# Numerical optimization by the PFEMCT - SIF method of the crack propagation of a linear elastic material

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*Abstract:*

*Introduction/purpose: This study investigates the influence of contour numbers surrounding the crack tip on stress intensity factors (SIFs) using the Propagation Finite Element Method Crack Tip Stress Intensity Factor (PFEMCT-SIF) approach. It also compares the maximum circumferential stress criterion (MCSC) and the Richard criterion for crack propagation prediction.*

*Methods: A finite element code written in Visual Fortran was developed to model crack tips with 3, 5, 10, 15 and 20 contours using 4-node quadratic CPE4 elements. Abaqus software was utilized to calculate SIFs and crack orientation angles. Horizontal and inclined cracks were analyzed in a steel plate under tensile loading. The results were validated against analytical solutions and previous numerical studies.*

*Results: The 10-contour model showed the best agreement with analytical SIF values. Increasing contour numbers improved SIF accuracy for horizontal cracks, but excessive refinement led to divergence for inclined cracks. The MCSC and the Richard criterion produced comparable crack trajectories, with the MCSC demonstrating slightly higher precision.*

*Conclusions: The PFEMCT-SIF method effectively evaluates SIFs and predicts crack propagation paths. A 10-contour crack tip model balances accuracy and computational efficiency. The study highlights the importance of optimizing crack tip mesh refinement in fracture mechanics simulations.*

*Key words: crack tip, crack propagation, SFEMCT-SIF method, MCSC and Richard criterion, contours.*

## Introduction

In fracture mechanics, crack propagation modeling by different numerical methods plays a beneficial role in solving various problems.

It is significantly used in fracture mechanics, structures calculation, materials fatigue and damage, etc. It also makes it possible to treat certain thermal, energetic and electromagnetic fatigue problems. Precise prediction of the fatigue life of components or structures creates a great interest in modeling and mechanical engineering applications. In this regard, there is now an increasing need to be able to accurately simulate initial cracks in structures. To solve various problems in fracture mechanics, application of numerical methods proves to be useful. Therefore, numerical methods have been widely developed in recent years, and remain among the most used methods and offer solutions for a very large number of applications.

Alshoaibi (2015) characterized the singularity and the stress intensity factors around the crack tip; he also used the motion extrapolation method to simulate crack propagation in 2D by the finite element method of a linear elastic plate.

Rao & Rahman (2000) used the EFGM method to eliminate the Lagrange multipliers drawbacks, typically used in the Galerkin formulations without elements. Boulenouar et al. (2014) used the displacement extrapolation technique to obtain the SIF at the crack front. Zaleha et al. (2007) evaluated the displacement extrapolation technique (DET) for the prediction of stress intensity factors. Cho (2015) evaluated the Petrov-Galerkin natural elements method (PG-NE), and proposed mixed modes to calculate the stress intensity factors of a two-dimensional inclined crack. (Boulenouar et al, 2016b) presented a study based on numerical examples demonstrating the efficiency, the robustness and the precision of the calculation algorithm allowing to predict the crack propagation path. Yaylaci (2016) proposed a comparative study between the finite element method (FEM) and the analytical method of a composite material with planar layers containing a perpendicular internal crack. However, Hamdi et al. (2007) used the strain energy density criterion on filled rubber materials to predict the initial orientation of a central crack in the case of large strains. Thus, this criterion was validated in the two cases of fragile and ductile materials: fragile materials by Theocaris (1984), Boulenouar et al. (2013, 2016a), Ayatollahi & Sedighiani (2012), Pegorin et al. (2012), Choi et al. (2006) and ductile materials (Komori, 2005; Chow & Jilin, 1985; Carpinteri, 1984).

Another study proposed by Boulenouar & Bendida (2019) is based on the implementation of the displacement correlation technique (DCT) and the maximum circumferential stress (MCS) theory in a finite element code by the use of the Ansys Parametric Design Language (APDL).

A comparative study between the finite element method (FEM) and the analytical method was proposed by Yaylaci (2016) to solve a plane problem of a laminated composite material containing an internal perpendicular crack using Ansys software in elements finished for 2D analysis. On the other hand, a crack propagation study was presented by Bentahar & Benzaama (2023) and Bentahar et al. (2024) to evaluate the stress intensity factor by the FEM method.

For a precise estimation of stress intensity factors in mode I, II and the mixed mode, Sajith et al. (2018) proposed a simple and effective technique based on finite elements which uses nodal displacements of crack faces for a precise estimation of the stress intensity factors in mode I, II and in the mixed mode.

Malekan et al. (2018) presented a work based on the XFEM method to model nucleation and crack propagation in structures made of linear or non-linear materials. In addition, different energies were evaluated by Bentahar et al. (2021a) and by Bentahar (2023a, 2023b) using the XFEM method.

To obtain the stress intensity factors at the crack front, Benamara et al. (2017) proposed a study for homogeneous materials based on the displacement extrapolation technique (DET).

Using a Fortran program, Alshoaibi (2018) presented a numerical simulation-based study of crack growth, using the extrapolation technique of displacement to obtain singular stresses in the crack front and the values of the stress intensity factors. In addition, this method is used to find out the crack direction by the maximum circumferential stress criterion.

The aim of this study is to model the effect of the number of contours at the crack tip on the stress intensity factor by the (SFEMCT-SIF) method using the maximum circumferential stress criterion (MCSC) and the Richard criterion for 3, 5, 10, 15, and 20 contours around the crack tip region. In addition, this work uses Fortran to program correctly and control evenly the number of contours around the crack tip.

## Law of fatigue crack propagation in 2D

To estimate the growth rate of fatigue cracks, Paris & Erdogan (1963) used the most common model, that of a crack stressed in pure mode I, with an applied load; the direction in this case is compatible.

It propagates depending on the material parameter and the environment. The value ΔKseuil lower than the amplitude of the stress intensity factor for a loading cycle $\Delta K_I = K_{Imax} - K_{Imin}$. (Figure 1) shows the three propagation regimes.

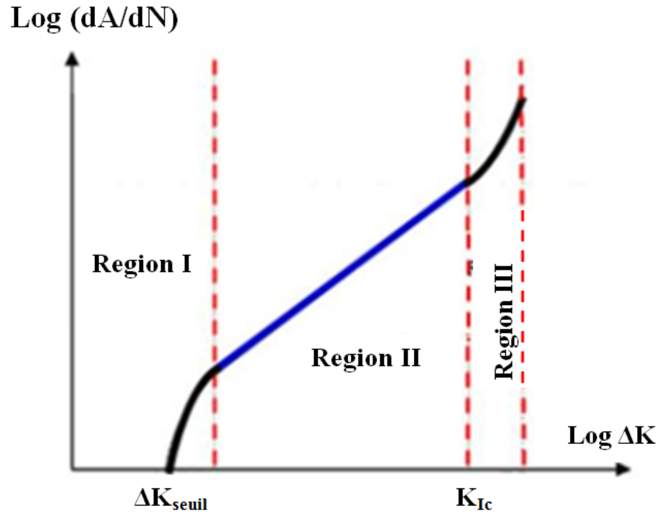


*Figure 1 – Explanatory diagram of the three crack propagation regions in case of mode I*

• In region I, the cracking rates in this regime are very low.

We observe the coalescence of microcracks and the formation of one or more macrocracks; for this, the value of ΔK is greater than $\Delta K_{seuil}$.

• In region II, the cracking rate is linear. In this regime on the log − log diagram, the Paris law, defined by equation (1), establishes a linear relationship between log(da) and log(ΔK) – this is a so-called stable propagation regime

$$\frac{d_a}{d_N} = C(\Delta K)^m \quad (1)$$

where ΔK is the variation of the stress intensity factor during a cycle that induces an advance da of the crack, and C and m are two parameters of the material defining respectively the position and the slope of the Paris line.

• In region III, there is a so-called unstable propagation regime. It is characterized by a strong acceleration of the crack and ΔK tends towards the toughness of the material $K_{Ic}$ which is experimentally observed during the rupture of the part.

The propagation speed is here higher than that expected in the Paris regime (region II).

## Crack propagation criteria

In order to simulate crack propagation under the linear elastic condition, the crack path direction must be determined. There are several methods used to predict the direction of the crack trajectory such as the maximum normal stress theory (or the maximum circumferential stress theory (Erdogan & Sih, 1963) and the minimum strain energy density theory criteria (Sih, 1974).

### *Maximum circumferential stress criterion (MCSC)*

The maximum circumferential stress criterion (MCSC) is a criterion that requires the calculation of the stress intensity factors $K_I$ and $K_{II}$ to determine this bending angle, on the one hand, and, on the other hand, it indicates that the crack always propagates in the direction of the maximum σθθ. This is because the direction of crack growth is directly determined by the local stress field along a small circle of the radius r centered at the crack tip:

$$tan\frac{\theta}{2} = \left[\frac{1}{4}\frac{K_I}{K_{II}} \pm \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right] \qquad (2)$$

where $K_I$ and $K_{II}$ are, respectively, the stress intensity factors corresponding to mode I and II loading.

### *Criterion of Richard 2D*

This criterion was developed empirically by Richard (1985) and Rossmanith (1984). Here, the comparative stress intensity factor Kv is defined by equation (3):

$$K_v = \frac{K_I}{2} + \frac{1}{2}\sqrt{K_I^2 + 5.366K_{II}^2} = K_{Ic} \qquad (3)$$

KV is based on the stress intensity factors $K_I$ and $K_{II}$, and the fracture toughness $K_{IC}$ is related to $K_V$ and crack growth is discontinuous when $K_V$ is greater than $K_{IC}$. This criterion provides an excellent approximation of the fracture limit surface of the maximum tangential stress criterion of Erdogan & Sih (1963). The crack pucker angle θ can be determined by equation (4):

$$\theta = \mp\left[140^{\circ}\frac{|K_{II}|}{|K_I|+|K_{II}|} - 70^{\circ}\left(\frac{|K_{II}|}{|K_I|+|K_{II}|}\right)^2\right] \quad (4)$$

when the bending angle is θ< 0, $K_{II}$ > 0, on the one hand, and, on the other hand, $K_I$ always > 0.

## Influence of the number of singularity area contours

### *Crack origin*

The CT point is the crack front center and the starting point for creating the other nodes by this method. The coordinates of this point are given by equation (5):

$$\begin{Bmatrix} X(CT) = a \\ Y(CT) = 0 \end{Bmatrix} \quad (5)$$

where
(a) is the crack length, and
(CT) is the crack tip designation.

### *Singularity zone*

The singularity zone is found in the crack front zone (plasticization zone) where the stresses gradients and deformations are the most important; there, singular elements were used.

For the Abaqus simulation, the CPE4 type elements were chosen. This element type is used for 2D models, quadrilateral with four nodes, and it is well suited for simulation. Thus, singular elements are used around the crack front. These singular types "quarter point" elements are collapsed quadratic elements.

These elements are obtained by following different stages:

1. Move the knots from the side connecting the middle to the crack front "quarter-point" (L/4) closer to the crack front, making from this mode the singularity effect reproduction 1/√r.

2. Collapse one of the sides (2D) or one of the faces (3D) belonging to the crack front so that these nodes collapsed on this side or this face will have the same coordinates. The reason for this transformation from a quadrilateral or a quadrangular prism to a triangle or a triangular prism is that the first only shows a singular behavior around the singular point of the crack while the second reproduces singularity in its whole domain.

3. Restrict the coincident nodes on the collapsed side or face for a joint movement, i.e., completely united, which allows the reproduction of the singularity 1/√r - an elastic-linear behavior characteristic.

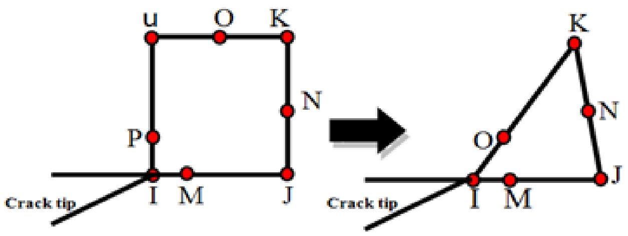


*Figure 2 – Illustration of a quadrilateral element reduced to obtain a triangular element, (Bentahar et al, 2021b)*

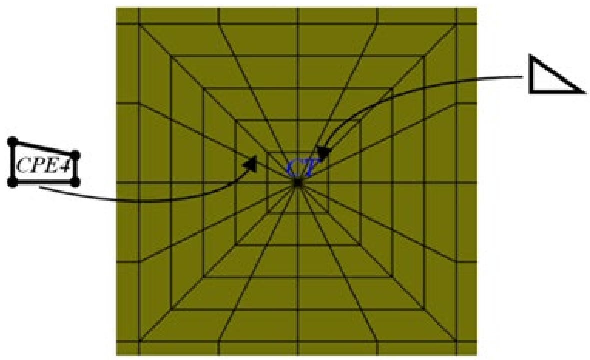


*Figure 3 – Element types chosen for modeling.*

## *Singularity zone outlines*

From the point node (CT) of the crack point, the singular elements created by equation (6):

$$\begin{cases} x_i = X(CT) - j\frac{L}{t}\cos((i-1)\theta) \\ y_i = Y(CT) - j\frac{L}{t}\sin(i\theta) \end{cases} \quad (6)$$

i = 1.16 (number of elements in each contour);
j = 1, t (contour number); and
θ = π/16 (angle division).

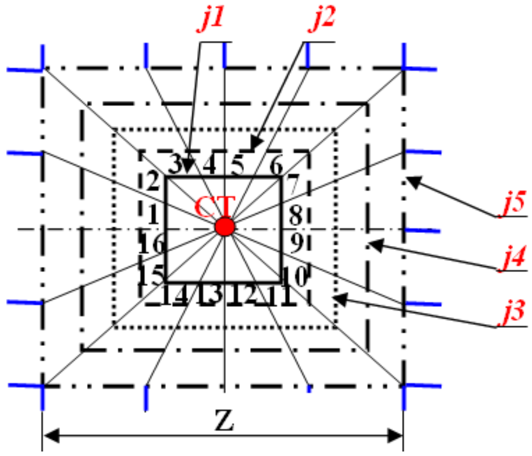


*Figure 4 – Singularity zone with its contours and its elements - illustration.*

## Stress field in the crack front vicinity

Figure 4 illustrates the stress field near a crack point with the polar coordinates (r, θ) in the crack tip (CT) by equation (7).

$$\sigma_{i,j}^{I,II}(r,\theta)=\frac{K_{I,II}}{\sqrt{2\pi r}}f_{ij}(\theta) \tag{7}$$

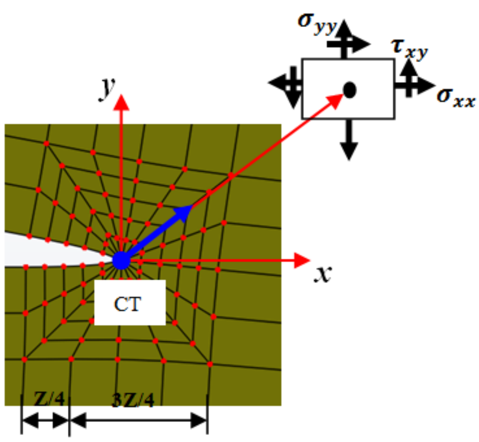


*Figure 4 – Stress field in the crack tip (CT) - properties*

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The position of the stress field in the polar coordinates (r, θ) for an isotropic linear elastic material at the crack tip is determined by equation (8):

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad (8)$$

Tada et al. (2000) proposed a general equation to describe the stress field in 2D in the crack front vicinity; the latter is defined by the stress intensity factor K.

## Stress intensity factor

The stress intensity factor K is the only important parameter that allows to know the state of stress and deformation at any crack point, Fiordalisi (2014). For the crack in the opening position, the relationship between the distal stress perpendicular to the crack axis σ and the stress intensity factor $K_I$, is given analytically by equation (9) proposed by Ewalds & Wanhill (1984) and the correction factor (F) is given by equation (10):

$$K_I = F\sigma\sqrt{a\pi} \quad (9)$$

where F is the correction factor given by:

$$F = 1.12 - 0.23\left(\frac{a}{W}\right) + 10.6\left(\frac{a}{W}\right)^2 - 21.7\left(\frac{a}{W}\right)^3 + 30.4\left(\frac{a}{W}\right)^4 \quad (10)$$

where the stress intensity factor $K_{II}$ is calculated by the relation:

$$K_I sin\theta + K_{II}(3cos\theta - 1) \quad (11)$$

## Numerical model and simulation

### *Horizontal crack propagation*

The structure considered has a length B of 12 mm and a width W of 10 mm. The horizontal crack length a is 3.5 mm and the length of the front Z is 1.5 mm. The parametric mesh consists of 478 square CPE4 type elements with four nodes. The total number of degrees of freedom is equal to 1016. The Fortran program for creating the mesh that will be analyzed by the Abaqus finite element code has been applied. The steel structure with E = 210 GPa and v = 0.3 is subjected to a uniform tensile stress σ = 120 MPa. To study and characterize the stress field in the crack tip vicinity, a several front contour numbers are proposed for the optimization of this area and therefore, the stress intensity factor in the mixed mode is calculated. The mesh including the numbers of 3, 5, 10, 15, and 20 contours is analyzed while keeping the same type of CPE4 elements at 4 nodes.

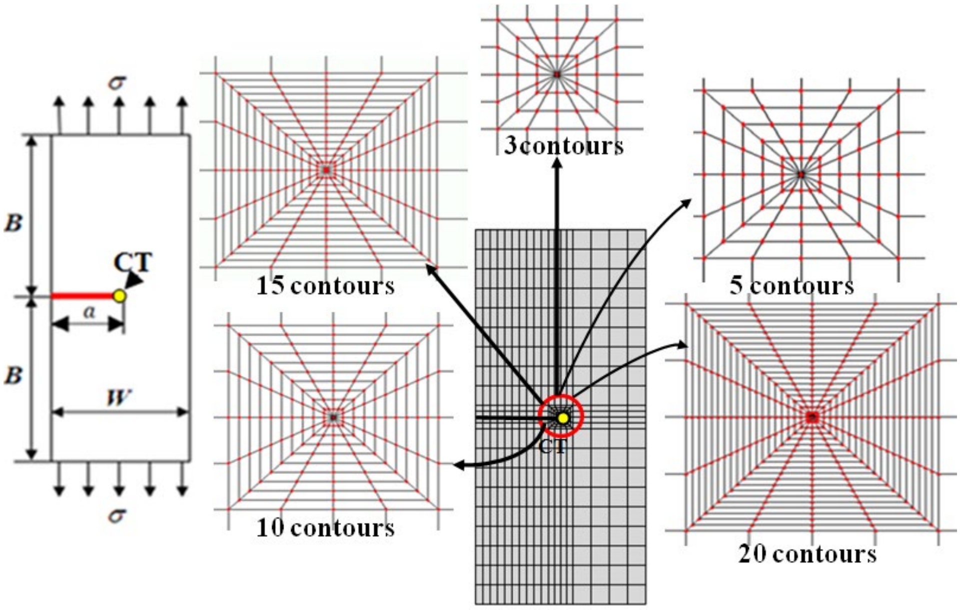



*Figure 5 – a) Geometry model with boundary conditions and the proposed structure dimensions for a horizontal crack of the length a and b) PFEMCT-SIF model with different numbers of contours for the singularity zone*

Figures 6a, b, and c illustrate a very long horizontal crack with different crack tip shapes used by Alshoaibi & Ariffin (2008), FRANC2D and the PFEMCT-SIF method. On the other hand, numerically, Figure 7 shows the three comparison methods presented by a) Phongthanapanich & Dechaumphai (2004), b) Rao & Rahman (2000) and c) PFEMCT-SIF method, in the case of a horizontal crack with an angle α = 0º and a = 3.5 mm. In addition, this study has different contours numbers (3, 5, 10, 15, and 20 contours). Table 1 below presents the comparison between the results obtained by the method (EFGM) proposed by Rao & Rahman (2000) and the results obtained by the PFEMCT-SIF method.

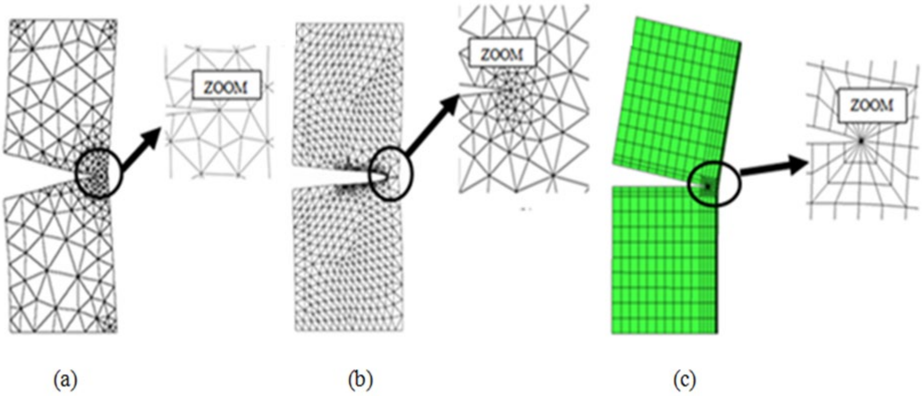



*Figure 6 – Crack tip model: a) Alshoaibi & Ariffin (2008), b) FRANC2D, and c) PFEMCT-SIF method*

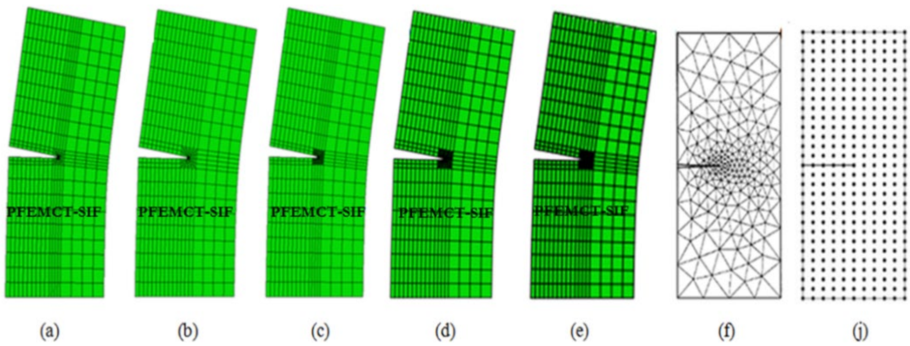
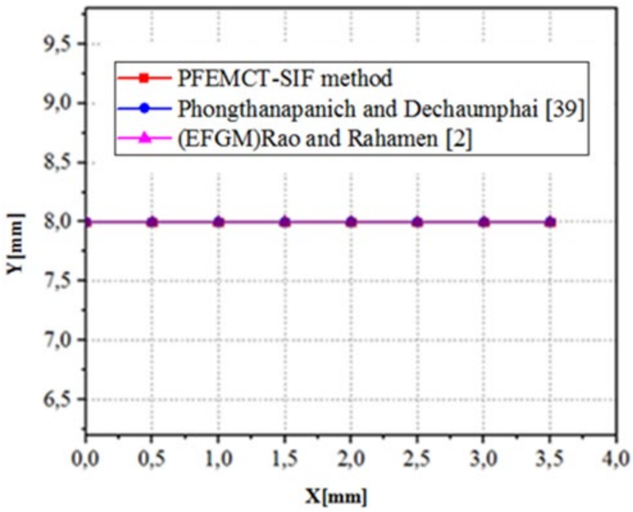


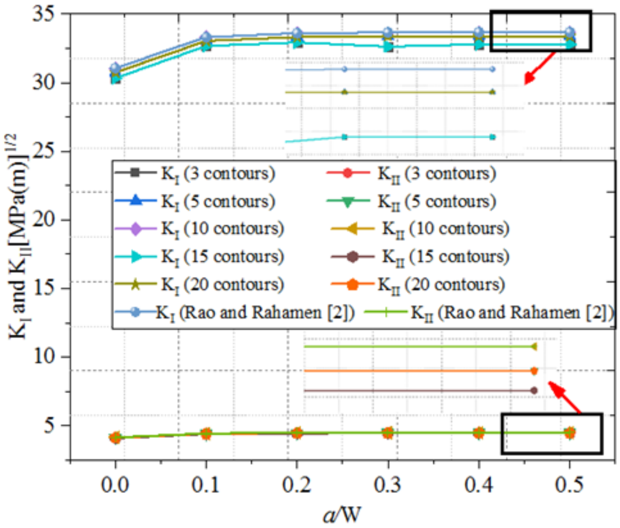

*Figure 7 – Crack propagation in the case of α = 0º: a) 3 contours, b) 5 contours, c) 10 contours, d) 15 contours, and e) 20 contours, (f) Phongthanapanich & Dechaumphai (2004), and (j) Rao & Rahamen (2000)*

*Table 1 – Comparison of the results obtained by the PFEMCT-SIF method and the method proposed by Rao & Rahman (2000) concerning the stress intensity factors $K_I$ and $K_{II}$ in the case of different numbers of contours (3, 5, 10, 15, and 20 contours).*

| PFEMCT-SIF method | | | | | | | | | | (Rao & Rahman, 2000) | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 contours | | 5 contours | | 10 contours | | 15 contours | | 20 contours | | | |
| $K_I$ | $K_{II}$ | $K_I$ | $K_{II}$ | $K_I$ | $K_{II}$ | $K_I$ | $K_{II}$ | $K_I$ | $K_{II}$ | $K_I$ | $K_{II}$ |
| 30.32 | 4.099 | 30.73 | 4.128 | 31.05 | 4.151 | 30.32 | 4.099 | 30.73 | 4.128 | 31.05 | 4.151 |
| 32.66 | 4.408 | 33.03 | 4.427 | 33.30 | 4.440 | 32.66 | 4.408 | 33.03 | 4.427 | 33.30 | 4.440 |
| 32.92 | 4.453 | 33.31 | 4.475 | 33.60 | 4.492 | 32.92 | 4.453 | 33.31 | 4.475 | 33.60 | 4.492 |
| 32.62 | 4.465 | 33.36 | 4.482 | 33.65 | 4.499 | 32.62 | 4.465 | 33.36 | 4.482 | 33.65 | 4.499 |
| 32.78 | 4.471 | 33.37 | 4.484 | 33.67 | 4.500 | 32.78 | 4.471 | 33.37 | 4.484 | 33.67 | 4.500 |
| 32.78 | 4.471 | 33.37 | 4.484 | 33.67 | 4.500 | 32.78 | 4.471 | 33.37 | 4.484 | 33.67 | 4.500 |




(a)




(b)

*Figure 8 – (a) Crack trajectory obtained by the three methods and (b) comparison between the results obtained by the method proposed by Rao & Rahman (2000) and the the PFEMCT-SIF for different numbers of contours.*

Figure 8 shows the comparison of the crack propagation in mode I, of the length a = 3.5mm between the three methods (EFGM) proposed by Rao & Rahman (2000), Adaptive Delaunay triangulation proposed by

Phongthanapanich & Dechaumphai (2004) and the PFEMCT-SIF method presented for different numbers of contours (3, 5, 10, 15, and 20 contours). In the case of α = 0º, the crack trajectory remains very comparable.

Figure 8b shows the evaluation of the SIF between the PFEMCT-SIF method presented for different numbers of contours and the method proposed by Rao & Rahman (2000); it can be noted that the comparison between these results allows to conclude that the numerical model used correctly describes the stress and strain field near the crack tip (CT) in the conditions of pure mode I. The comparison was made in the cases of the ratios a/w = 0.1, 0.2, 0.3, 0.4 and 0.5. It can be noted that the results obtained are the best.

### *Inclined horizontal crack propagation*

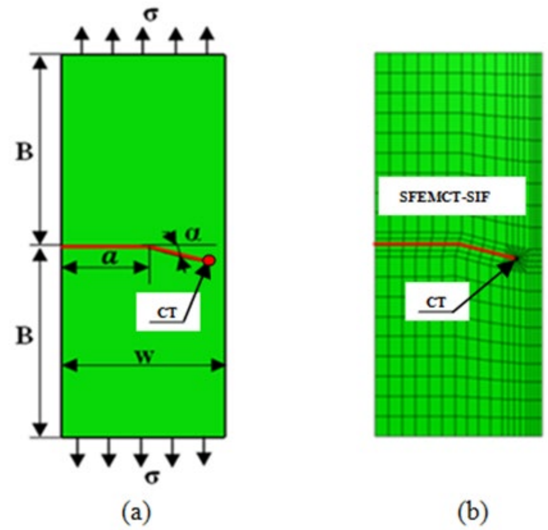



*Figure 9 – a) Inclined horizontal crack proposed model geometry of the length a and the inclination angle α for five propagations, and b) inclined horizontal crack PFEMCT-SIF model*

Figure 10 illustrates the crack propagation trajectory concerning the angle of crack α= -15.60º with 3, 5, 10, 15, and 20 contours, by the PFEMCT-SIF method in the first propagation.

Figure 11 presents the crack trajectory comparison in modes I and II in the case of a = 6.8 mm between the three methods: the EFGM proposed by Rao & Rahman (2000), the adaptive Delaunay triangulation method proposed by Phongthanapanich and Dechaumphai (2004) and the

PFEMCT-SIF method presented in this study for different numbers of contours (3, 5, 10, 15, and 20 contours).

Figure 12 shows the comparison of the crack trajectory between the study presented by Phongthanapanich & Dechaumphai (2004) and Rao & Rahman (2000) and the PFEMCT-SIF method. First of all, the angle α varies between 0º and 2.533º (Figure 9), i.e., the crack propagates horizontally along the opening mode (modes I and II), as shown in Figure 11.

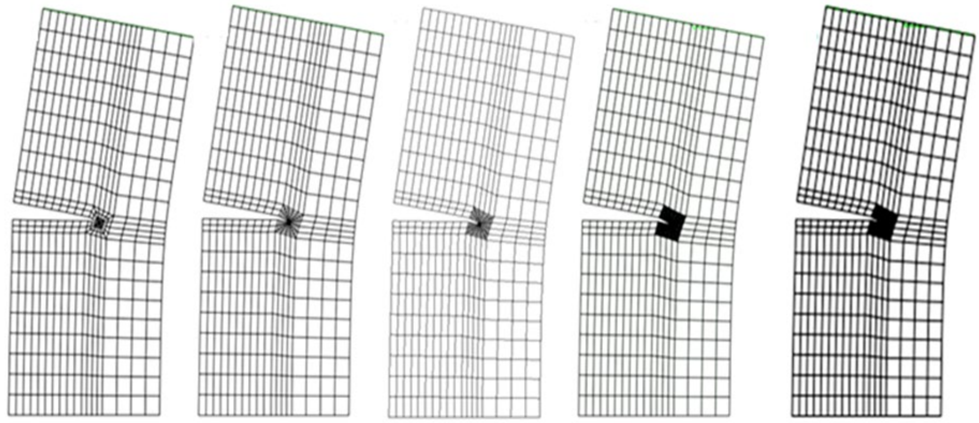


*Figure 10 – Crack propagation trajectory α = -15.60º: a) 3 contours, b) 5 contours, c) 10 contours, d) 15 contours, and e) 20 contours, by the PFEMCT-SIF method in the first propagation*

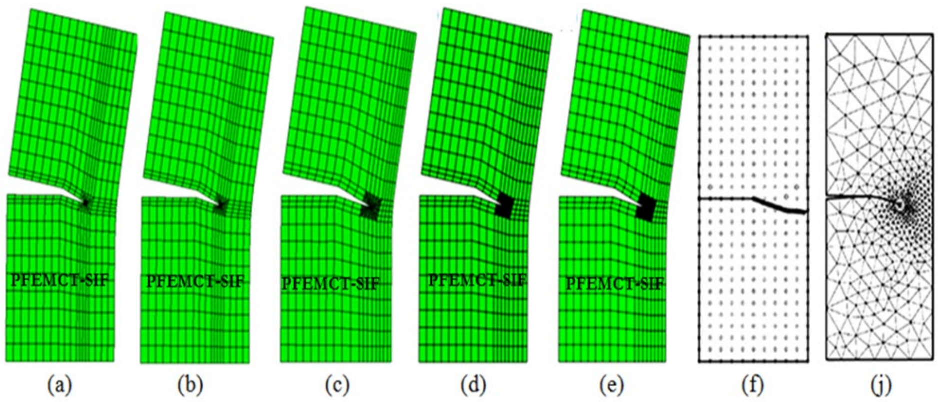



*Figure 11 – Crack propagation trajectory by the PFEMCT-SIF method: a) 3 contours, b) 5 contours, c) 10 contours, d) 15 contours, and e) 20 contours, (f) Phongthanapanich & Dechaumphai (2004) and (j) Rao & Rahman (2000)*

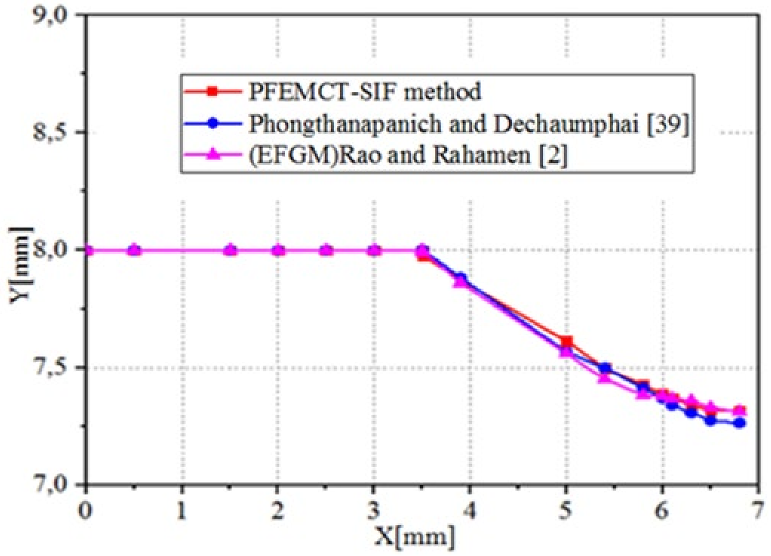



*Figure 12 – Comparison of the results between the three methods: Rao & Rahman (2000), Phongthanapanich & Dechaumphai (2004), and the PFEMCT-SIF method*

The comparison in Figure 12 shows a good correlation between the three methods, especially in mode I up to a slit length of (a= 3.5) mm. The calculations of Y are based on equation (5). Moreover, the results of the comparison between the analytical method and the PFEMCT-SIF method are shown in Tables 2 and 3, respectively.

*Table 2 – Comparison of the results obtained by the PFEMCT-SIF method and the analytical method for the stress intensity factors $K_I$ for different numbers of contours (3, 5, 10, 15, and 20 contours)*

| $K_I$ [MPa.(m)$^{1/2}$] PFEMCT-SIF method | | | | | Analytical method | |
|---|---|---|---|---|---|---|
| 3 contours | 5 contours | 10 contours | 15 contours | 20 contours | $K_I$ | α[◦] |
| 32.92 | 33.03 | 33.60 | 33.66 | 33.75 | 33.4 | -15.60 |
| 42.63 | 43.20 | 43.63 | 43.58 | 43.87 | 42.9 | -0.8237 |
| 55.51 | 56.38 | 57.03 | 56.93 | 57.47 | 56.95 | 0.9473 |
| 75.88 | 77.31 | 78.41 | 78.16 | 79.25 | 75.68 | 1.691 |
| 112.2 | 114.9 | 115.3 | 116.4 | 118.9 | 114.51 | 2.533 |

*Table 3 – Comparison of the results obtained by the PFEMCT-SIF method and the analytical method for the stress intensity factors $K_{II}$ for different numbers of contours (3, 5, 10, 15, and 20 contours)*

| $K_{II}$ [MPa.(m)$^{1/2}$] PFEMCT-SIF method | | | | | Analytical method | |
|---|---|---|---|---|---|---|
| 3contours | 5 contours | 10 contours | 15 contours | 20 contours | $K_{II}$ | α[∘] |
| 4.453 | 4.475 | 4.492 | 4.483 | 4.499 | 4.324 | -15.60 |
| 0.2590 | 0.2419 | 0.2278 | 0.2359 | 0.2094 | 0.2404 | -0.8237 |
| -0.3796 | -0.4120 | -0.4385 | -0.4267 | -0.4807 | -0.4006 | 0.9473 |
| -1.017 | -1.071 | -1.113 | -1.095 | -1.187 | -1.056 | 1.691 |
| -1.975 | -2.067 | -2.000 | -2.111 | -2.283 | -2.038 | 2.533 |

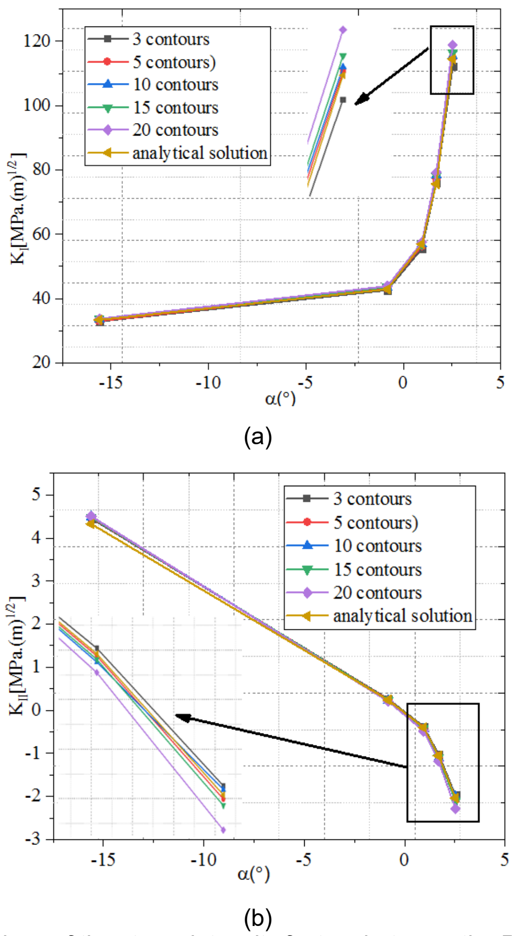



*Figure 13 – Comparison of the stress intensity factors between the PFEMCT-SIF method and the analytical method proposed by Ewalds & Wanhill (1984): a) $K_I$ and b) $K_{II}$*

In Figures 13a and 13b, it can be noted that the results of the crack length effect on the stress intensity factor $K_I$ (Figure 13a) and on $K_{II}$ (Figure 13b) clearly illustrate the proportionality between the results of the study presented and the analytical method for equations (9), (10) and (11) proposed by Ewalds & Wanhill (1984). The results agree well in the case of the ten-contour model; regarding the case of $K_I$ and $K_{II}$, they are somewhat distant in the case of the 20-contour model. This is due to the fact that whenever the crack is in a tilting state, the crack tip contours are affected and a divergence from the reference values occurs.

This comparison is made in the case of the length from 3.5mm to 5.5mm. In addition, the $K_I$ values increase with increasing the crack length and the $K_{II}$ values decrease. These forms of the results were obtained by Boulenouar et al (2014), Alshoaibi & Ariffin (2008) in different cases of crack propagation for both modes I and II.

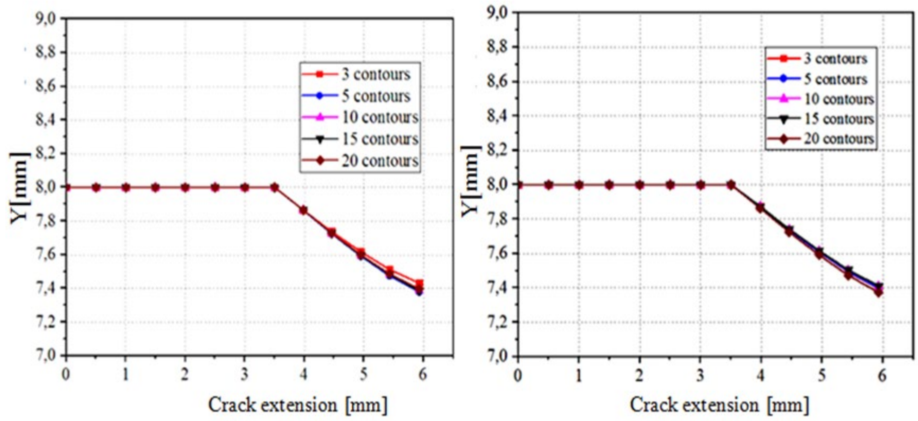
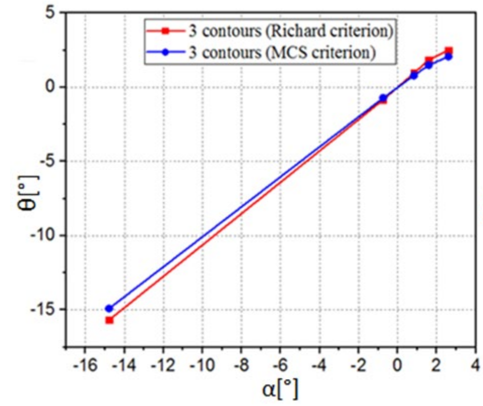
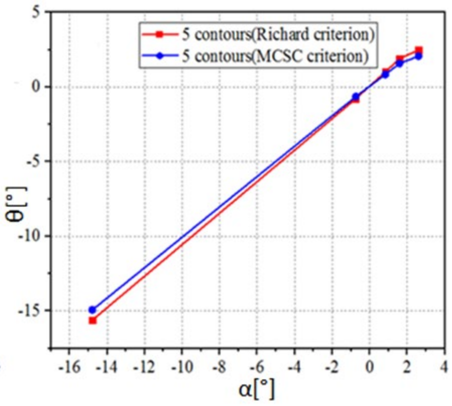



*Figure 14 – Crack propagation path PFEMCT-SIF method by: a) Richard criterion and b) MCS criterion*

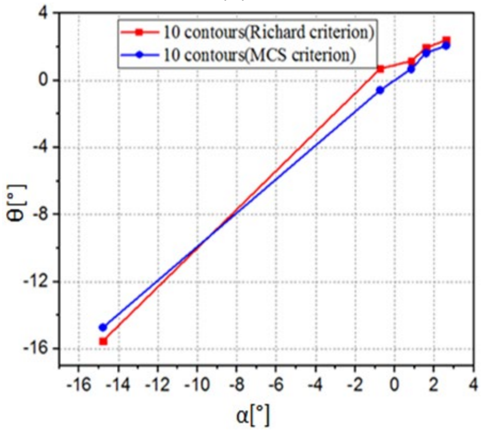
Figure 14 illustrates the crack propagation trajectory by two criteria of crack propagation: Figure 14a presents the results obtained by the Richard criterion, and Figure 14b shows the results obtained by the MCS criterion. It is important to predict the evolution of the crack trajectory during the propagation by the two criteria. The numbers of 5, 10, 15, and 20 contours give very proportional results between them. A slight difference is obtained in the case of 3 contours by the Richard criterion, on the one hand, while, on the other hand, 5, 10, and 15 contours of the crack tip give a good correlation between them. However, a slight difference is obtained in the case of 20 contours in the MCS criterion; in this context, the results obtained by Bouchard (2000) and Bouchard et al. (2003) found that the MCS criterion gives good results compared to other crack propagation criteria.



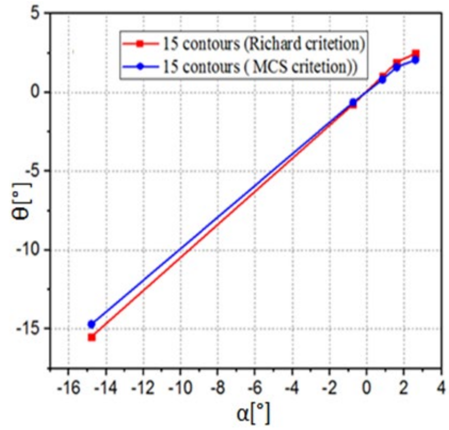

(a)



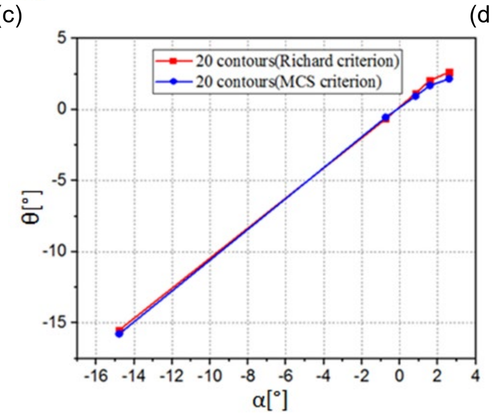

(b)




(c)




(d)




(e)

*Figure 15 – Comparison between the MCS and the Richard criterion by the PFEMCT-SIF; a) 3 contours, b) 5 contours, c) 10 contours, d) 15 contours, and e) 20 contours*

Figure 15 illustrates the inclination angle variation (α) according to the twist angle (θ) and calculated by the Richard criterion and the MCS criterion. Equations (2) and (4) are used and estimated at each crack length. The results obtained by the PFEMCT-SIF are comparable between the two criteria, (Richard's and that of the maximum tangential stress). We have chosen the criterion of the maximum circumferential stress (MCS). The use of the MCS criterion gives a crack propagation trajectory very close to that obtained using the Richard criterion, as shown in Figures 15a, b, c, d, and e which have presented the comparison for different numbers of the contours - 3,5,10,15, and 20 contours, respectively.

## Conclusion

It can be noted that:

This study had two important parts to compare. The first one is based on the comparison in terms of crack trajectory - it can be seen that the results obtained from this comparison are very consistent with the studies presented by Phongthanapanich & Dechaumphai (2004) and Rao & Rahman (2000). The second part of this study is based on the evaluation of the stress intensity factor and compared by the work of Rao & Rahman (2000). Two cases have been considered, a crack with a horizontal angle α = 0º and a crack with a horizontal angle α ranging from -15.60º to 2.533º. In fact, this study is based on the effect of the number of contours around the crack point (CT) on the stress intensity factor (SIF).

Singular quarter-point elements proposed by Barsoum (1976) are used to numerically model the singularity of the stress and strain fields in the vicinity of the crack tip to calculate the SIF by the PFEMCT-SIF method. The MCS criterion presents more precise results than that of Richard's criterion, in which, for $K_{II}$> 0, of the angle θ <0 and vice versa, while $K_I$ always> 0.

In general, the stress intensity factor $K_I$ is increased with the increase in the crack length (a). However, the stress intensity factor $K_{II}$ is decreased with increasing the crack length (a).

This study was very useful in terms of Fortran programming, and it is not easy to program a structure that contains a square end of a crack that contains several numbers of contours, by using the PFEMCT-SIF method to evaluate the stress intensity factor SIF.

As the crack begins to extend and tilt at an angle α, the crack tip elements become interconnected with each other.

Moreover, it can be seen that in the first case where α = 0, it is found that the more refined the crack front, the better the proportionality with the validation method proposed by Rao & Rahman (2000).

In the second case, when the angle α, is inclined, the best and closest number of contours around the crack front with the validation method proposed by Rao & Rahman (2000) is 10 contours. Therefore, we can say that this is the average case between the cases used and it helps to evaluate and find acceptable values for the stress intensity factor (SIF) by the PFEMCT-SIF method.

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## Optimización numérica por el método PFEMCT-SIF de la propagación de grietas en un material elástico lineal

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CAMPO: ingeniería mecánica
TIPO DE ARTÍCULO: artículo científico original

*Resumen:*

*Introducción/objetivo: Este estudio investiga la influencia de los números de contorno que rodean la punta de la grieta en los factores de intensidad de tensión (SIF) utilizando el enfoque del factor de intensidad de tensión de* *la punta de la grieta del método de elementos finitos de propagación (PFEMCT-SIF). También compara el criterio de tensión circunferencial máxima (MCSC) y el criterio de Richard para la predicción de la propagación de grietas.*

*Métodos: Se desarrolló un código de elementos finitos escrito en Visual Fortran para modelar puntas de grietas con 3, 5, 10, 15 y 20 contornos utilizando elementos CPE4 cuadráticos de 4 nodos. Se utilizó el software Abaqus para calcular los SIF y los ángulos de orientación de las grietas. Se analizaron grietas horizontales e inclinadas en una placa de acero sometida a carga de tracción. Los resultados se validaron frente a soluciones analíticas y estudios numéricos previos.*

*Resultados: El modelo de 10 contornos mostró la mejor concordancia con los valores analíticos de SIF. El aumento de los números de contorno mejoró la precisión de SIF para grietas horizontales, pero el refinamiento excesivo provocó divergencia para grietas inclinadas. El MCSC y el criterio de Richard produjeron trayectorias de grietas comparables, y el MCSC demostró una precisión ligeramente superior.*

*Conclusión:* El método PFEMCT-SIF evalúa eficazmente las SIF y predice las trayectorias de propagación de grietas. Un modelo de punta de grieta de 10 contornos equilibra la precisión y la eficiencia computacional. El estudio destaca la importancia de optimizar el refinamiento de la malla de la punta de la grieta en las simulaciones de mecánica de fracturas.

*Palabras claves: punta de grieta, propagación de grietas, método SFEMCT-SIF, MCSC y criterio de Richard, contornos.*

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Численная оптимизация распространения трещины в линейно-упругом материале методом PFEMCT-SIF

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РУБРИКА ГРНТИ: 30.19.00 Механика деформируемого твердого тела
ВИД СТАТЬИ: оригинальная научная статья

*Резюме:*

*Введение/цель: В данной статье исследуется влияние числа контуров, окружающих вершину трещины, на коэффициенты интенсивности напряжений (SIFS) с использованием метода конечных элементов распространения коэффициента интенсивности напряжений в вершине трещины (PFEMCT-SIF).*

*В статье также сравниваются критерий максимального окружного напряжения (MCSC) и критерий по Ричарду для прогнозирования распространения трещин.*

*Методы: В ходе исследования был разработан конечно-элементный код, написанный в Visual Fortran, для моделирования вершин трещин с 3, 5, 10, 15 и 20 контурами и с использованием 4-узловых четырехугольных элементов CPE4. Программное обеспечение Abaqus использовалось для расчета размеров трещин и углов ориентации трещин. Были проанализированы горизонтальные и наклонные трещины в стальном листе, подвергнутом растягивающей нагрузке. Результаты были подтверждены аналитическими решениями и предыдущими численными исследованиями.*

*Результаты: В результате исследования 10-контурная модель показала наилучшее соответствие с аналитическими значениями SIF. Увеличение числа контуров улучшило точность SIF у горизонтальных трещин, в то время как чрезмерная плотность сетки привела к расхождению в наклонных трещинах. Критерий MCSC и критерий Ричарда дали сопоставимые траектории трещин, причем MCSC оказался более точным.*

*Выводы: Метод PFEMCT-SIF эффективно оценивает параметры SIF и прогнозирует пути распространения трещин. Также выявлено, что 10-контурная модель вершины трещины обеспечивает баланс между точностью и вычислительной эффективностью. В исследовании подчеркивается значимость оптимизации плотности сетки вершин трещин при моделировании механики разрушения.*

*Ключевые слова: вершина трещины, распространение трещины, метод SFEMCT-SIF, критерий MCSC и критерий Ричарда, контуры.*

## Нумеричка оптимизација ширења прслине у линеарном еластичном материјалу помоћу метода PFEMCT-SIF

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ОБЛАСТ: механика
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

*Сажетак:*

*Увод/циљ: У овој студији испитује се утицај броја контура које окружују врх прслине на факторе интензитета напона (stress intensity factor - SIF) коришћењем приступа PFEMCT-SIF (Propagation Finite Element Method Crack Tip Stress Intensity Factor). Такође, пореде се критеријум максималног ободног напрезања (maximum circumferential stress criterion – MCSC) и критеријум по Richard-у за предвиђање ширења прслине.*

*Методе: Код коначних елемената, написан у Visual Fortran-у, развијен је за моделовање врхова прслина са 3, 5, 10, 15 и 20 контура помоћу квадратних елемената чистог оптерећења са 4 чвора (CPE4). Софтвер Abaqus коришћен је за израчунавање фактора интензитета напона и углова оријентације прслина. Анализиране су хоризонталне и нагнуте прслине на челичној плочи при напону на затезање. Резултати су потврђени поређењем са аналитичким решењима и претходним нумеричким студијама.*

*Резултати: Модел са 10 контура показао је најбоље слагање са аналитичким вредностима фактора интензитета напона. Повећање броја контура побољшало је прецизност фактора интензитета напона код хоризонталних прслина, док је превелика густина мреже довела до дивергенције код нагнутих прслина. Оба критеријума (MCSC и Richard) дала су упоредиве путање прслине, при чему је критеријум максималног ободног напрезања показао нешто већу прецизност.*

*Закључак: Метода PFEMCT-SIF ефикасно процењује факторе интензитета напона и предвиђа путеве ширења прслина. Модел врха прслине са 10 контура представља баланс између прецизности и рачунарске ефикасности. Истакнута је важност оптимизације густине мреже врха прслине у симулацијама механике лома.*

*Кључне речи: врх прслине, ширење прслине, метода SFEMCT-SIF, критеријум MCSC и критеријум по Richard-у, контуре.*

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