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A new iteration scheme for approximating fixed points of some generalized nonexpansive mappings

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Abstract:

Introduction/purpose: A new three-step iteration process, which converges faster than the Mann iteration and the S-iteration, is introduced, as well as some convergence results for approximation of fixed points of the Suzuki generalized nonexpansive mappings and nearly asymptotically nonexpansive mappings have been established.

Methods: The authors provide a specific three-step iterative method $\{x_n\}$ *in a Banach space, defined as a sequence of convex combinations of the current iterate and its images under the mapping T, with control sequences* $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\} \subseteq (0,1)$. *The results are proved in the setting of uniformly convex Banach spaces, where T is assumed to be either a Suzuki generalized nonexpansive mapping or a nearly asymptotically nonexpansive mapping. The authors obtain both weak and strong convergence theorems by using demiclosedness principles, Suzuki generalized nonexpansive mapping properties, and suitable lemmas on the behavior of the iterates. To compare the rates of convergence, they perform numerical experiments (usually implemented in MATLAB) where the proposed three-step iteration is run in parallel with the Thakur and S-iterations. The iterates are graphed to display the error convergence per iteration.*

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Results: The new iteration scheme converges faster than the S-iteration scheme if the mapping is a contraction. The new iteration scheme converges to a fixed point of a Suzuki generalized nonexpansive mapping under suitable conditions. The new iteration scheme converges to a fixed point of a nearly asymptotically nonexpansive mapping under suitable conditions.

Conclusions: The three-step iteration algorithm is proved, both theoretically and numerically, to converge faster than the Mann iteration and the S iteration (and sometimes faster than several other existing methods) for the considered types of mappings. The authors prove weak and strong convergence theorems for fixed points of the Suzuki generalized nonexpansive mappings and nearly asymptotically nonexpansive mappings in uniformly convex Banach spaces, thus generalizing, extending, and unifying several existing fixed-point approximation results in the literature.

Key words: fixed points, iteration, uniformly convex Banach space, nonexpansive mapping, Suzuki generalized nonexpansive mapping, nearly asymptotically nonexpansive mappings, reflexive Banach space.

Introduction

Let $(X, \|.\|)$ be a normed linear space, C a nonempty subset of X and $T : C \to C$ a mapping. Then T is said to be Lipschitzian if for any given $n \in \mathbb{N}$, there exists a real number $L_n \geq 0$ such that

$$\|T^n x - T^n y\| \leq L_n \|x - y\|, \text{ for all } x, y \in C$$

If $L_1 < 1$, then T is called contraction. If $L_1 = 1$, then it is called nonexpansive mapping. If $L_n \geq 1$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} L_n = 1$, then it is called asymptotically nonexpansive (Goebel & Kirk, 1972, p.172). If $x = Tx, x \in X$, then x is called a fixed point of T. To approximate the fixed points of different kinds of mappings, the following iteration schemes are used:

The Picard Iteration Scheme:

$$\begin{aligned} x_1 &\in C, \\ x_{n+1} &= Tx_n, \quad n \in \mathbb{N}. \end{aligned}$$

The Mann iteration scheme: C is a convex subset of X,

$$\begin{aligned} x_1 &\in C, \\ x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T x_n, \\ \alpha_n &\in [0,1), \quad n \in \mathbb{N}. \end{aligned}$$

The S-iteration scheme: In 2007, Agarwal et al. introduced this iteration scheme, which is as follows:
C is a convex subset of X. Choose $w_1 \in C$ and define a sequence $\{w_n\}$ in C as follows:

$$\begin{aligned} w_{n+1} &= \alpha_n T y_n + (1-\alpha_n) T w_n, \\ y_n &= (1-\beta_n) w_n + \beta_n T w_n, \quad n \in \mathbb{N}. \end{aligned} \tag{1}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $(0,1)$.

They showed that the rate of convergence of this process is the same as that of the Picard iteration but that it converges faster than the Mann iteration process (Mann, 1953) for contraction mappings.
Thakur et al. introduced the following iteration scheme in 2016 and showed that it has a faster rate of convergence than the Picard, Mann, S-iteration and some other iterations.

The Thakur Iteration Scheme:

$$\begin{aligned} x_1 &\in C, C \text{ is convex} \\ x_{n+1} &= \alpha_n T y_n + (1-\alpha_n) T z_n, \\ z_n &= (1-\gamma_n) y_n + \gamma_n T y_n, \\ y_n &= (1-\beta_n) x_n + \beta_n T x_n, \\ \alpha_n, \beta_n, \gamma_n &\in (0,1), \quad n \in \mathbb{N}. \end{aligned}$$

Our aim is to introduce an iteration process whose rate of convergence is faster than that of the S-iteration process, and hence faster than the Mann iteration. The new iteration scheme will also be compared numerically with the the Thakur iteration scheme. In addition, we prove some convergence results using this iteration process for Suzuki generalized nonexpansive mappings and nearly asymptotically nonexpansive mappings.

Preliminaries

Sahu introduced the class of nearly Lipschitzian mappings (Sahu, 2005, p. 654), which is a generalization of the class of Lipschitzian mappings.

Definition 1. *Let C be a nonempty subset of a normed linear space X and $\{a_n\}$ be a sequence in $[0, \infty)$ converging to zero. A mapping $T : C \to C$ is said to be nearly Lipschitzian with respect to $\{a_n\}$ if for any given $n \in \mathbb{N}$, there exists a real number $L_n \geq 0$ such that*

$$\|T^n x - T^n y\| \leq L_n(\|x - y\| + a_n) \text{ for all } x, y \in C \tag{2}$$

For any fixed n, the infimum of constants L_n for which (2) holds is called the nearly Lipschitz constant and will be denoted by k_n.

Definition 2. *the nearly Lipschitzian mapping T with sequence $\{(a_n, k_n)\}$ is said to be nearly asymptotically nonexpansive (Sahu, 2005, p. 654) if $k_n \geq 1$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} k_n = 1$.*

In 2008, Suzuki (Suzuki, 2008, p.1089) generalized the nonexpansive mapping as follows:

Definition 3. *Let T be a mapping on a subset C of a Banach space E. Then T is said to satisfy the condition (C) if*

$$\frac{1}{2}||x - Tx|| \leq ||x - y|| \text{ implies } ||Tx - Ty|| \leq ||x - y||$$

for all $x, y \in C$.

Let us call a mapping satisfying the above condition a Suzuki generalized nonexpansive mapping.

Definition 4. *A self mapping T defined on a nonempty subset C of a Banach space E is said to be a quasi-nonexpansive mapping if $F(T) \neq \emptyset$ and*

$$\|Tx - p\| \leq \|x - p\|, \text{ for all } x \in C, p \in F(T).$$

Definition 5. *Let E be a Banach space and $S = \{x \in E : \|x\| = 1\}$, then the norm of E is said to be Fréchet differentiable if for each $x \in S$, the limit*

$$\lim_{t\to 0} \frac{\|x + ty\| - \|x\|}{t}$$

is attained uniformly for $y \in S$.

Definition 6. *A Banach space* E *is said to satisfy the Opial condition (Opial, 1967, p.592) if for each sequence* $\{x_n\}$ *converging weakly to a point* $x \in E$, *we have*

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|$$

for all $y \in X$ *such that* $y \neq x$.

Definition 7. *A Banach space* E *is said to have the Kadec-Klee property if the following condition holds:*
If $\{x_n\}$ *is a sequence in* E *such that* $\{x_n\}$ *converges weakly to* x *and* $\|x_n\|$ *converges strongly to* $\|x\|$, *then* $\{x_n\}$ *converges strongly to* x.

Definition 8. *Let* C *be a nonempty subset of a Banach space* E, *then a mapping* $T : C \to E$ *is called demiclosed (Opial, 1967, p.591) if its graph in* $C \times E$ *is closed in the topology of a Cartesian product induced in* $C \times E$ *by the weak topology in* C *and the strong topology in* E. *That is, if* $\{x_n\}$ *is a sequence in* C, *converging weakly to an* $x_0 \in C$ *and* $\{Tx_n\}$ *converging strongly to a* $y_0 \in X$, *then* $Tx_0 = y_0$.

Definition 9. *A Banach space* E *is said to be uniformly convex if for given* $\epsilon \in (0, 2]$, *there exists a* $\delta = \delta(\epsilon) > 0$ *such that*

$$\|\tfrac{x+y}{2}\| \leq 1 - \delta \text{ whenever } \|x\| \leq 1, \|y\| \leq 1 \text{ and } \|x - y\| \geq \epsilon.$$

Throughout this paper, $\omega_w(\{x_n\})$ will denote the set of all weak subsequential limits of $\{x_n\}$, $\mathbb{N}$ will denote the set of all natural numbers and $F(T)$ will denote the set of all fixed points of mapping T.
The following proposition and lemmas will be used in our theorems later on.

Proposition 1. *(2008) Let* E *be a Banach space,* $\phi \neq C \subseteq E$ *and* $T : C \to C$ *be a mapping.*

(i) If T *is a nonexpansive mapping, then it is a Suzuki generalized nonexpansive mapping.*

(ii) If T *is a Suzuki generalized nonexpansive mapping and has a fixed point, then it is a quasi-nonexpansive mapping.*

(iii) If T is a Suzuki generalized nonexpansive mapping, then

$$\|x - Ty\| \leq 3\|Tx - x\| + \|x - y\| \text{ for all } x, y \in C.$$

Lemma 1. *(Schu, 1991, p. 1555) Let $(E, ||.||)$ be a uniformly convex Banach space, $0 < b < c < 1, a > 0, \{t_n\}$ is a sequence in $[b, c]$, $\{x_n\}, \{y_n\}$ are sequences in E such that $\lim_{n\to\infty} \sup ||x_n|| \leq a$, $\lim_{n\to\infty} \sup ||y_n|| \leq a$ and $\lim_{n\to\infty} ||t_n x_n + (1 - t_n)y_n|| = a$. Then $\lim_{n\to\infty} ||x_n - y_n|| = 0$.*

Lemma 2. *(Suzuki, 2008, p.1093) Let T be a mapping on a weakly closed subset C of a Banach space E with the Opial property. Assume that T is a Suzuki generalized nonexpansive mapping. Then $I - T$ is demiclosed at zero.*

Lemma 3. *(Sahu & Beg, 2008, p.141) Let E be a uniformly convex Banach space satisfying the Opial condition, C a nonempty closed convex subset of E and T: C $\rightarrow$ C a uniformly continuous nearly asymptotically nonexpansive mapping. Then I-T is demiclosed at zero.*

Lemma 4. *(Agarwal et al., 2007, pp. 65-66) Let C be a non-empty convex subset of a Banach space E and let $H_n : C \to E \;\; (n = 1, 2, ...)$ be mappings with $\bigcap_{n\in\mathbb{N}} F(H_n) \neq \phi$ satisfying*

$$||H_n x - H_n y|| \leq L_n ||x - y|| + \rho_n$$

for all $x, y \in C$ and $n \in \mathbb{N}$, where $\{L_n\}$ and $\{\rho_n\}$ are sequences of real numbers such that

(i) $L_n \geq 1$ for all $n \in \mathbb{N}$ and $\sum_{n=1}^{\infty}(L_n - 1) < \infty$,

(ii) $\rho_n \geq 0$ for all $n \in \mathbb{N}$ and $\sum_{n=1}^{\infty} \rho_n < \infty$.

Let $\{x_n\}$ be a sequence in C defined by

$$x_{n+1} = H_n x_n \text{ for all } n \in \mathbb{N}.$$

Then the following holds:

(a) $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in \bigcap_{n\in\mathbb{N}} F(H_n)$;

(b) If E is uniformly convex, then $\lim_{n\to\infty} \|tx_n + (1 - t)q_1 - q_2\|$ exists for all $q_1, q_2 \in \bigcap_{n\in\mathbb{N}} F(H_n)$ and $t \in [0, 1]$;

(c) *If E is a real uniformly convex Banach space with Fréchet differentiable norm, then $\lim_{n\to\infty} \langle x_n, J(q_1 - q_2)\rangle$ exists for all $q_1, q_2 \in \bigcap_{n\in\mathbb{N}} F(H_n)$, where J is a normalized duality mapping.*

Lemma 5. *(Agarwal et al., 2007, pp. 68-69) Let E be a reflexive Banach space satisfying the Opial condition, C a nonempty, closed, convex subset of E and $T : C \to E$ a mapping such that*

1. $F(T) \neq \emptyset$,
2. $I - T$ *is demiclosed at zero.*

Let $\{x_n\}$ be a sequence in C satisfying the following properties:

(D_1) $\lim_{n\to\infty} ||x_n - p||$ *exists for all* $p \in F(T)$;

(D_2) $\lim_{n\to\infty} ||x_n - Tx_n|| = 0$.

Then $\{x_n\}$ converges weakly to a fixed point of T.

Lemma 6. *(Agarwal et al., 2007, p. 69) Let C be a nonempty, closed, convex subset of a real Banach space E and $T : C \to C$ a mapping such that*

(a) $F(T) \neq \emptyset$

(b) $I - T$ *is demiclosed at zero.*

Let $\{x_n\}$ be a sequence in C which satisfies property (D_2) and one of the following conditions:

(i) *E is uniformly convex with the Fréchet differentiable norm and*

$$\lim_{n\to\infty} \langle x_n, J(q_1 - q_2)\rangle \text{ exists for all } q_1, q_2 \in F(T)$$

(ii) *E is reflexive, E^* has the Kadec-Klee property and*

$$\lim_{n\to\infty} \|tx_n + (1-t)q_1 - q_2\| \text{ exists}$$

for all $q_1, q_2 \in \omega_w(\{x_n\})$ and for all $t \in [0, 1]$.

Then $\{x_n\}$ converges weakly to a fixed point of T.

The following theorem regarding the existence of a fixed point of a nonexpansive mapping was proved independently by Browder, Kirk and Göhde in 1965.

Theorem 1. *Let C be a nonempty, closed, bounded and convex subset of a uniformly convex Banach space and $T : C \to C$ a nonexpansive mapping. Then T has a fixed point.*

The above theorem was generalized by Goebel and Kirk in 1972, which follows.

Theorem 2. *Every asymptotically nonexpansive self-mapping of a nonempty closed bounded and convex subset of a uniformly convex Banach space has a fixed point.*

The following theorem regarding the existence of a fixed point of the Suzuki generalized nonexpansive mapping was proved by Suzuki in 2008.

Theorem 3. *Let C be a weakly compact convex subset of a uniformly convex Banach space and $T : C \to C$ a Suzuki generalized nonexpansive mapping. Then T has a fixed point.*

C-iteration scheme and convergence analysis

We introduce the following iteration scheme:
Let C be a convex subset of a linear space X and T be a mapping from C into itself. Choose $x_1 \in X$ and define a sequence $\{x_n\}$ as follows

$$\begin{aligned} x_{n+1} &= \alpha_n T y_n + (1-\alpha_n) T z_n, \\ y_n &= (1-\beta_n) x_n + \beta_n T x_n, \\ z_n &= (1-\gamma_n) T x_n + \gamma_n T^2 x_n, \quad n \in \mathbb{N}. \end{aligned}$$

where $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ are sequences in $(0,1)$. We will call it the C-iteration scheme. x_{n+1} can be written as

$$x_{n+1} = C(x_n, \alpha_n, \beta_n, \gamma_n, T), n \in \mathbb{N} \tag{3}$$

where

$$\begin{aligned} C(x_n, \alpha_n, \beta_n, \gamma_n, T) =& \alpha_n T[(1-\beta_n)x_n + \beta_n T x_n] + (1-\alpha_n) T[(1-\gamma_n) T x_n \\ & + \gamma_n T^2 x_n]. \end{aligned}$$

If $\{x_n\}$ and $\{w_n\}$ are two iteration schemes converging to the same fixed point p, then according to Rhoades (1976), $\{x_n\}$ is better than $\{w_n\}$ if $||x_n - p|| \leq ||w_n - p||$ for all $n \in \mathbb{N}$

Proposition 2. *Let C be a nonempty closed convex subset of a Banach space E and $T : C \to C$ a contraction mapping with Lipschitz constant*

k *and a unique fixed point* p. *Let* $\{w_n\}$ *and* $\{x_n\}$ *are sequences obtained by S-iteration and C-iteration schemes respectively, where* $w_1 = x_1$. *Then* $||x_n - p|| \leq ||w_n - p||$ *for all* $n \in \mathbb{N}$*, i.e.* C *iteration is better than the S iteration.*

Proof. Agarwal et al. (2007) proved that $||w_{n+1} - p|| \leq k[1 - (1 - k)\alpha_n\beta_n]||w_n - P||$ for all $n \in \mathbb{N}$. If $a_n = k[1 - (1 - k)\alpha_n\beta_n]$ then the above inequality can be rewritten as

$$\begin{aligned} ||w_{n+1} - p|| &\leq a_n||w_n - p|| \\ &\leq a_n a_{n-1} \ldots a_1||w_1 - p|| \end{aligned} \tag{4}$$

As $y_n = (1 - \beta_n)x_n + \beta_n Tx_n$, we obtain

$$\begin{aligned} ||y_n - p|| &= ||(1 - \beta_n)(x_n - p) + \beta_n(Tx_n - p)|| \\ &\leq (1 - \beta_n)||(x_n - p)|| + \beta_n||(Tx_n - p)|| \\ &\leq (1 - \beta_n)||(x_n - p)|| + k\beta_n||(x_n - p)|| \\ &= \{(1 - \beta_n) + k\beta_n\}||(x_n - p)|| \\ &= \{1 - (1 - k)\beta_n\}||(x_n - p)|| \end{aligned} \tag{5}$$

As $z_n = (1 - \gamma_n)Tx_n + \gamma_n T^2 x_n$, we obtain

$$\begin{aligned} ||z_n - p|| &= ||(1 - \gamma_n)(Tx_n - p) + \gamma_n(T^2x_n - p)|| \\ &\leq (1 - \gamma_n)||(Tx_n - p)|| + \gamma_n||(T^2x_n - p)|| \\ &\leq k(1 - \gamma_n)||(x_n - p)|| + k^2\gamma_n||x_n - p)|| \\ &= \{1 - (1 - k)\gamma_n k\}||x_n - p||. \end{aligned} \tag{6}$$

Now

$$\begin{aligned} ||x_{n+1} - p|| &= ||\alpha_n(Ty_n - p) + (1 - \alpha_n)(Tz_n - p)|| \\ &\leq \alpha_n k||(y_n - p)|| + k(1 - \alpha_n)||(z_n - p)|| \end{aligned} \tag{7}$$

From (5), (6) and (7), we obtain

$$\begin{aligned} ||x_{n+1} - p|| \leq &\alpha_n k[\{1 - (1 - k)\beta_n\}||(x_n - p)||] \\ &+ k(1 - \alpha_n)[\{1 - (1 - k)\gamma_n k\}||x_n - p||] \\ \leq &k[1 - (1 - k)\{\alpha_n\beta_n + (1 - \alpha_n)\gamma_n k\}]||x_n - p|| \end{aligned} \tag{8}$$

VOJNOTEHNIČKI GLASNIK / MILITARY TECHNICAL COURIER, 2026, Vol. 1, Issue 74

If $b_n = k[1-(1-k)\{\alpha_n\beta_n + (1-\alpha_n)\gamma_n k\}]$, then above inequality can be written as

$$\begin{aligned} ||x_{n+1} - p|| &\leq b_n||x_n - p|| \\ &\leq b_n b_{n-1}......b_1||x_1 - p|| \end{aligned} \tag{9}$$

It is clear that $b_n \leq a_n$, thus from (4) and (9), we obtain

$$||x_{n+1} - p|| \leq ||w_{n+1} - p|| \tag{10}$$

Thus, the C - iteration converges better than the S-iteration. □

In the following example, we compare the rate of convergence of various iteration processes.

Example 1. Let $E = \mathbb{R}, C = [1, 100]$ and $T : C \to C$ be a mapping defined by $T(x) = \sqrt{x^2 - 10x + 30}$. Choosing $\alpha_n = \beta_n = \gamma_n = \frac{1}{2}$ and starting with the initial value 30, the successive iterations of various schemes are shown in the following table:

All the three sequences converge to 3, which is the fixed point of T. The table shows that the C iteration converges fastest among the others mentioned in the table.

Table 1 – Comparison of the rate of convergence

Step	Thakur Iteration	S-Iteration	C-Iteration
1	30	30	30
2	21.4448278	23.88014761	20.23163969
3	13.01007821	17.804770146	10.66340962
4	5.068991397	11.81891627	3.19808755
5	2.762120635	6.09946412	2.99531744
6	3.014495235	2.41807081	3.00008615
7	2.99905689	3.23583758	2.99999840
8	3.00006111	2.91017776	3.00000003
9	2.999996039	3.03518482	2.99999999
10	3.000000257	2.98635715	3.00000000
11	2.99999998	3.00531152	3.00000000
12	3.000000001	2.99793532	3.00000000
13	3.00000000	3.00080307	3.00000000
14	3.00000000	2.99968772	3.00000000
15	3.00000000	3.00012145	3.00000000
16	3.00000000	2.99995277	3.00000000
17	3.00000000	3.00001837	3.00000000
18	3.00000000	2.99999286	3.00000000
19	3.00000000	3.00000278	3.00000000
20	3.00000000	2.99999892	3.00000000
21	3.00000000	3.00000042	3.00000000
22	3.00000000	2.99999984	3.00000000
23	3.00000000	3.00000006	3.00000000
24	3.00000000	2.99999998	3.00000000
25	3.00000000	3.00000001	3.00000000
26	3.00000000	2.999999996	3.00000000
27	3.00000000	3.000000001	3.00000000
28	3.00000000	2.999999999	3.00000000
29	3.00000000	3.00000000	3.00000000

Remark 1. In Proposition 2, if T is quasi-nonexpansive with a fixed point p, then inequalities (5), (6) and (8) become

$$
\begin{aligned}
||y_n - p|| &\leq ||x_n - p||, && (11)\\
||z_n - p|| &\leq ||x_n - p|| \text{ and} && (12)\\
||x_{n+1} - p|| &\leq ||x_n - p|| && (13)
\end{aligned}
$$

respectively.

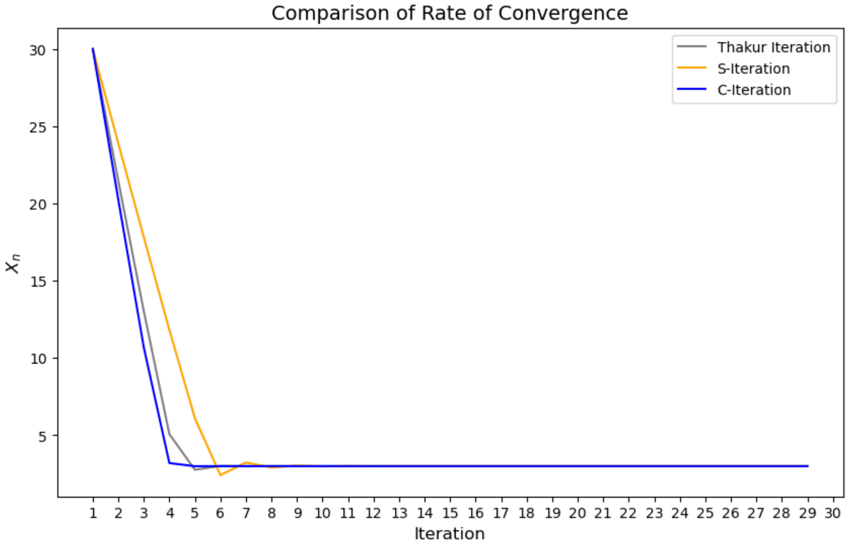

Figure 1 – Comparison of the rate of convergence

Now we will define the modified C-iteration as follows:
Let C be a convex subset of a normed linear space X and T be a mapping from C into itself. Choose $x_1 \in C$ and define a sequence $\{x_n\}$ in C as follows:

$$x_{n+1} = C(x_n, \alpha_n, \beta_n, \gamma_n, T^n), n \in \mathbb{N}. \tag{14}$$

That is

$$x_{n+1} = \alpha_n T^n[(1-\beta_n)x_n + \beta_n T^n x_n] + (1-\alpha_n)T^n[(1-\gamma_n)T^n x_n + \gamma_n T^{n+1} x_n]$$

where $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ are sequences in $(0, 1)$. Equivalently, it can be written as

$$x_{n+1} = \alpha_n T^n y_n + (1-\alpha_n)T^n z_n \tag{15}$$
$$y_n = (1-\beta_n)x_n + \beta_n T^n x_n \tag{16}$$
$$z_n = (1-\gamma_n)T^n x_n + \gamma_n T^{n+1} x_n, \quad n \in \mathbb{N} \tag{17}$$

Fixed point results for the Suzuki generalized nonexpansive mappings

The following Proposition and Lemma will be used to prove our main results in this section.

Proposition 3. *Let E be a Banach space, C a nonempty, convex subset of E, $T : E \to E$ be a quasi nonexpansive mapping with $F(T) \neq \emptyset.\{x_n\}$ be a sequence in E defined by C-iteration scheme. Then $\lim_{n\to\infty} ||x_n - p||$ exists for all $p \in F(T)$.*

Proof. If T is a quasi nonexpansive mapping with a fixed point p, then from (13), we obtain

$$||x_{n+1} - p|| \leq ||x_n - p||$$

for all $n \in \mathbb{N}$. That is, the sequence of real numbers $\{||x_n - p||\}$ is non-increasing and bounded below, so $\lim_{n\to\infty} ||x_n - p||$ exists for all $p \in F(T)$. □

Lemma 7. *Let E be a uniformly convex Banach space, C a nonempty, convex subset of E, T be a Suzuki generalized nonexpansive self mapping of C. Let $\{x_n\}$ be defined by the C-iteration scheme, given by (3), where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[a, b]$ for some $a, b \in \mathbb{R}$ with $0 < a \leq b < 1$. If $F(T) \neq \phi$, then $\lim_{n\to\infty} ||x_n - Tx_n|| = 0$.*

Proof. Since T is a Suzuki generalised nonexpansive mapping with $F(T) \neq \phi$, it is a quasi-nonexpansive by Proposition 1. Hence, by Propostion 3, $\lim_{n\to\infty} ||x_n - p||$ exists for all $p \in F(T)$. Let this limit be l. From (11) and (12), we obtain

$$\limsup_{n\to\infty} ||y_n - p|| \leq l \text{ and} \tag{18}$$

$$\limsup_{n\to\infty} ||z_n - p|| \leq l. \tag{19}$$

Since $||Tx_n - p|| \leq ||x_n - p||, ||Ty_n - p|| \leq ||y_n - p||, ||Tz_n - p|| \leq ||z_n - p||$, we obtain

$$\limsup_{n\to\infty} ||Tx_n - p|| \leq l; \tag{20}$$

$$\limsup_{n\to\infty} ||Ty_n - p|| \leq l; \tag{21}$$

$$\limsup_{n\to\infty} ||Tz_n - p|| \leq l. \tag{22}$$

Now

$$l = \limsup_{n\to\infty} ||x_{n+1} - p|| = \lim_{n\to\infty} ||\alpha_n(Ty_n - p) + (1 - \alpha_n)(Tz_n - p)||$$

thus, by Lemma 1, we obtain

$$\lim_{n\to\infty} ||Ty_n - Tz_n|| = 0. \tag{23}$$

Now

$$\begin{aligned}||x_{n+1}-p|| &= ||Tz_n - p + \alpha_n(Ty_n - Tz_n)|| \\ &\leq ||Tz_n - p|| + \alpha_n||(Ty_n - Tz_n)||.\end{aligned}$$

This implies

$$l \leq \lim_{n\to\infty} \inf ||Tz_n - p|| \tag{24}$$

This implies using (22) that

$$\lim_{n\to\infty} ||Tz_n - p|| = l.$$

Now, we have

$$\begin{aligned}||Tz_n - p|| &\leq ||Tz_n - Ty_n|| + ||Ty_n - p|| \\ &\leq ||Tz_n - Ty_n|| + ||y_n - p||.\end{aligned}$$

This implies

$$l \leq \lim_{n\to\infty} \inf ||y_n - p||$$

This inequality with (18) yields

$$\lim_{n\to\infty} ||y_n - p|| = l. \tag{25}$$

Moreover

$$l = \lim_{n\to\infty} ||y_n - p|| = \lim_{n\to\infty} ||(1-\beta_n)(x_n - p) + \beta_n(Tx_n - p)|| \tag{26}$$

Thus, by Lemma 1, we obtain

$$\lim_{n\to\infty} ||x_n - Tx_n|| = 0.$$

□

Theorem 4. *Let E be a uniformly convex Banach space satisfying the Opial condition, C a nonempty, weakly closed, convex subset of E. T is a Suzuki generalized nonexpansive self mapping of C. Let $\{x_n\}$ be defined by the C-iteration scheme, given by (3), where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[a, b]$ for some $a, b \in \mathbb{R}$ with $0 < a \leq b < 1$. If $F(T) \neq \emptyset$, then $\{x_n\}$ converges weakly to a fixed point of T.*

Proof. Since T is a Suzuki generalized nonexpansive with $F(T) \neq \emptyset$, it is a quasi nonexpansive mapping by Proposition 1 and hence $\{x_n\}$ has the property (D_1) and (D_2) by Proposition 3 and Lemma 7 respectively. $I - T$ is demiclosed by Lemma 2. Since E is uniformly convex, it is reflexive; hence, using Lemma 5, we conclude that sequence $\{x_n\}$ converges weakly to a fixed point of T. □

Corollary 1. *Let E be a uniformly convex Banach space satisfying the Opial condition, C a nonempty, closed, bounded and convex subset of E. T be a nonexpansive self mapping of C. Let $\{x_n\}$ be defined by the C-iteration scheme, given by (3), where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[a, b]$ for some $a, b \in \mathbb{R}$ with $0 < a \leq b < 1$. Then $\{x_n\}$ converges weakly to a fixed point of T.*

Proof. $F(T) \neq \phi$ by Theorem 1. C is closed, therefore it is weakly closed. T is nonexpansive, therefore it is a Suzuki generalized nonexpansive map. Thus, the result follows from Theorem 4. □

Theorem 5. *Let E be a uniformly convex Banach space, C a nonempty, compact, convex subset of E. Let T be a Suzuki generalized nonexpansive self mapping of C. Let $\{x_n\}$ be defined by the C-iteration scheme, given by (3), where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[a, b]$ for some $a, b \in \mathbb{R}$ with $0 < a \leq b < 1$. Then $\{x_n\}$ converges strongly to a fixed point of T.*

Proof. $F(T) \neq \phi$ by Theorem 3, therefore by Lemma 7, we obtain $\lim_{n\to\infty} \|Tx_n - x_n\| = 0$. Since C is compact, the sequence $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ strongly converging to a point $p \in C$. By Proposition 1, we obtain

$$\|x_{n_k} - Tp\| \leq 3\|Tx_{n_k} - x_{n_k}\| + \|x_{n_k} - p\| \quad \text{for all} \quad k \geq 1$$

Letting $k \to \infty$, we obtain $Tp = p$. Since $\lim_{n\to\infty} \|x_n - p\|$ exists by Proposition 3, $\{x_n\}$ converges strongly to p. □

Fixed point results for nearly asymptotically nonexpansive mappings

The following Lemma will be used to prove our main results in this section.

Lemma 8. *Let E be a uniformly convex Banach space, C a nonempty, closed, convex subset of E. Let $T : C \to C$ be a nearly asymptotically nonexpansive mapping with a sequence $\{(a_n, k_n)\}$ and $F(T) \neq \phi$ such that $\sum_{n=1}^{\infty} a_n < \infty$ and*
$\sum_{n=1}^{\infty} (k_n\sqrt{k_1} - 1) < \infty$. Let $\{x_n\}$ be defined by the modified C-iteration given by (14), where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[a, b]$ for some $a, b \in \mathbb{R}$ with $0 < a \leq b < 1$. Then

(i) $\lim_{n\to\infty} ||x_n - p||$ *exists;*

(ii) $\lim_{n\to\infty} ||x_n - T^n x_n|| = 0$*;*

(iii) $\{x_n\}$ *satisfies property* (D_2)*, if T is uniformly continuous.*

Proof. (i) We define a mapping $H_n : C \to C$ by

$$H_n(x) = \alpha_n T^n[(1 - \beta_n)x + \beta_n T^n x] + (1 - \alpha_n)T^n[(1 - \gamma_n)T^n x + \gamma_n T^{n+1}x]$$

where $n \in \mathbb{N}$. Now

$$\begin{aligned}
||H_n x - H_n y|| \leq& \alpha_n[k_n(||(1 - \beta_n)(x - y) + \beta_n(T^n x - T^n y)|| + a_n)] \\
&+ (1 - \alpha_n)[k_n(||(1 - \gamma_n)(T^n x - T^n y) + \gamma_n(T^{n+1}x - T^{n+1}y)|| \\
&+ a_n)] \\
\leq& \alpha_n[k_n\{(1 - \beta_n)||x - y|| + \beta_n k_n(||x - y|| + a_n) + a_n\}] \\
&+ (1 - \alpha_n)[k_n\{(1 - \gamma_n)k_n(||x - y|| + a_n) + \gamma_n k_{n+1}(||x - y|| \\
&+ a_{n+1}) + a_n\}] \\
\leq& [k_n(1 - \beta_n) + \beta_n k_n^2 + k_n\{(1 - \gamma_n)k_n + \gamma_n k_{n+1}\}]||x - y|| \\
&+ 2k_n^2 a_n + a_n k_n + k_n k_{n+1} a_{n+1} \\
\leq& [\alpha_n k_n^2 + (1 - \alpha_n)k_n \max\{k_n, k_{n+1}\}]||x - y|| + 2k_n^2 a_n + a_n k_n \\
&+ k_n k_{n+1} a_{n+1} \\
\leq& [\max\{k_n^2, k_n \max\{k_n, k_{n+1}\}\}]||x - y|| + 2k_n^2 a_n + a_n k_n \\
&+ k_n k_{n+1} a_{n+1} \\
\leq& [\max\{k_n^2, k_n k_{n+1}\}]||x - y|| + 2k_n^2 a_n + a_n k_n + k_n k_{n+1} a_{n+1} \\
=& L_n||x - y|| + \rho_n
\end{aligned}$$

where $L_n = \max\{k_n^2, k_nk_{n+1}\}$ and $\rho_n = 2k_n^2a_n + a_nk_n + k_nk_{n+1}a_{n+1}$
If $K = \sup\{k_n : n \in \mathbb{N}\}$ then

$$\begin{aligned}\sum_{n=1}^{\infty}(L_n - 1) =& \sum_{n=1}^{\infty}(\max\{k_n^2, k_nk_{n+1}\} - 1) \\ \leq& \sum_{n=1}^{\infty}(\max\{k_n^2, k_n^2k_1\} - 1) \\ \leq& \sum_{n=1}^{\infty}(k_n^2k_1 - 1) \\ \leq& (K\sqrt{k_1} + 1)\sum_{n=1}^{\infty}(k_n\sqrt{k_1} - 1) \\ <& \infty\end{aligned}$$

and

$$\sum_{n=1}^{\infty}\rho_n = (2K^2 + K)\sum_{n=1}^{\infty}a_n + K^2\sum_{n=1}^{\infty}a_{n+1} < \infty.$$

It can be easily shown that $F(T) \subseteq F(H_n)$ for all $n \in \mathbb{N}$. Therefore by Lemma 4 $\lim_{n\to\infty} ||x_n - p||$ exists where $p \in F(T)$.
(ii) Let $\lim_{n\to\infty} ||x_n - p|| = l$.
Since

$$\begin{aligned}||y_n - p|| &\leq (1 - \beta_n)||x_n - p|| + \beta_n||T^nx_n - p|| \\ &\leq (1 - \beta_n)||x_n - p|| + \beta_nk_n(||x_n - p|| + a_n) \\ &\leq k_n||x_n - p|| + k_na_n.\end{aligned}$$

Therefore we obtain

$$\lim_{n\to\infty} \sup ||y_n - p|| \leq l. \tag{27}$$

This implies

$$\lim_{n\to\infty} \sup ||T^ny_n - p|| \leq \lim_{n\to\infty} \sup\{k_n(||y_n - p|| + a_n)\} \leq l \tag{28}$$

Moreover,

$$||z_n - p|| \leq (1 - \gamma_n)||T^nx_n - p|| + \gamma_n||T^{n+1}x_n - p||$$

$$\leq (1-\gamma_n)k_n(||x_n-p||+a_n)+\gamma_n k_{n+1}(||x_n-p||+a_n)$$
$$\leq \max\{k_n, k_{n+1}\}||x_n-p|| + a_n \max\{k_n, k_{n+1}\}$$

which gives

$$\lim_{n\to\infty} \sup ||z_n - p|| \leq l \tag{29}$$

and hence

$$\lim_{n\to\infty} \sup ||T^n z_n - p|| \leq \lim_{n\to\infty} \sup\{k_n(||z_n-p||+a_n)\} \leq l. \tag{30}$$

Since

$$l = \lim_{n\to\infty} ||x_{n+1}-p|| = \lim_{n\to\infty} ||\alpha_n(T^n y_n - p) + (1-\alpha_n)(T^n z_n - p)||,$$

therefore by Lemma 1, we obtain

$$\lim_{n\to\infty} ||T^n y_n - T^n z_n|| = 0. \tag{31}$$

Now, since

$$||x_{n+1}-p|| \leq \alpha_n ||T^n y_n - T^n z_n|| + ||T^n z_n - p||,$$

thus

$$l \leq \lim_{n\to\infty} \inf ||T^n z_n - p||. \tag{32}$$

From (30) and (32), we obtain

$$\lim_{n\to\infty} ||T^n z_n - p|| = l. \tag{33}$$

Since

$$||T^n z_n - p|| \leq ||T^n z_n - T^n y_n|| + ||T^n y_n - p|| \leq ||T^n z_n - T^n y_n|| + k_n(||y_n - p|| + a_n),$$

we obtain

$$l \leq \lim_{n\to\infty} \inf ||y_n - p||. \tag{34}$$

From (27) and (34), we obtain

$$\lim_{n\to\infty} ||y_n - p|| = l.$$

Since

$$l = \lim_{n\to\infty} ||y_n - p|| = \lim_{n\to\infty} ||(1-\beta_n)(x_n - p) + \beta_n(T^n x_n - p)||$$

therefore by Lemma 1, we obtain

$$\lim_{n\to\infty} ||x_n - T^n x_n|| = 0. \tag{35}$$

(iii) Since $||T^n x_n - p|| \leq k_n(||x_n - p|| + a_n)$,

$$\lim_{n\to\infty} \sup ||T^n x_n - p|| \leq l \ \text{ and } \ \lim_{n\to\infty} \sup ||T^{n+1} x_n - p|| \leq l. \tag{36}$$

Since $||T^n z_n - p|| \leq k_n(||z_n - p|| + a_n)$, using (33), we obtain

$$l \leq \lim_{n\to\infty} \inf ||z_n - p||. \tag{37}$$

(29) and (37) give

$$\lim_{n\to\infty} ||z_n - p|| = l.$$

This gives using (17) that

$$l = \lim_{n\to\infty} ||z_n - p|| = \lim_{n\to\infty} ||(1-\gamma_n)(T^n x_n - p) + \gamma_n(T^{n+1} x_n - p)$$

This gives using (36) and Lemma 1 that

$$\lim_{n\to\infty} ||T^n x_n - T^{n+1} x_n|| = 0. \tag{38}$$

Now

$$\begin{aligned} ||z_n - x_n|| &= ||(1-\gamma_n)T^n x_n + \gamma_n T^{n+1} x_n - x_n|| && (39) \\ &\leq ||x_n - T^n x_n|| + \gamma_n ||T^n x_n - T^{n+1} x_n||. && (40) \end{aligned}$$

This implies using (35) and (38) that

$$\lim_{n\to\infty} ||z_n - x_n|| = 0 \tag{41}$$

Thus

$$\lim_{n\to\infty} ||T^n z_n - T^n x_n|| \leq \lim_{n\to\infty} k_n(||z_n - x_n|| + a_n) = 0$$

This implies

$$\lim_{n\to\infty} ||T^n z_n - T^n x_n|| = 0 \tag{42}$$

Now from (15), we have

$$||x_{n+1} - T^n x_n|| = ||\alpha_n T^n y_n + (1-\alpha_n)T^n z_n - T^n x_n||$$

$$\leq \quad \alpha_n||T^n y_n - T^n z_n|| + ||T^n z_n - T^n x_n||$$

This gives using (31) and (42)

$$\lim_{n\to\infty} ||x_{n+1} - T^n x_n|| = 0. \tag{43}$$

By (35) and (43), we have

$$0 = \lim_{n\to\infty} ||x_{n+1} - x_n|| \leq \lim_{n\to\infty} ||x_{n+1} - T^n x_n|| + \lim_{n\to\infty} ||T^n x_n - x_n|| = 0 \tag{44}$$

By uniform continuity of T and (35), we have

$$\lim_{n\to\infty} ||T^{n+1} x_n - T x_n|| = 0. \tag{45}$$

Now

$$\begin{aligned} ||x_n - Tx_n|| \leq & ||x_n - x_{n+1}|| + ||x_{n+1} - T^{n+1}x_{n+1}|| + ||T^{n+1}x_{n+1} - T^{n+1}x_n|| \\ & + ||T^{n+1}x_n - Tx_n|| \\ \leq & ||x_n - x_{n+1}|| + ||x_{n+1} - T^{n+1}x_{n+1}|| + k_{n+1}(||x_{n+1} - x_n|| + a_n) \\ & + ||T^{n+1}x_n - Tx_n||. \end{aligned}$$

This implies using (35), (44) and (45), that

$$\lim_{n\to\infty} ||x_n - Tx_n|| = 0.$$

□

Theorem 6. *Let E be a uniformly convex Banach space satisfying the Opial condition, C a nonempty, closed, convex subset of E. $T : C \to C$ be a uniformly continuous nearly asymptotically nonexpansive mapping with the sequence $\{(a_n, k_n)\}$ and $F(T) \neq \phi$ such that $\sum_{n=1}^{\infty} a_n < \infty$ and $\sum_{n=1}^{\infty} (k_n\sqrt{k_1} - 1) < \infty$. Let $\{x_n\}$ be defined by the modified C-iteration given by (14) where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[a, b]$ for some $a, b \in \mathbb{R}$ with $0 < a \leq b < 1$. Then $\{x_n\}$ converges weakly to a fixed point of T.*

Proof. E is uniformly convex, so it is reflexive. The sequence $\{x_n\}$ has properties (D_1) and (D_2) by Lemma 8. $I - T$ is demiclosed at zero by Lemma 3 and $F(T) \neq \emptyset$ by assumption. Therefore, by Lemma 5, $\{x_n\}$ converges weakly to a fixed point of T. □

Corollary 2. *Let E be a uniformly convex Banach space satisfying the Opial condition, C a nonempty, closed, convex subset of E. $T : C \to C$ be a uniformly continuous asymptotically nonexpansive mapping with sequence $\{k_n\}$ such that $\sum_{n=1}^{\infty}(k_n\sqrt{k_1} - 1) < \infty$. Let $\{x_n\}$ be defined by the modified C-iteration given by (14) where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[a, b]$ for some $a, b \in \mathbb{R}$ with $0 < a \le b < 1$. Then $\{x_n\}$ converges weakly to a fixed point of T.*

Proof. Since $F(T) \neq \phi$ by Theorem 2 and every asymptotically nonexpansive mapping is nearly asymptotically nonexpansive mapping, the proof follows by Theorem 6. □

Theorem 7. *Let E be a real uniformly convex Banach space. Let E has the Fréchet differentiable norm or E^* has the Kadec-Klee property, C a nonempty, closed, convex subset of E. $T : C \to C$ be a uniformly continuous nearly asymptotically nonexpansive mapping with sequence $\{(a_n, k_n)\}$ such that $\sum_{n=1}^{\infty} a_n < \infty$, $\sum_{n=1}^{\infty}(k_n\sqrt{k_1} - 1) < \infty, I - T$ is demiclosed at zero and $F(T) \neq \phi$. Let $\{x_n\}$ be defined by the modified C-iteration given by (14) where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[a, b]$ for some $a, b \in \mathbb{R}$ with $0 < a \le b < 1$. Then $\{x_n\}$ converges weakly to a fixed point of T.*

Proof. $\{x_n\}$ has properties (D_1) and (D_2) by Lemma 8. By Lemma 4, we have

(a) $\lim_{n\to\infty} \|tx_n + (1 - t)q_1 - q_2\|$ exists for all $q_1, q_2 \in F(T)$, $t \in [0, 1]$;

(b) $\lim_{n\to\infty} \langle x_n, J(q_1 - q_2)\rangle$ exists for all $q_1, q_2 \in F(T)$ if E has Fréchet differentiable norm.

$I - T$ is demiclosed by assumption. E is reflexive as it is uniformly convex. Hence the result follows from Lemma 6. □

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Nova iteraciona šema za aproksimaciju fiksnih tačaka nekih generalizovanih neekspanzivnih preslikavanja

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OBLAST: matematika
KATEGORIJA (TIP) ČLANKA: originalni naučni rad

Sažetak:

Uvod/cilj: U radu je predstavljen novi trostepeni iterativni postupak koji konvergira brže od Manove iteracije i S-iteracije, a utvrđeni su i rezultati konvergencije za aproksimaciju fiksnih tačaka generalizovanih Suzukijevih neekspanzivnih preslikavanja i skoro asimptotski neekspanzivnih preslikavanja.

Metode: Autori daju specifičnu trostepenu iterativnu metodu $\{x_n\}$ *u Banahovom prostoru, definisanu kao niz konveksnih kombinacija trenutnog iterata i njegovih slika pod preslikavanjem T, sa kontrolnim nizovima* $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\} \subseteq (0,1)$. *Rezultati su dokazani u okviru uniformno konveksnih Banahovih prostora gde se pretpostavlja da je T Suzukijevo generalizovano neekspanzivno preslikavanje ili skoro asimptotski neekspanzivno preslikavanje. Autori su dobili teoreme o slaboj i jakoj konvergenciji koristeći principe demizatvorenosti, osobine Suzukijevih gene-*

ralizovanih neekspanzivnih preslikavanja i odgovarajuće leme o ponašanju iterata. Za poređenje brzine konvergencije sprovedeni su numerički eksperimenti (obično implementirani u MATLAB-u) u kojima se predložena trostepena iteracija vrši paralelno sa Takurovom i S-iteracijom. Iterati su grafički prikazani kako bi se pokazala konvergencija greške po iteraciji.*

Rezultati: Nova iteraciona šema konvergira brže od S-iteracione šeme ako je preslikavanje kontrakcija. Nova iteraciona šema konvergira ka fiksnoj tački Suzukijevog generalizovanog neekspanzivnog preslikavanja pod odgovarajućim uslovima. Šema takođe konvergira ka fiksnoj tački skoro asimptotski neekspanzivnog preslikavanja pod odgovarajućim uslovima.

Zaključci: Dokazano je, i teorijski i numerički, da trostepeni iteracioni algoritam konvergira brže od Manove iteracije i S-iteracije (a ponekad i brže od nekoliko drugih postojećih metoda) za razmatrane tipove preslikavanja. Autori su dokazali teoreme o slaboj i jakoj konvergenciji za fiksne tačke Suzukijevih generalizovanih neekspanzivnih preslikavanja i skoro asimptotski neekspanzivnih preslikavanja u uniformno konveksnim Banahovim prostorima, čime se generalizuju, proširuju i objedinjuju brojni postojeći rezultati aproksimacije fiksnih tačaka u literaturi.

Ključne reči: fiksne tačke, iteracija, uniformno konveksan Banahov prostor, neekspanzivno preslikavanje, Suzukijevo generalizovano neekspanzivno preslikavanje, skoro asimptotski neekspanzivna preslikavanja, refleksivni Banahov prostor.

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