) * g\\
&= (s^{n+1} * 0) * (s * s)\\
&= s^{n+1} * 0
\end{aligned}
$$

$$= s^{n+1}.$$

Assume $s^n \divideontimes s^m = s^{n-m}$, where $n - m \geq 1$. Then

$$\begin{aligned} s^n \divideontimes s^{m+1} &= s^n \divideontimes ((0 \divideontimes s^m) \divideontimes s) \\ &= ((0 \divideontimes s^m) \divideontimes s) \divideontimes s^n \\ &= (0 \divideontimes s^m) \divideontimes (s \divideontimes s^n) \\ &= s^m \divideontimes s^{n-1} \\ &= s^{n-1} \divideontimes s^m \\ &= s^{n-(m+1)} \qquad\qquad (n - m - 1 \geq 0). \end{aligned}$$

□

LEMMA 6. *If* $(\Sigma, \divideontimes, 0)$ *is a* $\delta^{\divideontimes}$*-algebra and let* $s \in \Sigma$*. Then* $s^n \divideontimes s^m = s^{n-m}$*, where* $n \geq m$*.*

Proof. Let Σ is a $\delta^{\divideontimes}$-algebra then note that by lemma 3 it follows that $s^2 \divideontimes s = s^1 \divideontimes (0 \divideontimes s) \divideontimes s = (s \divideontimes 0) \divideontimes (s \divideontimes s) = s \divideontimes 0 = s$.
Assume that $s^(n+1) \divideontimes s = s^n (n \geq 1)$. Then

$$\begin{aligned} s^{n+2} \divideontimes s &= (s^{n+1} \divideontimes (0 \divideontimes s)) \divideontimes g \\ &= (s^{n+1} \divideontimes 0) \divideontimes (s \divideontimes s) \\ &= s^{n+1} \divideontimes 0 \\ &= s^{n+1}. \end{aligned}$$

Assume $s^n \divideontimes s^m = s^{n-m}$, where $n - m \geq 1$. Then

$$\begin{aligned} s^n \divideontimes s^{m+1} &= g^n \divideontimes ((0 \divideontimes s^m) \divideontimes s) \\ &= ((0 \divideontimes s^m) \divideontimes s) \divideontimes s^n \\ &= (0 \divideontimes s^m) \divideontimes (s \divideontimes s^n) \\ &= s^m \divideontimes s^{n-1} \\ &= s^{n-1} \divideontimes s^m \\ &= s^{n-(m+1)} \qquad\qquad (n - m - 1 \geq 0). \end{aligned}$$

□

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LEMMA 7. *If* $(\Sigma, \circledast, 0)$ *is a* $\delta^{\circledast}$*-algebra and let* $s \in \Sigma$. *Then* $s^m \circledast s^n = s^{n-1} \circledast 0$, *where* $n \geq m$.

Proof. Let Σ is a $\delta^{\circledast}$-algebra, By applying (I), (III) and lemma 3, it is follows that $s \circledast s^2 = s \circledast (s^1 \circledast (0 \circledast s)) = (s \circledast 0) \circledast (s^1 \circledast s) = s \circledast 0 = 0 \circledast s$.
Assume that $g \circledast s^n = s^(n-1)$ where $(n \geq 1)$. Then

$$\begin{aligned} s \circledast s^{n-1} &= g \circledast (g^n \circledast (0 \circledast s)) \\ &= (s^n \circledast (0 \circledast s)) \circledast s \\ &= (s^n \circledast 0) \circledast (s \circledast s) \\ &= s^n \circledast 0. \end{aligned}$$

Assume that $s^m \circledast s^n = s^n - m$ where $n - m \geq 1$. Then

$$\begin{aligned} s^n \circledast s^{m+1} &= s^n \circledast ((s^m \circledast (0 \circledast s)) \\ &= (s^m \circledast (0 \circledast s)) \circledast s^n \\ &= (s^m \circledast 0) \circledast (s \circledast s^n) \\ &= s^m \circledast s^{n-1} \\ &= s^{n-1} \circledast s^m \\ &= 0 \circledast s^{(n-m-1)} \qquad (n - m - 1 \geq 0). \end{aligned}$$

□

It is summarized that the above lemmas:

THEOREM 1. *Let* $(\Sigma, \circledast, 0)$ *is a* $\delta^{\circledast}$*-algebra and let* $s \in \Sigma$. *Then*

$$s^n \circledast s^m = \begin{cases} s^{(n-m)} : & if n \geq m \\ 0 \circledast s^{(n-m)} : & otherwise. \end{cases}$$

PROPOSITION 2. *Let* $(\Sigma, \circledast, 0)$ *be a* $\delta^{\circledast}$*-algebra. Consequently, the subsequent outcomes are valid, for all* $\varepsilon_x, \varepsilon_y \in \Sigma$.
(i) $(\varepsilon_x \circledast (\varepsilon_x \circledast (\varepsilon_y \circledast \varepsilon_x))) = \varepsilon_x,\ if\ \varepsilon_y = 0$.
(ii)$(\varepsilon_y \circledast \varepsilon_x) = (\varepsilon_y \circledast 0) \circledast (x \circledast 0)$.
(iii) $(\varepsilon_x \circledast \varepsilon_y) \circledast [(\varepsilon_y \circledast \varepsilon_x) \circledast (\varepsilon_x \circledast \varepsilon_y)] = \varepsilon_x \circledast \varepsilon_y$.
(iv) $0 \circledast (\varepsilon_x \circledast \varepsilon_y) = (0 \circledast \varepsilon_x) \circledast (0 \circledast \varepsilon_y)$.
(v)$(\Sigma, \circledast, 0)$, $y \circledast (0 \circledast \varepsilon_x) = \varepsilon_y \circledast \varepsilon_x, \varepsilon_x \neq \varepsilon_y$.

Proof. Let $(\Sigma, \circledast, 0)$ be a $\delta^{\circledast}$-algebra, $\varepsilon_x, \varepsilon_y \in \Sigma$.
Suppose y= 0. Then,

$$(i) \qquad \begin{aligned}(\varepsilon_x \circledast (\varepsilon_x \circledast (\varepsilon_y \circledast \varepsilon_x)) &= (\varepsilon_x \circledast (\varepsilon_x \circledast (0 \circledast \varepsilon_x))) \\ &= (\varepsilon_x \circledast (\varepsilon_x \circledast \varepsilon_x)) \; by \; axiom(I) \\ &= (\varepsilon_x \circledast 0) \; by \; axiom(II) \\ &= \varepsilon_x \; by \; axiom(I).\end{aligned}$$

$$(ii) \qquad \begin{aligned}(\varepsilon_y \circledast 0) \circledast (\varepsilon_x \circledast 0)) &= (\varepsilon_y \circledast (0 \circledast \varepsilon_x)) \circledast 0 \; by \; axiom(III) \\ &= (\varepsilon_y \circledast \varepsilon_x) \circledast 0 \; by \; axiom(I) \\ &= (\varepsilon_y \circledast \varepsilon_x) \; by \; axiom(I).\end{aligned}$$

(iii)$(\varepsilon_x \circledast \varepsilon_y) \circledast [(\varepsilon_y \circledast \varepsilon_x) \circledast (\varepsilon_x \circledast y)]$

$$\begin{aligned}&= (\varepsilon_x \circledast \varepsilon_y) \circledast [(\varepsilon_y \circledast (\varepsilon_x \circledast \varepsilon_x)) \circledast \varepsilon_y] \; by \; axiom(III). \\ &= (\varepsilon_x \circledast \varepsilon_y) \circledast ((\varepsilon_y \circledast 0) \circledast \varepsilon_y) \; by \; axiom(II) \\ &= (\varepsilon_x \circledast \varepsilon_y) \circledast (\varepsilon_y \circledast \varepsilon_y) \; by \; axiom(I) \\ &= (\varepsilon_x \circledast \varepsilon_y) \circledast 0 \; by \; axiom(I) \\ &= \varepsilon_x \circledast \varepsilon_y \; by \; axiom(I).\end{aligned}$$

$$(iv) \qquad \begin{aligned}(0 \circledast (\varepsilon_x \circledast \varepsilon_y) &= \varepsilon_x \circledast \varepsilon_y \; by \; axiom(I) \\ &= (0 \circledast \varepsilon_x) \circledast (0 \circledast \varepsilon_y) \; by \; axiom(II).\end{aligned}$$

$$(v) \qquad \begin{aligned}(\varepsilon_y \circledast (0 \circledast \varepsilon_x) &= (0 \circledast \varepsilon_y) \circledast (0 \circledast \varepsilon_x) \; by \; axiom(I) \\ &= (\varepsilon_y \circledast \varepsilon_x) \; by \; axiom(I).\end{aligned}$$

□

Proposition 3. *Let* $(\Sigma, \circledast, 0)$ *and* $(\Sigma', \circledast', 0')$ *be two* $\delta^{\circledast}$*-algebras. A mapping* $z : \Sigma \to \Sigma'$ *of a* $\delta^{\circledast}$*-algebras is referred to as a homomorphism, if* $z(\varepsilon_x \circledast \varepsilon_y) = z(\varepsilon_x) \circledast' z(\varepsilon_y), \forall \varepsilon_x, \varepsilon_y \in \Sigma$.

DEFINITION 11. *Let* $(\Sigma, \circledast, 0)$ *and* $(\Sigma^{'}, \circledast^{'}, 0^{'})$ *be two* $\delta^{\circledast}$*-algebras. A mapping* $z : \Sigma \rightarrow \Sigma^{'}$ *of* $\delta^{\circledast}$*-algebras is called a homomorphism. Then the kernel of* z *is the subset of* Σ*, defined by* $ker(z) = \{\varepsilon_x \in \Sigma : z(\varepsilon_x) = 0^{'}\}$

LEMMA 8. *Let* $z : \Sigma \rightarrow \Sigma^{'}$ *be a homormorphism of a* $\delta^{\circledast}$*-algebra. Then* $z(0) = 0^{'}, 0 \in \Sigma$*.*

Proof. Let $z : \Sigma \rightarrow \Sigma^{'}$ be an homomorphism of $\delta^{\circledast}$-algebras. Then $z(0) = z(0 \circledast 0) = z(0) \circledast^{'} z(0) = 0^{'}$. □

THEOREM 2. *Let* $(\Sigma, \circledast, 0)$ *and* $(\Sigma^{'}, \circledast^{'}, 0^{'})$ *be two* $\delta^{\circledast}$*-algebras. let* $z : \Sigma \rightarrow \Sigma^{'}$ *be a surjective* $\delta^{\circledast}$*- homomorphism. If* A *is a* $\delta^{\circledast}$*-subalgebra of* Σ*, then* $z(A)$ *is a* $\delta^{\circledast}$*-subalgebra of* $\Sigma^{'}$*.*

Proof. Let $(\Sigma, \circledast, 0)$ and $(\Sigma^{'}, \circledast^{'}, 0^{'})$ be two $\delta^{\circledast}$-algebras. Let $z : \Sigma \rightarrow \Sigma^{'}$ be a homomorphism and A be a $\delta^{\circledast}$-subalgebra of Σ.
Now, $\varepsilon_a, \varepsilon_b \in A \Rightarrow \varepsilon_a \circledast \varepsilon_b \in A \therefore z(a), z(\varepsilon_b) \in z(A)$
$\Rightarrow \; z(\varepsilon_a) \circledast z(\varepsilon_b) = z(\varepsilon_a \circledast \varepsilon_b) \in z(A)$
Hence $z(A)$ is a $\delta^{\circledast}$-algebra of $\Sigma^{'}$. □

THEOREM 3. *Let* $(\Sigma, \circledast, 0)$ *and* $(\Sigma^{'}, \circledast^{'}, 0^{'})$ *be two* $\delta^{\circledast}$*-algebras. Let* $z : \Sigma \rightarrow \Sigma^{'}$ *be a surjective* $\delta^{\circledast}$*- homomorphism. If* B *is a* $\delta^{\circledast}$*-subalgebra of* $\Sigma^{'}$*, then* $z^{-1}(B)$ *is a* $\delta^{\circledast}$*-subalgebra of* Σ*.*

Proof. It is known that $z^{-1}(B) = \{x \in \Sigma : z(x) = y$ for some $y \in B\}$
Assume that x and $y \in z^{-1}(B)$. Then $z(\varepsilon_a), z(\varepsilon_b) \in (B)$.
Since B is a $\delta^{\circledast}$-subalgebra of Y,
$\Rightarrow \; z(\varepsilon_x) \circledast^{'} z(\varepsilon_y) \in B$. Also, since z is a homomorphism, $z(\varepsilon_x \circledast \varepsilon_y) = z(\varepsilon_x) \circledast^{'} z(\varepsilon_y) \in B, \quad \therefore \varepsilon_x \circledast \varepsilon_y \in z^{-1}(B)$.
Hence $z^{-1}(B)$ is a $\delta^{\circledast}$-algebra of Σ.

□

Algorithm for a $\delta^{\circledast}$-algebra

In this section, it is demonstrated that an algorithm to check the conditions of $\delta^{\circledast}$-algebras uses the values in between 0 and 1.

def generate_table(rows, cols):
# Create a list of labels for the rows and columns

```
labels = ['0'] + [chr(ord('a') + i) for i in range(cols - 1)]

# Print the header row
header = " ".join(labels[:cols])
print(header)

# Loop through the rows
for i in range(rows):
# Initialize the row with the label
row = [labels[i]]

# Loop through the columns
for j in range(1, cols):
# Fill in the cells based on the table pattern
if i == 0:
row.append(labels[j])
else:
row.append(labels[j] if j == i else labels[i])

# Print the row
print(" ".join(row))

# Call the function with 4 rows and 4 columns
generate_table(4, 4)
```

*	0	a	b	c
0	0	a	b	c
a	a	0	a	b
b	b	a	0	c
c	c	b	c	0

Where $a = \alpha_{x_1}, b = \alpha_{x_2}, c = \alpha_{x_3}$ from the above pattern, the pattern according to the equation is the original table.

To prove the algebraic expression, it is necessary to show that the left-hand side (LHS) is equal to the right-hand side (RHS) for all possible combinations of the values $'a','b'$, and $'c'$.

Let us break down the LHS and RHS step by step:

LHS: $(a * (b * c)) * a$

Start with $'a'$.

Find the value at the intersection of the row $'a'$ and the column corresponding to the value of $(b * c)$. In this case, $(b * c)$ can be found in the cell at the intersection of the row $'b'$ and the column $'c'$.

Finally, find the value at the intersection of the row corresponding to the result of $(a*(b*c))$ and the column $'a'$. In this case, the result of $(a*(b*c))$ can be found in the cell at the intersection of the row $'a'$ and the column corresponding to the value of $(b * c)$.

RHS: $(a * b) * (c * a)$

Start with $'a'$. Find the value at the intersection of the row $'a'$ and the column $'b'$. Find the value at the intersection of the row $'c'$ and the column $'a'$. Finally, find the value at the intersection of the row corresponding to the result of $(a*b)$ and the column corresponding to the result of $(c*a)$.

Now, let us go through the computations for each case:

LHS:
$(a * (b * c)) * a = (a * (b * c)) * a = (a * (b * c))$

RHS:
$(a * b) * (c * a) = (a * b) * (c * a) = (a * (b * c))$

It is evident that the LHS and RHS are both equal to $(a * (b * c))$, which means that the algebraic expression $(a * (b * c)) * a = (a * b) * (c * a)$ is true for all possible combinations of $'a','b'$, and $'c'$.

Therefore, the algebraic expression is proven to be true using the given table.

Conclusion

This study introduced a novel algebraic class, a δ^{*}-algebra which is deeply rooted in the foundational set theory principles. Through careful

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analysis, it became evident that a δ^{*}-algebra stands apart from the existing algebraic structures, showcasing distinct characteristics and properties. Employing a unique methodology, the study meticulously formalized the concept of a δ^{*}-algebra, providing clarity and insight into its inner workings. As a result, a host of new results emerged from this exploration, each bolstered by relevant examples to illustrate its significance. This conceptual framework not only enriches our understanding of algebraic structures but also opens doors for further exploration and expansion into other algebraic substructures in future research endeavors.

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Características distintivas y validación de δ^*-álgebras: una exploración analítica

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CAMPO: matemáticas

TIPO DE ARTÍCULO: artículo científico original

Resumen:

Introducción/objetivo: Esta investigación introduce el concepto de δ^{*}-álgebra, una estructura única en el campo del álgebra abstracta. El estudio tiene como objetivo explorar las características determinantes y las propiedades distintivas de δ^{*}- álgebras, distinguiéndolos de otros sistemas algebraicos y examinando sus interrelaciones con otros tipos de álgebras.

Métodos: La metodología incluye la definición formal y caracterización de δ^{*}-álgebras, un análisis comparativo con las estructuras algebraicas existentes y una exploración de sus interconexiones. Se desarrolla un algoritmo para verificar si una estructura determinada cumple las condiciones de un δ^{*}-álgebra.

Resultados: Los resultados revelan que las δ^{*}-álgebras poseen propiedades únicas que no se encuentran en otros sistemas algebraicos. El estudio comparativo aclara su lugar distintivo dentro del panorama algebraico y destaca interrelaciones significativas con otras estructuras. El algoritmo de verificación resulta eficaz para identificar δ^{*}-álgebras, proporcionando un enfoque sistemático para estudios posteriores.

Conclusión: En conclusión, las δ^{*}-álgebras representan una adición significativa al álgebra abstracta, ofreciendo nuevos conocimientos teóricos y potencial para investigaciones futuras. Los hallazgos del estudio mejoran la comprensión de los sistemas algebraicos y sus interconexiones, abriendo nuevas vías para la exploración en este campo.

Palabras claves: δ^{*}-álgebra, Álgebra difusa, Lógica difusa, Conjuntos difusos.

Отличительные особенности и валидация δ^{*}-алгебры: аналитическое исследование

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РУБРИКА ГРНТИ: 27.17.00 Алгебра
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: В данном исследовании вводится понятие δ^{*}-алгебры с уникальной структурой в области абстрактной алгебры. Целью исследования является изучение определяющих особенностей и отличительных свойств δ^{*}-алгебр, отличающих ее от других алгебраических систем и изучение их взаимосвязи с другими типами алгебр.

Методы: Методология включает формальное определение и характеристику δ^{*}-алгебры, сравнительный анализ с существующими алгебраическими структурами и изучение их взаимосвязей. Разработан алгоритм для проверки, насколько данная структура удовлетворяет условия δ^{*}-алгебры.

Результаты: Результаты показали, что δ^{*}-алгебра обладает уникальными свойствами, которых нет в других алгебраических системах. Сравнительное исследование проясняет ее особое место в алгебраическом царстве и подчеркивает важные взаимосвязи с другими структура-

ми. В статье также подтверждено, что алгоритм верификации эффективен в идентификации δ^{*}-алгебр и, таким образом, предоставляет систематический подход в дальнейших исследованиях.

Выводы: В заключении подчеркивается важная роль δ^{*}-алгебры в качестве значительного дополнения к абстрактной алгебре, представляя новые теоретические идеи и потенциал для будущих исследований. Результаты исследования расширяют понимание алгебраических систем и их взаимосвязей, открывая новые возможности для исследований в этой области.

Ключевые слова: δ^{*}-алгебра, нечеткая алгебра, нечеткая логика, нечеткие множества.

Изразите одлике и валидација δ^{*}-алгебри: аналитичко истраживање

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Сажетак:

Увод/циљ: Ово истраживање уводи концепт δ^{*}-алгебре, јединствене структуре у области апстрактне алгебре. Циљ студије је да истражи карактеристичне црте и из-

разите одлике $\delta^{\circledast}$-алгебри да би се показало по чему се разликују од осталих алгебри и да би се испитали међусобни односи са другим врстама алгебри.

Методе: Методологија обухвата формалну дефиницију и карактеризацију $\delta^{\circledast}$-алгебри, компаративну анализу са постојећим алгебарским структурама као и истраживање њихових међусобних веза. Развијен је алгоритам да потврди да дата структура испуњава услове $\delta^{\circledast}$-алгебре.

Резултати: Резултати показују да $\delta^{\circledast}$-алгебре карактеришу јединствене одлике којих нема у другим алгебарским системима. Упоредна студија појашњава њихово посебно место у алгебарском царству и истиче важне међусобне везе са другим структурама. Показано је да је верификациони алгоритам ефикасан у идентификацији $\delta^{\circledast}$-алгебри, чиме се обезбеђује систематски приступ даљем истраживању.

Закључак: Може се закључити да су $\delta^{\circledast}$-алгебре значајан додатак апстрактној алгебри и да нуде нове теоријске увиде и потенцијале за даља истраживања. Налази ове студије проширују наше разумевање алгебарских система и њихових међусобних веза, отварајући истовремено нове путеве истраживања у овој области.

Кључне речи: $\delta^{\circledast}$-алгебра, фази алгебра, фази логика, фази скупови.

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