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Models of microeconomic dynamics: bifurcations and complex system behavior algorithms

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Abstract:

Introduction/purpose: Studying the dynamics of the mutual influence of supply and demand is relevant in connection with the financial losses that arise due to uncertainty in demand and forecast errors. The work aims to build a mathematical model of the dynamics of this interaction for the market of one product.

Methods: The paper proposes a mathematical model of the states of the supply-demand system, within the framework of which the processes occurring in this system are considered from the perspective of the methodology of economic synergetics. The mathematical model of dynamics has the form of a system of two differential equations with quadratic nonlinearity.

Results: The use of the proposed model to reproduce various dynamic states of market self-regulation processes made it possible to identify the *hierarchy of transition from stable dynamic regimes to unstable ones with the appearance of corresponding bifurcations. The main attention was paid to studying the behavior of the system at the boundaries of the stability region.*

Conclusion: The existence of a saddle-node bifurcation of limit cycles has been revealed, which suggests the appearance of stable self-oscillations in the case of a "soft" cycle and unstable ones in the case of a "hard" cycle. When studying a bifurcation of codimension two - "double zero" - special dynamic structures were discovered, determined by the properties of global bifurcations. This type of behavior is characterized by self-oscillations with a low frequency, which gives rise to the so-called "ultra-long waves" of the economic state.

Key words: dynamics of the supply-demand system, time lag, limit cycle, bifurcation, chaos.

Introduction

The formation of market equilibrium in the supply-demand system for a particular product in quantitative terms, as well as the equilibrium price for this product, has long attracted the attention of researchers. The problem of constructing a market equilibrium model is currently the focus of research programs of the world's leading scientific centers specialized in microeconomic analysis. The ability to analyze and make informed forecasts regarding the dynamics of the interaction of supply and demand allows decision makers to optimize resource allocation, ensure consumer satisfaction, reduce risks, and improve production efficiency.

Neoclassical economic theory offers two main models to explain the processes leading to the formation of market equilibrium in the supply-demand system. These are the Walras model and the Marshall model (Davar, 2015; Donzelli, 2008; Arena & Caldari, 2024 and etc.). In his model, Leon Walras analyzed the establishment of equilibrium between supply and demand occurring in the short term. According to his model, when the price increases, the quantity demanded will decrease, as a result of which the quantity supplied will exceed the equilibrium value. The market for a particular product is in equilibrium if, at the prices prevailing on the market for all goods, the quantity of the product required to satisfy the demand of potential buyers is equal to the quantity supplied by potential sellers. In contrast to the approach proposed by Walras, Alfred Marshall considered the price of the product as the driving force which is leading the market to a state of equilibrium. If the demand price exceeds the supply price, then, according to the Marshall model, such a price difference stimulates producers to increase supply, and buyers will be able to expand their demand until the price is established at a new, higher equilibrium level. This is true for a longer period. The last statement from the standpoint of the theory of the firm regarding equilibrium in the supply-demand system is formulated as follows: at the moment of market equilibrium, the price of output must be equal to the marginal cost of the enterprise. The equilibrium price, as well as the equilibrium volume of goods (supply), are determined by the intersection of the supply and demand curves. This is the so-called Marshall Cross Diagram (Figure 1).

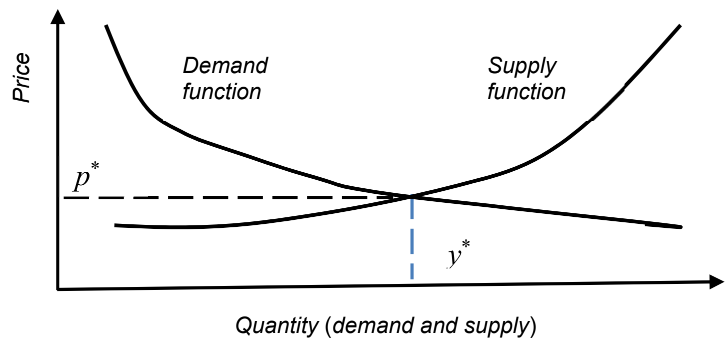

Figure 1 – Market equilibrium as the point of the intersection of the demand curve and the supply curve

It should be noted that both the Walras model and the Marshall model consider market equilibrium as the final result of the interaction of the demand and supply functions, while at the present stage of economic theory development, it is of interest to analyze the dynamics of the processes leading to the establishment of this equilibrium.

Mathematical models of varying complexity are proposed to describe economic dynamics (Li & Ma, 2020; Voronin et al, 2020; Fu et al, 2023; Chen et al, 2024 and etc.), but this problem is still far from being resolved. The founder of the study of economic dynamics is Griffith Conrad Evans, who proposed a mathematical model of monopoly. In his model, Evans used a first-order differential equation with constant coefficients to describe the demand function, which depends on price dynamics. Within the framework of this model, cyclical price fluctuations in the market of one product are considered under the assumption that the price changes smoothly over time. This model has been developed in a number of studies and still continues to attract interest (Nahorski & Ravn, 2000; Pomin, 2018; Dilenko & Tarakanov, 2020 and etc.). But in real conditions, price changes occur abruptly, and it is especially important to take this fact into account when constructing mathematical models in which the forecast is carried out for a short-term period (He, 2018; He et al, 2021; Zabolotnii & Mogilei, 2023). In addition, the presence of a delay that occurs in the process of establishing the equilibrium price may cause instability of the system and lead to fluctuations. In particular, when analyzing mathematical models of nonlinear systems with delay, which were developed for systems of different nature, such complex phenomena as bifurcations and chaos were discovered (Liao et al, 2007; Wei & Yu, 2011; Le et al, 2012 and etc.).

The study of real economic processes allows us to conclude that changes in the price of goods and production volume have a mutual influence and cannot be considered in isolation from each other. In a number of recent articles (Voronin et al, 2020; He et al, 2021; Zabolotnii & Mogilei, 2023 and etc.), much attention is paid specifically to the dynamics of the processes of interaction between supply and demand. Thus, an increase in operating costs or a shortage of raw materials may cause production delays, as a result of which the supply of goods may not correspond to immediate demand, i.e., there is some delay (Cai, 2005; Hattaf et al, 2017; Davizón et al, 2023). For example, there may be a time lag between the adoption of an investment decision and its implementation. The result of this delay is the emergence of stable fluctuations of price around the market equilibrium, the emergence of large growth cycles and even the possibility of a sudden market collapse (Levi et al, 2018; Chen et al, 2024). This chaotic behavior in the system is the result of the appearance of the Hopf bifurcation, when the delay reaches a critical value (Li et al, 2019; Elkarmouchi et al, 2024).

The purpose of this work is to construct a mathematical model that would allow analyzing the dynamics of the balance between supply and demand. In this work, we will limit ourselves to the simplest situation, when there is only one type of product and its implementation is carried out on one market. Models of such a dimension were studied by both traditional and modern methods of mathematical stability theory with the corresponding conclusions about the behavior of the economic system near the equilibrium position (and no more!), which gives only approximate information about the evolution of the object under consideration. The mathematical model of dynamics proposed in the work also allows us to consider the processes occurring in the market of several goods, but not in a quantitative, but in a qualitative form with the corresponding order parameters. At the same time, changing the system parameters allows us to observe a wide range of market dynamics, namely, the equilibrium state, periodic and chaotic behavior.

Method used and the basic mathematical model

In microeconomic analysis, when constructing mathematical models of the market equilibrium, the following notations are traditionally used: $p = p(t)$ is the price of a unit of goods depending on time t; $y = y(t)$ is the volume of products, which also depends on time; $D = D(p; y)$ is the volume of demand in the market; $S = S(p; y)$ is the volume of supply of goods produced; $P_d = P_d(p; y)$ is the market demand price for a product; and $P_s = P_s(p; y)$ is the market offer price from the manufacturer.

The principles of constructing dynamic models consist of ways to describe time lag factors on both the demand and supply sides. The simplest assumption regarding the delay, if the analysis is carried out in discrete time, is a concentrated delay (lag) of supply from demand for one-time interval (the lag T_1):

$$D(p; y; t) = S(p; y; t - T_1) . \tag{1}$$

Equality (1) occurs when a certain period of time is required to produce a given volume of goods. This period of time is called the production lag. In this case, as a rule, it is assumed that there are no inventories, i.e., to meet demand, all manufactured products are supplied to the market in full. It should also be emphasized that the manufacturer builds his expectations of the future price based on the existing price, i.e., actually focuses on the price of the previous period T_2:

$$P_s(p; y; t) = P_d(p; y; t - T_2) . \tag{2}$$

If we consider the processes of interest to us in continuous time, then we should replace distributed delays with continuously distributed ones. One example of this type of model with a continuously distributed delay is Voltaire's system of integral equations for determining the price $p(t)$ and the volume of a product $y(t)$:

$$\begin{cases} D(p;y;t) = \int\limits_0^t K_1(t;\tau) S(p;y;\tau) d\tau; \\ P_s(p;y;t) = \int\limits_0^t K_2(t;\tau) P_d(p;y;\tau) d\tau. \end{cases} \tag{3}$$

Functions of two variables $K_1(t;\tau)$ and $K_2(t;\tau)$, called integral equation kernels, determine the shape of the distributed delays.

In this article, which is a further development of the study of one of the authors (Voronin & Chernyshov, 2007), in a qualitatively basic mathematical model we will consider a system of two differential equations that describe the evolution of the mutual influence of prices and volumes of goods produced:

$$\begin{cases} \alpha \dfrac{dp}{dt} = D(p) - y; \\ \beta \dfrac{dy}{dt} = p - \dfrac{dC(y)}{dy}, \end{cases} \qquad (4)$$

where p – the unit price of the product; y – the volume of production of goods in physical terms; $D(p)$ – the market demand for a manufactured product, depending only on the price at the current moment in time (here and now); $C(y)$ – the production cost; $\dfrac{dC(y)}{dy} = P_s(y)$ – the supply price, which is equal to the marginal cost of production; and α, β – the parameters that have the meaning of the characteristic times for dynamic variables.

The first of the equations of system (4), which is a system of two ordinary differential equations, is essentially a reflection of the classical market pricing scheme in the form of Leon Walras. Its basis is the price formation mechanism which is focused on finding a position of equilibrium between supply and demand. If the volume of demand exceeds the quantity of supply, the price of a unit of goods increases, and in the opposite case, it decreases. The second equation of system (4) describes the process of establishing an equilibrium between the demand price (the actual price of a unit of goods) and the supply price (marginal production costs). The logic of this process provides for the fact of imbalance with the need to regulate the volume of production of goods. Accordingly, if the unit price of a product is greater than the producer's marginal cost, then the firm's profit increases. Conversely, in the opposite case, there is a need to limit production capacity. Significant assumptions were made when constructing the model. The first of these assumptions should be considered the hypothesis about producing only one type of product. The second assumption relates to simplifying the market structure since either the absence of competition is assumed or its impact is considered insignificant. However, despite the above simplifications, system (4) has quite complex behavioral properties, which will be the subject of this study.

Results and discussion

A substantive analysis of the qualitative behavior of system (4) should begin with determining the price of a unit of goods p^* and the value of the output y^*, corresponding to the equilibrium position. To do this, we solve a system of two algebraic equations with two unknowns:

$$\begin{cases} D(p) = y; \\ p = P_s(y). \end{cases} \tag{5}$$

Let us assume that the algebraic system (5) has at least one positive solution p^* and y^*. The demand function $D(p)$ will also be assumed to be a nonlinear function of the price and that in a small neighborhood of the equilibrium price value p^* there is an expansion of this function in a Taylor series up to and including cubic terms:

$$D(p) = d_0 + d_1(p - p^*) + \frac{d_2(p-p^*)^2}{2} + \frac{d_3(p-p^*)^3}{6} + 0\left((p-p^*)^3\right). \tag{6}$$

In equation (6), the coefficients d_i ($i = \overline{0,3}$) have the meaning of the corresponding derivatives of the demand function at the point p^*.

The cost function is usually represented as a quadratic function of the variable y:

$$C(y) = \frac{s_1 y^2}{2} + s_0 y + C_0. \tag{7}$$

In equation (7), the coefficients in each term of the function $C(y)$ are constant values. Having differentiated the cost function by the output volume variable y, one obtains:

$$P_s(y) = s_1 y + s_0. \tag{8}$$

Then, from (5), one finds the relationship between the equilibrium value of the price p^* and the equilibrium value of the production volume y^*:

$$\begin{cases} s_1 D(p^*) + s_0 - p^* = 0; \\ y^* = \dfrac{p^* - s_0}{s_1}. \end{cases} \tag{9}$$

At the next stage, it seems advisable to move in system (4) to new variables $\hat{p} = p - p^*$ and $\hat{y} = y - y^*$, which have the meaning of the deviation of the original variables p and y from their equilibrium values. To reduce the number of parameters in system (4), the time scale is changed by introducing the coefficient $\gamma = \dfrac{\alpha}{\beta}$. System (4), in this case, takes the form:

$$\begin{cases} \dfrac{d\widehat{p}}{dt} = d_1\widehat{p} + d_2\dfrac{\widehat{p}^2}{2} + d_3\dfrac{\widehat{p}^3}{6} - \widehat{y}; \\ \dfrac{d\widehat{y}}{dt} = \gamma\left(\widehat{p} - s_1\widehat{y}\right). \end{cases} \tag{10}$$

From system (10), it is easy to obtain the equations for calculating the equilibrium values p^* and y^*:

$$\begin{cases} \widehat{y}^* = \dfrac{\widehat{p}^*}{s_1}; \\ (s_1d_1 - 1)\widehat{p}^* + \dfrac{s_1d_2}{2}\left(\widehat{p}^*\right)^2 + \dfrac{s_1d_3}{6}\left(\widehat{p}^*\right)^3 = 0. \end{cases} \tag{11}$$

Obviously, one of the solutions of this system is trivial: $\widehat{p}^* = 0$ and $\widehat{y}^* = 0.$

The second equation in system (11) is transformed to the form:

$$\widehat{p}^*\left(s_1d_3\left(\widehat{p}^*\right)^2 + 3s_1d_2\widehat{p}^* + 6(s_1d_1 - 1)\right) = 0. \tag{12}$$

It is obvious that equation (12), in addition to the trivial solution $\widehat{p}^* = 0,$ can have two more roots:

$$\widehat{p}^*_{1,2} = \frac{-3d_2 \pm \sqrt{9d_2^2 - 24d_3(d_1 - 1/s_1)}}{2d_3}. \tag{13}$$

For the value p^* to be valid, the following condition must be true:

$$9d_2^2 - 24d_3d_1 + \frac{24d_3}{s_1} \geq 0. \tag{14}$$

If $s_1d_1 = 1$, then from (13) it follows that $\widehat{p}^*_1 = \widehat{p}^*_0 = 0$ and $\widehat{p}^*_2 = -\dfrac{3d_2}{d_3}$. This means that there is a double zero root.

To analyze the stability of the trivial equilibrium position $\widehat{p}^*_0$ and $\widehat{y}^*_0$ of system (10), let us construct a characteristic equation to determine the eigenvalues of the linear part:

$$\lambda^2 + (\gamma s_1 - d_1)\lambda + \gamma(1 - d_1s_1) = 0. \tag{15}$$

Quadratic equation (15) has negative real parts if the conditions for stability of the equilibrium position are met:

$$\begin{cases} \gamma s_1 < d_1; \\ d_1s_1 < 1. \end{cases} \tag{16}$$

The system of inequalities (16) allows the construction of the stability regions in the parameter space $\gamma,\ s_1,\ d_1$. An example is the image of the stability region on the plane $\gamma 0 s_1$ (Figure 2).

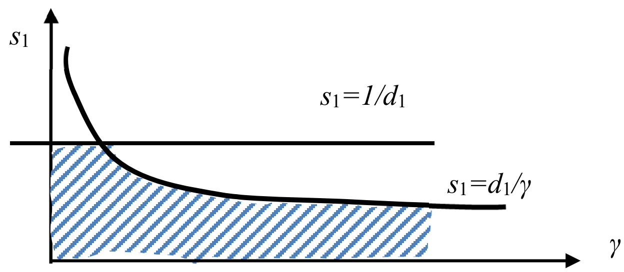

Figure 2 – Stability region at a fixed value d_1 on the plane $\gamma 0 s_1$

It is of significant interest to study the dynamics of system (10) on one of the boundaries of the stability region, namely $\gamma = d_1 / s_1$. Let us assume that $\gamma s_1 \approx d_1$, and the measure of approximation is determined by the small parameter μ:

$$\mu = d_1 - \gamma s_1. \tag{17}$$

By its nature, the parameter γ is dynamic, i.e., it is associated with the characteristic times of transition processes in price and output. Let us express this parameter through purely static characteristics of the demand and supply price functions, i.e., through d_1 and s_1, respectively:

$$\gamma = \frac{d_1 - \mu}{s_1}. \tag{18}$$

In this case, the characteristic equation (15) takes the form:

$$\lambda^2 - \mu\lambda + \frac{d_1 - \mu}{s_1}(1 - d_1 s_1) = 0. \tag{19}$$

Considering that d_1 and s_1 are positive numbers and the second condition is satisfied, which follows from the stability conditions of system (16), namely $d_1 s_1 < 1$, it makes sense to introduce the following notation:

$$\omega^2 = \frac{d_1}{s_1} - d_1^2 \quad \Rightarrow \quad \frac{1}{s_1} - d_1 = \frac{\omega^2}{d_1}. \tag{20}$$

Then quadratic equation (19) takes the form:

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$$\lambda^2 - \mu\lambda + \omega^2\left(1 - \frac{\mu}{d_1}\right) = 0. \tag{21}$$

Accordingly, one obtains a solution to the quadratic equation (21):

$$\lambda_{1,2} = \frac{\mu}{2} \pm \sqrt{\frac{\mu^2}{4} + \frac{\omega^2\mu}{d_1} - \omega^2}. \tag{22}$$

Neglecting those powers of the parameter μ that are higher than the first, the following linearization of the roots of the quadratic equation is obtained:

$$\lambda_{1,2} = \frac{\mu}{2} \pm i\omega\left(1 - \frac{\mu}{2d_1}\right). \tag{23}$$

Since the parameter μ is essentially a small variable quantity, it follows from relation (23) that the trivial equilibrium position of system (10) is the focus. Moreover, if the condition $\mu < 0$ is met, this focus is stable, but otherwise if $\mu > 0$, the focus is unstable. It is obvious that when a small parameter μ passes through zero, one can expect the appearance of a special periodic regime in the dynamic system (10), the implementation of which is due to the Hopf bifurcation. This regime can only be observed in a nonlinear system. It is called a self-oscillatory or limit cycle. Such cycles are characteristic of dissipative systems. In this regard, it is necessary to check an important condition of the Hopf bifurcation theorem concerning the derivative of eigenvalues (23) with respect to the parameter μ.

Having differentiated one of the roots represented by relation (23) with respect to the parameter μ, one obtains:

$$\frac{d\lambda}{d\mu} = \frac{1}{2} - \frac{\omega}{2d_1}. \tag{24}$$

Obviously, the real part of (24) is not equal to zero: $\mathrm{Re}\frac{d\lambda}{d\mu} = \frac{1}{2} \neq 0$. For system (10), this means the absence of conservatism conditions with an infinite number of periodic trajectories. In other words, in accordance with the conditions of Hopf's theorem, the appearance of one or several limit cycles near a trivial singular point is possible.

To find the basic characteristics of the limit cycle such as its frequency, amplitude, and direction of stability, the system of differential equations is reduced to a normal form, for which we introduce new phase variables: $\hat{p} = x_1$, $\hat{y} = d_1x_1 + \omega x_2$, and $t = \omega\tau$. Let us pretend that $\mu = 0$. We obtain the following system of two differential equations for the variables x_1 and x_2, which is the Poincaré normal form with respect to system (10):

$$\begin{cases} \dfrac{dx_1}{d\tau} = -x_2 + \dfrac{d_2}{\omega} \cdot \dfrac{x_1^2}{2} + \dfrac{d_3}{\omega} \cdot \dfrac{x_1^3}{6}; \\ \dfrac{dx_2}{d\tau} = x_1 - \dfrac{d_1 d_2}{\omega^2} \cdot \dfrac{x_1^2}{2} - \dfrac{d_1 d_3}{\omega} \cdot \dfrac{x_1^3}{6}. \end{cases} \quad (25)$$

System (25) will store all the information necessary to calculate the so-called first Lyapunov quantity l_1. This quantity determines the direction of stability of the limit cycle. According to fundamental research on the theory of bifurcations (Hassard et al, 1981), the following expression is obtained:

$$l_1(0) = \frac{d_3\omega^2 + d_2^2 d_1}{16\omega^4}, \quad (26)$$

where ω – the frequency of self-oscillations, which depends on the static parameters d_1 and s_1:

$$\omega = d_1 \sqrt{\frac{1}{s_1 d_1} - 1} > 0. \quad (27)$$

The analysis of expression (26) allows one to draw the following conclusions:

1) if $d_3\omega^2 + d_2^2 d_1 < 0$, then the limit cycle is stable and a “soft” mode of self-oscillation occurs.
2) if $d_3\omega^2 + d_2^2 d_1 > 0$, then the so-called “hard” periodic regime is observed, which is accompanied by a catastrophic loss of stability.

For us, the most interesting case is when the first Lyapunov quantity is a sign-alternating parameter close to zero, and accordingly, we introduce the following notation: $l_1 = \nu$. This situation is possible if the following conditions are met: $d_3\omega^2 + d_2^2 d_1 \approx 0$. From this condition, it follows that $d_3 \approx -\dfrac{d_2^2 d_1}{\omega^2}$. Since there is the inequality $d_1 > 0$, it follows that $d_3 < 0$. In terms of economic theory, this means that the demand function $D(p)$ has a saturation effect, i.e., the demand function cannot increase without limit as price decreases. From a mathematical point of view, the fact that the first Lyapunov value l_1 is close to zero means that stable and unstable limit cycles coexist in the system, which can be transformed into a double cycle by merging.

For a subsequent analysis of the two-parameter bifurcation of the limit cycle, it is necessary to calculate the so-called second Lyapunov quantity $l_2(0)$. Using the corresponding bifurcation formulas given in the work (Golubitsky & Langford, 1981), one obtains:

$$l_2(0) = \frac{d_1^3 d_3}{192\omega^3}\left(\frac{4d_1 d_2^2}{\omega^2} - \frac{3}{2}d_3\right). \qquad (28)$$

Taking into account the fact that there is a relation $d_3 \approx -\frac{d_2^2 d_1}{\omega^2}$ formula (28) takes the form:

$$l_2(0) = -\frac{11 d_1^5 d_2^4}{384\omega^7}. \qquad (29)$$

From relation (29) it follows that $l_2(0) < 0$.

In (Kuznetsov, 2023), the corresponding normal form of a bifurcation of codimension two in polar coordinates is given, where r is the magnitude of the amplitude, and φ is the phase of the emerging limit cycles:

$$\begin{cases} \frac{dr}{dt} = r(\delta_1 + \delta_2 r^2 - r^4); \\ \frac{d\varphi}{dt} = 1. \end{cases} \qquad (30)$$

Moreover, $\delta_1 = \mu$ and $\delta_2 = \nu\sqrt{l_2(0)}$ are small alternating parameters.

The first equation of system (30) has three special solutions. The value $r = 0$ corresponds to the trivial equilibrium, and the remaining solutions must satisfy the biquadratic equation:

$$r^4 - \delta_2 r^2 - \delta_1 = 0. \qquad (31)$$

In order for all solutions of equation (31) to be positive, the following conditions must be met: $\delta_1 < 0, \quad \delta_2 > 0$ and $\delta_2^2 + 4\delta_1 > 0$. There is a Hopf bifurcation on the line $\delta_1 = 0$. For it, the first Lyapunov quantity has the following meaning: $l_1 = \delta_2$. If $\delta_2 < 0$, then this corresponds to birth of the "soft" limit cycle, and if $\delta_2 > 0$, this corresponds to the occurrence of hard self-oscillations. That is, both stable and unstable limit cycles coexist at the same time. On the line $\delta_2^2 + 4\delta_1 = 0$ under the condition $\delta_2 > 0$, both cycles, due to the compaction of trajectories, merge into one double cycle and disappear, i.e., there is a bifurcation of the "fold" type. These results of the bifurcation analysis are similar to those published for the double limit cycle in the work (Dorokhov et al, 2023).

Let us return to equation (12) and use it to determine the equilibrium price $\widehat{p}^*$. First, it is necessary to find out the number of real roots of cubic equation (12) under the conditions of the occurrence of a double cycle. With the equalities $d_3 = -\frac{d_2^2 d_1}{\omega^2}$ and $\omega^2 = \frac{d_1}{s_1} - d_1^2$, expression (12) is transformed to the form:

$$\widehat{p}\left(\widehat{p}^2 + 3n\widehat{p} + 6n^2\right) = 0, \quad (32)$$

where $n = \frac{d_1 s_1 - 1}{d_2 s_1}$.

Equation (32) can be represented in a different form by highlighting the complete square:

$$\widehat{p}\left(\left(\widehat{p} + \frac{3n}{2}\right)^2 + \frac{15n^2}{4}\right) = 0. \quad (33)$$

Obviously, (33) has only one trivial solution $\widehat{p}^* = 0$ if the specified restrictions on the parameters of equation (12) are met. But the bifurcation behavior of system (10) does not end there. Before exploring other bifurcations of codimension two, it is necessary to transform system (10) into another form. Let us reduce system (10), consisting of two differential equations, to one second-order differential equation with respect to the price $\widehat{p}$. After successive identical transformations, one obtains the following differential equation:

$$\frac{d^2\widehat{p}}{dt^2} = \\ = \left(d_1 - \gamma s_1\right)\frac{d\widehat{p}}{dt} + \gamma(s_1 d_1 - 1)\widehat{p} + \gamma s_1 d_2 \frac{\widehat{p}^2}{2} + d_2 \widehat{p}\frac{d\widehat{p}}{dt} + \gamma s_1 d_2 \frac{\widehat{p}^3}{6} + \frac{d_3 \widehat{p}^2}{2} \cdot \frac{d\widehat{p}}{dt}. \quad (34)$$

Let us assume that the parameters for the linear terms of differential equation (34) are small, i.e., $d_1 - \gamma s_1 = \mu_2$ and $\gamma(s_1 d_1 - 1) = \mu_1$. We take the following notation: $\widehat{p} = x_1$ and $\frac{d\widehat{p}}{dt} = x_2$. Using the new notation, we transform equation (34) into a system of two differential equations:

$$\begin{cases} \frac{dx_1}{dt} = x_2; \\ \frac{dx_2}{dt} = \mu_1 x_1 + \mu_2 x_2 + d_1 d_2 \frac{x_1^2}{2} + d_2 x_1 x_2 + d_1 d_3 \frac{x_1^3}{6} + d_3 \frac{x_1^2 x_2}{2}. \end{cases} \quad (35)$$

Considering that the parameter μ_2 is small, one can use the coefficient d_1 instead of the product γs_1 for nonlinear terms d_1.

It should be emphasized that system (10), which is represented in the variables $\hat{p}$ (price) and $\hat{y}$ (volume of production), has now been transformed into system (35) with relatively new variables x_1 (price) and x_2 (surplus demand). The linear part of system (35) corresponds to the matrix $\tilde{A} = \begin{pmatrix} 0 & 1 \\ \mu_1 & \mu_2 \end{pmatrix}$, for which the characteristic polynomial has the form: $\lambda^2 - \mu_2\lambda + \mu_1 = 0$. The equality $\mu_1 = \mu_2 = 0$ implies the presence of a Bogdanov–Takens bifurcation, the so-called "double zero" (Guckenheimer & Holmes, 1983). This means that the analysis of the stability of system (35) should be carried out in close proximity to the stability boundaries, namely $s_1 = \dfrac{1}{d_1}$ and $s_1 = \dfrac{d_1}{\gamma}$ (see Figure 2).

Let us analyze the dynamic properties of system (35). For this purpose, consider two cases. In the first case, assume that $d_3 = 0$. Then system (35) takes the form:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2; \\ \dfrac{dx_2}{dt} = \mu_1 x_1 + \mu_2 x_2 + d_1 d_2 \dfrac{x_1^2}{2} + d_2 x_1 x_2. \end{cases} \tag{36}$$

To pass to the Poincaré normal form for the Bogdanov–Takens bifurcation, one carries out a change of variables: $x_1 = R_1 V_1$, $x_2 = R_2 V_2$ and $t = R_3 \tau$. After substituting new variables into system (35) and performing algebraic transformations, one obtains:

$$\begin{cases} \dfrac{dV_1}{d\tau} = V_2; \\ \dfrac{dV_2}{d\tau} = \dfrac{4\mu_1}{d_1^2} V_1 + \dfrac{2\mu_2}{d_1} V_2 + V_1^2 + V_1 V_2, \end{cases} \tag{37}$$

where $R_1 = \dfrac{8}{d_1^3 d_2}$, $R_2 = \dfrac{16}{d_1^4 d_2}$ and $R_3 = \dfrac{2}{d_1}$.

Using the shift $V_1 = U_1 - \dfrac{2\mu_1}{d_1^2}$ and $V_2 = U_2$, one obtains the final form of the Poincaré normal form for system (37):

$$\begin{cases} \dfrac{dU_1}{d\tau} = U_2; \\ \dfrac{dU_2}{d\tau} = \xi_1 + \xi_2 U_2 + U_1^2 + U_1 U_2, \end{cases} \tag{38}$$

where $\xi_1 = -\dfrac{4\mu_1^2}{d_1^4}$ and $\xi_2 = \dfrac{2}{d_1}\left(\mu_2 - \dfrac{\mu_1}{d_1}\right)$.

In the works (Takens, 1974; Kopell & Howard, 1975) for a system similar to system (38), a complete topological analysis of the stability of the equilibrium position was performed when the corresponding bifurcations appeared. Accordingly, system (38) has two equilibrium positions: $U_1^* = \left(-\sqrt{-\xi_1};\ 0\right)$ and $U_2^* = \left(\sqrt{-\xi_1};\ 0\right)$. In this case, the point $\left(\sqrt{-\xi_1};\ 0\right)$ is a point of unstable equilibrium for $\xi_1 < 0$ and for any values of ξ_2. In turn, the equilibrium point $\left(-\sqrt{-\xi_1};\ 0\right)$ is an unstable focus for $\xi_1 < 0$ and $\xi_2 > \sqrt{-\xi_1}$; conversely, it is a stable focus for $\xi_1 < 0$ and $\xi_2 < \sqrt{-\xi_1}$. Thus, it should be assumed that the occurrence of the "saddle - node" bifurcation takes place on the line $\xi_1 = 0,\ \xi_2 \neq 0$, and the occurrence of the Bogdanov - Takens bifurcation takes place on the half-parabola $\xi_2 = \sqrt{-\xi_1},\ \xi_1 < 0$. Moreover, the analysis of the Hopf bifurcation stability indicates that the limit cycle is unstable, i.e., “hard” self-oscillations arise. A similar result was obtained in the work (Elkarmouchi et al, 2024), the authors of which propose a mathematical IS-LM model with two-time delays, which describes many equilibrium positions in the investment-savings markets and the money market. Numerical modeling revealed the presence of a bifurcated periodic solution, which occurs when the time delay exceeds a critical value.

It should also be emphasized that in system (38), there is a global bifurcation, where the limit cycle annihilates inside the separatrix loop of the saddle. Using a special scaling transformation, we convert system (38) into a system close to Hamiltonian (Carr, 1982), and at the same time we obtain an approximate global bifurcation equation:

$$\xi_1 \approx -\frac{49}{25}\xi_2^2,\ \ \xi_2 \geq 0. \tag{39}$$

Returning to the original small parameters μ_1 and μ_2, we can draw the following conclusions:

1) The “saddle-node” bifurcation takes place on the line $\mu_1 = 0$;

2) Hopf bifurcation takes place on the lines $\mu_2 = 0$ and $\mu_2 = \frac{2\mu_1}{d_1}$;

3) Global bifurcation exists on the half-lines $\mu_2 = \frac{12}{7} \cdot \frac{\mu_1}{d_1}$ and $\mu_2 = \frac{2}{7} \cdot \frac{\mu_1}{d_1}$ if the constraint is satisfied $\mu_2 > \frac{\mu_1}{d_1}$.

In this version, algebraic equation (12), when the condition $d_3 = 0$ is met, takes the form:

$$\frac{d_1 s_1 - 1}{s_1} \cdot \widehat{p}^* + \frac{d_2}{2} \cdot \left(\widehat{p}^*\right)^2 = 0, \tag{40}$$

and when the condition $\mu_1 = 0$, i.e., $d_1 s_1 = 1$, there is a twofold trivial equilibrium $\widehat{p}^* = 0,\ \widehat{y}^* = 0$.

Let us consider a different configuration of system (35), assuming, accordingly, in this case, there is no quadratic nonlinearity. Now let us rewrite system (35) as follows:

$$\begin{cases} \frac{dx_1}{dt} = x_2; \\ \frac{dx_2}{dt} = \mu_1 x_1 + \mu_2 x_2 + d_1 d_3 \frac{x_1^3}{6} + d_3 \frac{x_1^2 x_2}{2}. \end{cases} \tag{41}$$

It is easy to see that system (41) has central symmetry, or symmetry with respect to rotation through an angle of 180^0. Let us transform system (41) to the Poincaré normal form for the "double zero" bifurcation with cubic nonlinearities. Using the new variables: $x_1 = \sqrt{\frac{-2d_1}{3d_3}} y_1$, $x_2 = \frac{d_1}{3} \sqrt{\frac{-2d_1}{3d_3}} y_2$ and $t = \frac{3}{d_1} \tau$, let us reduce system (41) to the form:

$$\begin{cases} \frac{dy_1}{d\tau} = y_2; \\ \frac{dy_2}{d\tau} = \frac{9}{d_1^2} \mu_1 y_1 + \frac{3}{d_1} \mu_2 y_2 - y_1^3 - y_1^2 y_2. \end{cases} \tag{42}$$

The main results of the study of this system based on its mathematical model can be presented in the form of comments to the bifurcation diagram shown in Figure 3.

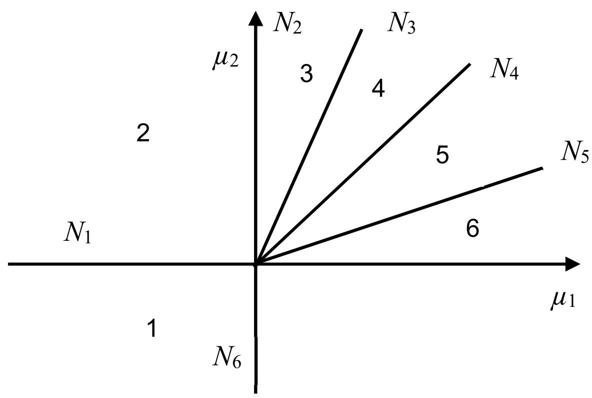

Figure 3 – Bifurcation diagram of "double zero"

System (42) has three equilibrium positions, one of which is trivial $E_0 = (0; 0)$, and if $\mu_1 > 0$ there are two nontrivial solutions: $E_{1,2} = \left(\pm\sqrt{\mu_1}; 0\right)$. In region 1 (see Figure 3), where $\mu_1 < 0$, there is a single trivial equilibrium E_0, which is a stable node that smoothly passes into the focus. On the left side of the straight line N_1, where $N_1 = \{(\mu_1; \mu_2) : \mu_1 < 0; \mu_2 = 0\}$, there is the Andronov–Hopf bifurcation with the appearance of a stable limit cycle. Two stable nodes E_1 and E_2 are separated from the trivial equilibrium position E_0 when the half-line $N_2 = \{(\mu_1; \mu_2) : \mu_1 = 0; \mu_2 > 0\}$ is crossing those on the border between regions 2 and 3 as a result of the "pitchfork" bifurcation. In region 3, all three equilibrium positions E_0, E_1 and E_2, are inside the "large" limit cycle. On the half-line $N_3 = \left\{(\mu_1; \mu_2) : \mu_2 = \frac{3}{d_1}\mu_1;\ \mu_1 > 0\right\}$ the foci E_1 and E_2 pass through the Andronov–Hopf bifurcation. This effect leads to the appearance of two small unstable limit cycles around the equilibrium positions E_1 and E_2. In other words, three limit cycles coexist in region 4. On the half-line $N_4 = \left\{(\mu_1; \mu_2) : \mu_2 = \frac{12}{5d_1}\mu_1;\ \mu_1 > 0\right\}$ as a result of the homoclinic bifurcation, "small" cycles are contracted to the trivial equilibrium position E_0 and form a symmetrical curve that has an external resemblance to

Bernoulli's lemniscate. On the half-line N_4 the saddle point E_0 has two closed homoclinic orbits. When passing through the half-line N_4, which is the boundary between regions 4 and 5, the destruction of "small" cycles occurs with the simultaneous appearance of a "large" cycle. In area 5, two "large" cycles with different types of stability simultaneously coexist: the outer one is stable, and the inner one is unstable. Both of these cycles merge and disappear along the half-line $N_5 = \left(\mu_2 = \frac{3k_0}{d_1}\mu_1;\ \mu_1 > 0 \right)$, where $k_0 = 0.752...$ is a transcendental number. The disappearance of these two cycles is explained by the presence of a saddle-node bifurcation of the limit cycle. With this, the cyclic behavior of system (41) is completely exhausted. All three equilibrium positions, E_0, E_1 and E_2, in area 6, merge on the half-line $N_6 = \{(\mu_1; \mu_2) : \mu_1 = 0; \mu_2 < 0\}$ as a result of the "pitchfork" bifurcation, and a return to area 1 occurs.

Regarding algebraic equation (12), it should be noted that in the context of the above assumptions, the three equilibrium positions merge to form a threefold equilibrium value.

In further research, while constructing a mathematical model, it is advisable to introduce a cyclic component into the demand function, which corresponds to seasonal fluctuations. In this case, one can expect the occurrence of resonance in the system, which can cause complex chaotic behavior of the system.

Conclusion

The mathematical model of a dynamic system proposed in the work describes the state of the market for one product as a result of the interaction of supply and demand functions with a time lag. The model is a system of two differential equations with quadratic nonlinearity, which made it possible, using qualitative analysis (along trajectories), to study the basic properties of this economic system. It was discovered that the nonlinear demand function ensures the presence of non-unique market equilibrium positions, which is a fact that is far from trivial. From the authors' point of view, the main problem for this dynamic system is to study the stability of equilibrium positions, taking into account the nonlinear interaction of its main variables which are the price of a product and the volume of its supply. Modeling of the behavioral properties of multi-product markets, which was carried out by Sir John Richard Hicks, Vasily Leontiev, and Oskar Ryszard Lange, is a much more complex task, the solution of which is complicated by the high dimensionality of the economic system, and the presence of nonlinearity of connections does not allow the use of analytical methods. However, the existing approximate methods of reducing multidimensional dynamic systems allow such systems to be reduced to a system with one or two degrees of freedom, but this problem requires independent study and was not considered in this work.

As a result of the research, the area of stability of the economic system was identified and the parameters of this area were determined, as well as an analysis of the behavior of the system at the boundaries of the area of stability was carried out and a bifurcation diagram was constructed for the characterization of the processes occurring at these boundaries. A number of types of changes in the stability of the equilibrium regimes with the appearance of characteristic bifurcations have been identified. This includes the saddle-node bifurcation, as well as the bifurcation of the birth or death of a limit cycle. In addition, there is a symbiosis of the above-mentioned bifurcations in the form of a bifurcation of codimension two, i.e., the so-called Bogdanov–Takens bifurcation arises. This is accompanied by a global restructuring of the phase portrait with the appearance of ultra-low frequency cycles. Such trends in the production and economic system evolution are a unique phenomenon manifested in the presence of so-called turning points, at which a change in the increasing phase of long waves to a decreasing one and vice versa occurs. All these factors should be taken into account when developing strategic plans for managing under conditions of a constantly changing market environment. The ability to anticipate the existence of such phenomena and eliminate their occurrence allows decision makers to optimize the allocation of resources thus increasing production efficiency and fully satisfying customer needs.

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Modelos de dinámica microeconómica: bifurcaciones y algoritmos de comportamiento de sistemas complejos

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CAMPO: matemáticas
TIPO DE ARTÍCULO: artículo científico original

Resumen:

Introducción/objetivo: El estudio de la dinámica de la influencia mutua de la oferta y la demanda es relevante en relación con las pérdidas financieras que surgen debido a la incertidumbre en la demanda y los errores de pronóstico. El trabajo tiene como objetivo construir un modelo matemático de la dinámica de esta interacción para el mercado de un producto.

Métodos: En el presente trabajo se propone un modelo matemático de los estados del sistema oferta-demanda, en cuyo marco se consideran los procesos que ocurren en dicho sistema desde la perspectiva de la metodología de la sinergética económica. El modelo matemático de la dinámica tiene la forma de un sistema de dos ecuaciones diferenciales con no linealidad cuadrática.

Resultados: La utilización del modelo propuesto para reproducir diversos estados dinámicos de los procesos de autorregulación del mercado permitió identificar la jerarquía de transición de regímenes dinámicos estables a inestables con la aparición de las correspondientes bifurcaciones. Se prestó especial atención al estudio del comportamiento del sistema en los límites de la región de estabilidad.

Conclusión: Se ha revelado la existencia de una bifurcación de los nodos de silla de los ciclos límite, lo que sugiere la aparición de autooscilaciones estables en el caso de un ciclo "suave" e inestables en el caso de un ciclo "duro". Al estudiar una bifurcación de codimensión dos – "doble cero" – se descubrieron estructuras dinámicas especiales, determinadas por las propiedades de las bifurcaciones globales. Este tipo de comportamiento se caracteriza por autooscilaciones de baja frecuencia, lo que da lugar a las llamadas "ondas ultralargas" del estado económico.

Palabras claves: dinámica del sistema de oferta y demanda, desfase temporal, ciclo límite, bifurcación, caos.

Модели микроэкономической динамики: бифуркационные алгоритмы поведения сложных систем

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РУБРИКА ГРНТИ: 06.35.51 Экономико-математические методы и модели
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Изучение динамики взаимного влияния спроса и предложения актуально в связи с финансовыми потерями, возникающими из-за неопределенности спроса и ошибок прогнозов. Целью работы является построение математической модели динамики этого взаимодействия для рынка одного товара.

Методы: В статье предложена математическая модель состояний системы спроса и предложения, в рамках которой процессы, происходящие в этой системе, рассматриваются с позиций методологии экономической синергетики. Математическая модель динамики имеет вид системы двух дифференциальных уравнений с квадратичной нелинейностью.

Результаты: Использование предложенной модели для воспроизведения различных динамических состояний процессов рыночного саморегулирования позволило выявить иерархию перехода от устойчивых динамических режимов к неустойчивым с возникновением соответствующих бифуркаций. Основное внимание уделено изучению поведения системы на границах области устойчивости.

Выводы: Выявлено существование узловой бифуркации предельных циклов, что означает возникновение устойчивых автоколебаний в случае «мягкого» цикла и неустойчивых в случае «жесткого» цикла. Также обнаружены особые динамические структуры, определяемые свойствами глобальных бифуркаций. Для этого типа поведения системы характерны автоколебания с низкой частотой, что приводит к возникновению так называемых «сверхдлинных волн» экономического состояния.

Ключевые слова: динамика систем спроса и предложения, временной лаг, предельный цикл, бифуркация, хаос.

Модели микроекономске динамике: алгоритми бифуркације и понашања сложених система

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ОБЛАСТ: математика
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Проучавање динамике узајамног утицаја понуде и потражње важно је када је реч о финансијским губицима услед неизвесне потражње и грешака у предвиђању. Циљ рада јесте да креира математички модел динамике ове интеракције за тржиште једног производа.

Методе: У раду се предлаже математички модел стања система понуде и потражње унутар оквира у којем се разматрају процеси који делују у овом систему са аспекта методологије економске синергије. Математички модел динамике има облик система две диференцијалне једначине са квадратном нелинеарношћу.

Резултати:Коришћење предложеног модела за репродукцију различитих динамичких стања процеса тржишне саморегулације омогућило је идентификацију хијерархије преласка из стабилних динамичких режима у нестабилне са појавом одговарајућих бифуркација. Највише пажње посвећено је проучавању понашања система на границама области стабилности.

Закључак: Откривено је постојање бифуркације седло-чвор граничних циклуса што указује на појаву аутоосцилација које су стабилне у случају „меког” циклуса, а нестабилне у случају „тврдог” циклуса. Приликом проучавања бифуркације кодимензије 2 – „двострука нула” откривене су специјалне динамичке структуре, одређене својствима општих бифуркација. Ова врста понашања карактерише се аутоосцилацијама ниске фреквенције што доводи до такозваних „ултрадугих” таласа економског стања.

Кључне речи: динамика система понуде и потражње, временско кашњење, гранични циклус, бифуркација, хаос.

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