

Multi-attribute approach for enhancing maintenance processes in maintenance depots under a type-2 fuzzy environment

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Abstract:

Introduction/purpose: The purpose of this research is to determine the priority of Key Performance Indicators (KPIs) in a precise and structured manner. By applying the fuzzy multi-attribute decision-making model, operational management can identify and prioritize activities that will enhance maintenance process reliability in the shortest possible time while simultaneously reducing costs.

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Methods: The relative importance of sub-processes and KPI values is represented using predefined linguistic terms modelled by interval type-2 fuzzy numbers (IT2FNs). These assessments are formulated as a fuzzy group decision-making framework. The weight vector is determined using the fuzzy geometric mean, while the ranking of KPIs is obtained through the Taxonomy method combined with IT2FNs, which represents the main scientific contribution of this research.

Results: Real-world data gathered from a maintenance depot were used to test the proposed model. The study effectively modelled uncertainty in KPI evaluations using seven predefined linguistic expressions mapped onto IT2FNs. A consistent weight vector was obtained using the fuzzy group decision-making approach. Effective KPI ranking was achieved through a combination of the Taxonomy method and IT2FNs, which helped pinpoint the most important areas for operational improvement. The method's ability to provide clear priorities to support reliability improvements while cutting costs was validated through its application.

Conclusion: The key contributions of this study are: (i) fuzzy algebra rules with IT2FNs are used to determine the group utility value, and (ii) the integration of the Taxonomy method with IT2FNs for an improved decision-making procedure.

Key words: key performance indicators, operational management, maintenance process reliability, interval type-2 fuzzy numbers, Taxonomy method, group decision-making.

Introduction

In recent decades, with the rapid advancement of technology, maintenance has become one of the key business processes in manufacturing companies, particularly within maintenance systems. The complexity of maintenance systems in terms of organization, technology, and resource diversity creates conditions for unforeseen losses, which are often invisible to management but directly impact the achievement of established goals.

Generally, the overhaul process can be defined as the restoration of resources through the application of approved technological procedures. It consists of complementary preventive and corrective maintenance processes. The overhaul process ensures the required reliability of assets in operation, ultimately achieving their expected operational readiness. At the same time, this process requires specialized verification and maintenance-related technical documentation.

Developing a model for measuring key performance indicators (KPIs) in the execution of core activities within complex maintenance

systems enables management to achieve target values more quickly and efficiently, which are of strategic importance for system functionality.

Research trends indicate that the relative importance of sub-processes and KPI values can be more precisely described using linguistic variables. The development of mathematical theories, such as type-2 fuzzy set theory (Zadeh, 1988), has enabled accurate quantification of these linguistic variables. One advantage of type-2 fuzzy sets over other kinds of fuzzy sets is that they offer more flexibility and freedom, enabling better handling of uncertainty (Mendel & John, 2002). The requirement for intricate and time-consuming computational processes is a major drawback of using type-2 fuzzy sets in real-world decision-making situations. Many authors recommend using interval type-2 fuzzy numbers, which are a special case of type-2 fuzzy sets, to overcome this limitation (Tadić & Komatina, 2025; Komatina et al., 2022; Celik, Yucesan, & Gul, 2021; Đurić et al., 2019; Aleksic et al., 2019; Zhong & Yao, 2017). The domain, granularity, and membership function are the primary features of fuzzy numbers. In the literature, interval type-2 triangular fuzzy numbers (IT2FNs) are most commonly used, as they effectively capture the imprecision inherent in natural language expressions. Granularity can be defined as the number of linguistic expressions used to describe the considered uncertainty. When determining granularity, decision-makers (DMs) must take into account the size of the problem. The domains of the used IT2FNs are defined on the real line, belonging to different intervals. In this research, existing uncertainties are described using seven linguistic expressions. The domains of IT2FNs, which model these uncertainties, are defined on the measurement scale [1-10].

Many authors suggest that the assessment of relative importance should be framed as a fuzzy group decision-making problem, as in (Komatina et al., 2025; Tadić et al., 2025; Tian, Wang, & Zhang, 2018). Additionally, many authors argue that when there are more attributes, as in this study, determining the weight vector is easier and more accurate through the application of subjective methods combined with fuzzy algebra rules (Komatina, 2025; Sudžum et al., 2024; Božanić et al., 2024; Xu et al., 2024; Komatina et al., 2023; Aleksić et al., 2023). In this paper, the fuzzy geometric mean operator is used to determine the unique assessment of the relative importance of sub-processes. The normalized weight vector is determined using a linear normalization procedure (Palczewski & Sałabun, 2019).

Ranking the KPIs, taking into account all sub-processes as well as their weights under a type-2 uncertain environment, can be considered a

MADM problem extended with IT2FNs, as in Aleksić & Tadić (2023), Celik et al. (2015), Mardani, Jusoh, & Zavadskas (2015). In many studies found in the literature (Keshavarz Ghorabae et al., 2017), determining the values of the elements of the fuzzy decision matrix is treated as a fuzzy group decision-making problem (Oğuz, 2024; Toptancı et al., 2023), as in this research. In general, attributes can be either of the benefit or cost type. In such cases, it is necessary to construct a normalized fuzzy decision matrix to determine the values, thereby increasing the computational effort. To reduce the computational load, some authors have introduced the assumption that, when assessing the value of alternatives, decision-makers should consider the attribute type (Li et al., 2022; Đurić et al., 2019). There are also studies in which the authors formulate the problem in such a way that the attributes are of the same type (Petrović et al., 2022; Aleksic et al., 2019). In this research, the sub-processes are of the same type, so there is no need to apply the normalization process. After constructing the weighted fuzzy decision matrix, the developed MADM method can be applied to determine the ranking. It should be noted that the choice of the MADM method can be considered a separate problem.

In the literature, there are several papers where the Taxonomy method has been extended with different types of fuzzy numbers and applied to determine the ranking of alternatives in various research domains. In Centobelli, Cerchione, & Esposito (2018), the Taxonomy method is proposed for evaluating operational tools at the level of each macro area. The paper shows how enterprises can use this tool to understand which category they belong to and support decision-making for introducing changes that lead to improved alignment levels. The weight vector of macro perspectives is determined using the Delphi method. At the level of each macro-perspective, operational tools are defined. The values of the defined operational tools are described by predefined linguistic terms. The modeling of these linguistic expressions is done using trapezoidal fuzzy numbers of the first order. Based on the results obtained, decision-makers can identify measures to support the competitiveness of local systems by improving management processes.

Taxonomy with spherical fuzzy sets is proposed by Diao, Cai, & Wei (2022). The paper discusses the problem of selecting car rental companies according to various criteria. The criteria weights are determined using the entropy method. The values of the criteria are described by spherical fuzzy sets (Kutlu Gündoğdu & Kahraman, 2019). The spherical fuzzy composite distance matrix is calculated according to the procedure proposed by the conventional Taxonomy method and

fuzzy algebra rules with spherical fuzzy sets (Szmidt & Kacprzyk, 2000). The ranking of the considered companies is determined according to the conventional Taxonomy method. The problem of ranking companies according to technological innovations using the Taxonomy method with T-Spherical Fuzzy Numbers is proposed by Liu & Wang (2022). The elements of the fuzzy decision matrix are modeled by T-Spherical Fuzzy Numbers (Mahmood et al., 2019). These fuzzy numbers represent an extension of Pythagorean fuzzy numbers (Peng & Yang, 2016) and intuitionistic fuzzy numbers (Atanassov, 1999), which can depict a neutral attitude in human thinking with a considerably large expression range. In Wei et al. (2020), the authors propose a fuzzy two-stage multi-attribute decision model with Pythagorean fuzzy numbers. The proposed model is tested on an example involving rating and selecting tourist destinations. The values of the elements of the fuzzy decision matrix are determined by the Pythagorean fuzzy weighted average operator (Peng & Yang, 2016). The weight vector is determined using the procedure proposed in the entropy method and the normalized Hamming distance (Zhang & Xu, 2014). The ranking of the considered tourist destinations is obtained by the conventional Taxonomy method combined with fuzzy algebra rules with Pythagorean fuzzy numbers.

The broader objective of this research can be interpreted as the integration of the following approaches: (i) modeling existing uncertainties in the relative importance of sub-processes and KPI values using IT2FNs, (ii) determining the relative importance of sub-processes as a fuzzy group decision-making problem; the fuzzy weights vector is determined using the fuzzy geometric mean combined with a linear normalization procedure, (iii) ranking KPIs using the proposed Taxonomy method with IT2FNs, and (iv) implementing appropriate methods to improve KPI values, thus enhancing the effectiveness and reliability of the maintenance depot.

The rest of the paper is organized as follows: Section 2 describes the proposed two-stage fuzzy model. A case study with real-world data is presented in Section 3, while Section 4 provides the conclusion.

Methodology

This section provides a brief overview of the proposed two-stage fuzzy model. In the first stage, the weights of sub-processes are determined using a subjective approach. In the second stage, the ranking of the considered KPIs is established, enabling decision-makers to select the KPIs that need to be improved in order to increase the operational

efficiency of the maintenance depot. A simple graphical representation of the proposed model is presented in Figure 1.

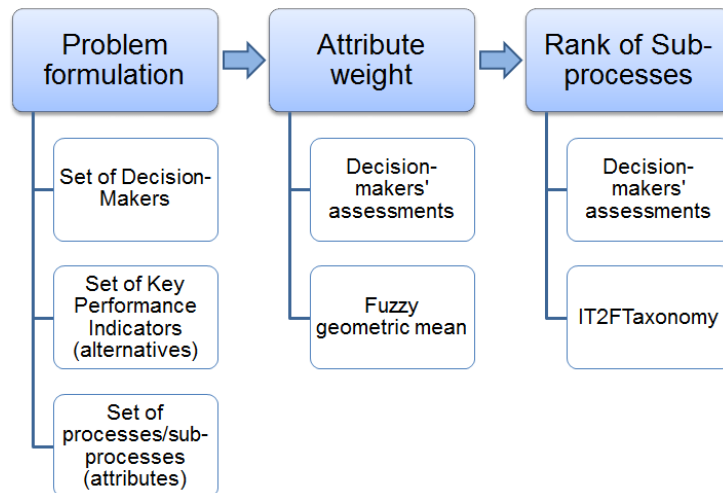


Figure 1 – The proposed model

In the following section, additional explanations of the proposed model are provided, along with detailed guidelines for applying this approach. The interval type-2 fuzzy algebra used is presented in the Appendix.

Defining the set of decision-makers

The assessment of the relative importance of the sub-processes and KPI values, as well as their evaluation, was conducted by ten DMs originating from the maintenance sector of six manufacturing companies, including the considered maintenance depot. In this case, a total of three maintenance managers, four maintenance engineers, and three maintenance operators participated.

The DMs were selected to ensure a balanced representation of all hierarchical levels within the maintenance sector, namely the managerial, engineering, and operational levels. Maintenance managers were selected from three of the six companies. They are responsible for strategic decisions within the maintenance department. Four maintenance engineers were chosen based on their practical experience, as well as three maintenance operators who are directly involved in the execution of daily maintenance activities. One of the key criteria for

selecting the decision-makers was that they possess a minimum of five years of work experience in the maintenance sector.

These DMs are formally represented by the set of indices $\{1, \dots, e, \dots, E\}$. The total number of DMs is denoted as E , and a DM index is represented as e , where $e, e = 1, \dots, E$.

In the case of KPI value assessment, the evaluation was conducted by five employees from the considered maintenance depot. The assessment team comprised the maintenance depot manager, two maintenance engineers, and two maintenance operators. In this case, the DMs are formally presented by the set of indices $\{1, \dots, j, \dots, J\}$. The total number of DMs is denoted as J , and a DM index is represented as j , where $j, j = 1, \dots, J$.

The presented two-step decision-making approach is aligned with the objectives of the study. In the first phase, DMs from six manufacturing companies participated in order to objectively assess the importance of sub-processes and minimize the impact of the specific situation and characteristics of just one organization. In the second phase, the evaluations were carried out exclusively by employees of the considered maintenance depot, as they were directly relevant to the considered case study.

Defining the set of alternatives

In the general case, DMs define a varying number of KPIs that are essential for monitoring the reliability of the maintenance process in the considered maintenance depot. The set of KPIs is represented by the index set $\{1, \dots, i, \dots, I\}$. The total number of KPIs is denoted as I , and the index of a KPI is represented as i , where $i = 1, \dots, I$.

In this study, the following KPIs are considered:

- Availability of protective equipment required in the maintenance process ($i = 1$),
- Customer satisfaction with the maintenance service ($i = 2$),
- Material supply of spare parts and consumables ($i = 3$),
- Implementation level of occupational safety measures in the maintenance process ($i = 4$),
- Availability of general and specialized tools and equipment ($i = 5$),
- Required knowledge and skills level of experts ($i = 6$),
- Coordination level with other units of the maintenance organization ($i = 7$),
- Ability to follow modern maintenance trends ($i = 8$),

- Compliance level of procedures ($i = 9$),
- Availability of experts ($i = 10$),
- Accessibility of technical documentation and literature ($i = 11$),
- Availability of workshop capacities ($i = 12$).

The term KPI is commonly used for quantitative and measurable indicators that directly affect system performance (e.g., downtime, effectiveness, efficiency, number of complaints, etc.). In this paper, it is important to note that the term is used in a broader sense. In this context, KPIs refer to all factors that have a significant impact on the performance of the maintenance sector, even if they are not necessarily part of performance metrics. Essentially, KPIs in this study include various factors and characteristics of the system. As previously mentioned, this interpretation and use of the term have been applied in relevant literature, and that approach was adopted in this research.

Defining the set of attributes

In this research, the considered attributes are sub-processes of the maintenance process in the maintenance depot. Generally, the set of considered sub-processes is represented by the index set $\{1, \dots, p, \dots, P\}$. The total number of sub-processes is denoted as P , and the index of a sub-process is represented as p , where $p = 1, \dots, P$.

The selected sub-processes of the considered maintenance depot are (Milovanović et al., 2024):

- Defect detection in the assembled state ($p = 1$),
- Dismantling of assemblies, subassemblies, and components ($p = 2$),
- Regeneration of spare parts ($p = 3$),
- Assembly, adjustment, and partial testing ($p = 4$),
- Final inspections ($p = 5$),
- Painting of equipment ($p = 6$),
- Testing ($p = 7$),
- Verification of the maintenance process and documentation completion ($p = 8$),
- Identification and risk assessment ($p = 9$).

Defining a set of linguistic variables for describing uncertain data

In this research, uncertain data, such as the relative importance of sub-processes and KPI values, are adequately assessed using a seven-

point scale. Determining the attribute weights is formulated as a fuzzy group decision-making problem. These predefined linguistic expressions are modeled using IT2FNs:

- Very low importance/values (L1): $((1, 1, 2.5; 1), (1, 1, 2; 0.8))$
- Low importance/values (L2): $((1, 2.5, 4; 1), (1.5, 2.5, 3.5; 0.8))$
- Fairly medium-low importance/values (L3): $((2, 3.5, 5; 1), (2.5, 3.5, 4.5; 0.8))$
- Medium importance/values (L4): $((4, 5.5, 7; 1), (4.5, 5.5, 6.5; 0.8))$
- Fairly medium-high importance/values (L5): $((6, 7.5, 9; 1), (6.5, 7.5, 8.5; 0.8))$
- High importance/values (L6): $((7, 8.5, 10; 1), (7.5, 8.5, 9.5; 0.8))$
- Very high importance/values (L7): $((8.5, 10, 10; 1), (9, 10, 10; 0.8))$

The domain values of the IT2FNs are defined within the interval [1-10]. A value of 1 indicates that the relative importance of sub-processes and KPI values have an almost negligible influence, whereas a value of 10 signifies a significantly strong influence in evaluating and selecting KPIs.

Determining the weight vector

In this research, determining the relative importance of the considered sub-processes of the maintenance process is formulated as a fuzzy group decision-making problem. It is assumed that all DMs have equal importance in assessing the relative importance of sub-processes. The procedure for determining the fuzzy weights vector is presented as follows:

Step 1. Each DM evaluates the relative importance of sub-process $p, p = 1, \dots, P$ using one of the predefined linguistic expressions, \tilde{W}_p^e .

Step 2. The aggregation of the DMs' assessments into a unique value, \tilde{W}_p is obtained applying the fuzzy geometric mean operator, so that:

$$\tilde{W}_p = \sqrt[E]{\prod_{e=1}^E \tilde{W}_p^e} \quad (1)$$

Step 3. The normalized fuzzy weights vector is obtained by applying a linear normalization procedure (Palczewski & Sałabun, 2019):

$$[\tilde{\omega}_p]_{1 \times P}$$

Determining the rank of KPIs using Taxonomy method with IT2FNs

The rank of KPIs is determined using the proposed IT2FNs based Taxonomy method.

Step 1. Each DM evaluates the value of KPIs at the level of each sub-process using one of the predefined linguistic expressions, \tilde{x}_{ip}^j .

Step 2. The aggregated assessments of DMs, $\tilde{\tilde{x}}_{ip}$ are obtained by using the fuzzy geometric mean, so that:

$$\tilde{\tilde{x}}_{ip} = \sqrt[J]{\tilde{x}_{ip}^j} \quad (2)$$

The fuzzy decision matrix can be presented as:

$$[\tilde{\tilde{x}}_{ip}]_{I \times P}$$

Step 3. Construct the weighted normalized decision matrix:

$$[\tilde{y}_{ip}]_{I \times P}$$

where:

$$\tilde{y}_{ip} = \tilde{\omega}_p \cdot \tilde{\tilde{x}}_{ip} \quad (3)$$

Step 4. Construct the standardized decision matrix:

$$[z_{ip}]_{I \times P}$$

where:

$$z_{ip} = \frac{d(\tilde{y}_{ip}, \tilde{y}_p)}{\sigma_p} \quad (4)$$

\tilde{y}_p is the average value of sub-process $p, p = 1, \dots, P$ and:

$$\tilde{y}_p = \frac{1}{I} \cdot \sum_{i=1, \dots, I} \tilde{y}_{ip} \quad (5)$$

The standard deviation is calculated as:

$$\sigma_p = \sqrt{\frac{1}{I-1} \cdot \sum_{i=1, \dots, I} (d(\tilde{y}_{ip}, \tilde{y}_p))^2} \quad (6)$$

Step 5. Construct the compromise decision matrix:

$$[c_{it}]_{I \times I}$$

where:

$$c_{ii'} = \sqrt{\sum_{p=1, \dots, P-1} (c_{ii'}^p - c_{ii'}^{p+1})^2} \quad (7)$$

Step 6. Check the homogeneity of alternatives according to the following procedure:

$$O_i = \min_{i'=1, \dots, I} c_{ii'} \quad (8)$$

The homogeneity check for alternatives is computed using the expression:

$$O_i = \bar{O} \pm 2 \cdot \sigma_0 \quad (9)$$

where:

$$\bar{O} = \frac{1}{I} \cdot \sum_{i=1, \dots, I} O_i \quad (10)$$

$$\sigma_0 = \sqrt{\frac{1}{I-1} \cdot (O_i - \bar{O})^2} \quad (11)$$

If all O_i values lie within a specific confidence interval, it can be considered that the alternatives are homogeneous.

Step 7. Development Model:

Determine the values of z_{0i} as:

$$z_{0i} = \max_{p=1, \dots, P} z_{ip} \quad (12)$$

The values of C_{i0} are calculated according to the formula:

$$C_{i0} = \sqrt{\sum_{p=1, \dots, P} (z_{ip} - z_{0i})^2} \quad (13)$$

where:

The average value is denoted as \bar{C} , and the standard deviation is denoted as σ .

Determine:

$$C_0 = \bar{C} + 2 \cdot \sigma \quad (14)$$

Step 8. Determine the development attribute values, F_i :

$$F_i = \frac{C_{i0}}{C_0} \quad (15)$$

Step 9. Rank the values of F_i in descending order. The alternative with the highest F_i value will be ranked first and can be considered the best.

The proposed Taxonomy method with IT2FNs

The estimated values of KPIs are presented in Table 2 (Step 1 of the proposed Algorithm).

Table 2 – The assessed KPI values

		<i>p</i> = 1	<i>p</i> = 2	<i>p</i> = 3	<i>p</i> = 4	<i>p</i> = 5	<i>p</i> = 6	<i>p</i> = 7	<i>p</i> = 8	<i>p</i> = 9
<i>j</i> = 1	<i>i</i> = 1	L6	L7	L6	L7	L6	L7	L7	L6	L4
<i>j</i> = 2		L7	L5	L6	L6	L6	L5	L7	L3	L6
<i>j</i> = 3		L6	L4	L6	L4	L6	L6	L7	L7	L6
<i>j</i> = 4		L7	L3	L5	L3	L7	L5	L7	L7	L2
<i>j</i> = 5		L6	L6	L7	L5	L7	L5	L7	L6	L5
<i>j</i> = 1	<i>i</i> = 2	L6	L6	L7	L6	L6	L6	L7	L6	L6
<i>j</i> = 2		L6	L6	L5	L7	L7	L7	L5	L6	L3
<i>j</i> = 3		L7	L6	L4	L3	L6	L5	L5	L4	L2
<i>j</i> = 4		L6	L4	L6	L4	L7	L7	L5	L4	L1
<i>j</i> = 5		L7	L7	L7	L6	L2	L5	L5	L6	L7
<i>j</i> = 1	<i>i</i> = 3	L7	L7	L7	L4	L5	L5	L6	L7	L6
<i>j</i> = 2		L7	L7	L7	L6	L6	L7	L5	L7	L6
<i>j</i> = 3		L7	L7	L6	L7	L6	L5	L5	L7	L6
<i>j</i> = 4		L7	L7	L6	L7	L6	L7	L5	L7	L6
<i>j</i> = 5		L7	L7	L2	L5	L6	L6	L6	L6	L6
<i>j</i> = 1	<i>i</i> = 4	L7	L7	L5	L7	L6	L4	L6	L5	L6
<i>j</i> = 2		L6	L4	L7	L7	L6	L7	L6	L6	L7
<i>j</i> = 3		L7	L1	L5	L6	L6	L5	L6	L5	L5
<i>j</i> = 4		L6	L7	L7	L5	L6	L6	L6	L6	L6
<i>j</i> = 5		L3	L6	L6	L5	L6	L7	L6	L4	L5
<i>j</i> = 1	<i>i</i> = 5	L7	L7	L7	L7	L7	L6	L7	L7	L7
<i>j</i> = 2		L5	L4	L5	L6	L7	L5	L7	L5	L4
<i>j</i> = 3		L7	L6	L7	L7	L6	L5	L7	L7	L7
<i>j</i> = 4		L5	L5	L6	L6	L7	L7	L7	L7	L6
<i>j</i> = 5		L6	L4	L7	L5	L7	L4	L7	L5	L7
<i>j</i> = 1	<i>i</i> = 6	L6	L6	L5	L7	L6	L5	L7	L5	L6
<i>j</i> = 2		L5	L7	L4	L6	L6	L7	L7	L5	L5
<i>j</i> = 3		L6	L5	L7	L5	L6	L4	L7	L7	L7
<i>j</i> = 4		L6	L7	L4	L7	L6	L5	L7	L5	L7
<i>j</i> = 5		L7	L7	L7	L6	L4	L4	L7	L7	L5
<i>j</i> = 1	<i>i</i> = 7	L5	L6	L4	L7	L6	L6	L6	L6	L3
<i>j</i> = 2		L6	L6	L4	L6	L6	L5	L6	L6	L4
<i>j</i> = 3		L6	L4	L4	L6	L6	L7	L6	L6	L6
<i>j</i> = 4		L6	L6	L4	L7	L6	L6	L6	L6	L6
<i>j</i> = 5		L5	L4	L4	L7	L5	L5	L6	L6	L6
<i>j</i> = 1	<i>i</i> = 8	L6	L5	L7	L6	L4	L5	L6	L2	L5
<i>j</i> = 2		L6	L7	L6	L6	L6	L6	L6	L6	L6
<i>j</i> = 3		L5	L5	L6	L4	L5	L7	L6	L6	L4

		p = 1	p = 2	p = 3	p = 4	p = 5	p = 6	p = 7	p = 8	p = 9
<i>j</i> = 4		L6	L7	L7	L6	L6	L6	L6	L2	L5
<i>j</i> = 5		L6	L5	L7	L5	L6	L4	L6	L6	L4
<i>j</i> = 1	<i>i</i> = 9	L7	L7	L6	L7	L7	L2	L5	L7	L6
<i>j</i> = 2		L7	L6	L6	L6	L7	L7	L7	L6	L6
<i>j</i> = 3		L7	L6	L6	L6	L7	L6	L7	L5	L6
<i>j</i> = 4		L6	L6	L4	L6	L4	L7	L7	L4	L6
<i>j</i> = 5		L6	L5	L6	L7	L5	L2	L7	L7	L6
<i>j</i> = 1		<i>i</i> = 10	L7	L7	L7	L5	L7	L2	L7	L7
<i>j</i> = 2	L7		L4	L7	L6	L6	L5	L5	L6	L6
<i>j</i> = 3	L7		L6	L7	L6	L7	L7	L4	L5	L5
<i>j</i> = 4	L7		L6	L7	L5	L4	L7	L7	L5	L4
<i>j</i> = 5	L7		L7	L7	L6	L6	L6	L6	L5	L6
<i>j</i> = 1	<i>i</i> = 11	L6	L7	L7	L7	L6	L5	L5	L3	L7
<i>j</i> = 2		L6	L5	L7	L5	L7	L7	L7	L3	L3
<i>j</i> = 3		L6	L5	L7	L5	L5	L6	L7	L5	L2
<i>j</i> = 4		L5	L7	L6	L7	L6	L7	L6	L3	L6
<i>j</i> = 5		L6	L7	L7	L7	L7	L3	L7	L7	L7
<i>j</i> = 1	<i>i</i> = 12	L6	L7	L6	L7	L6	L6	L7	L5	L4
<i>j</i> = 2		L7	L5	L7	L6	L5	L7	L5	L7	L5
<i>j</i> = 3		L7	L4	L6	L4	L6	L6	L7	L7	L6
<i>j</i> = 4		L7	L7	L5	L7	L7	L7	L7	L6	L7
<i>j</i> = 5		L6	L6	L7	L5	L7	L7	L7	L7	L5

The weighted aggregated KPI values are obtained using the fuzzy geometric mean and are presented in Tables 3 to 5 (Step 2 to Step 3 of the proposed Algorithm).

Table 3 – The weighted aggregated KPI values and fuzzy arithmetic mean

	p = 1	p = 2	p = 3
<i>i</i> = 1	$\left((5.07, 7.89, 10; 1), (5.97, 7.89, 9.31; 0.8) \right)$	$\left((2.11, 3.81, 5.64; 1), (2.68, 3.81, 4.95; 0.8) \right)$	$\left((4.23, 6.59, 9.20; 1), (5.13, 6.59, 8.35; 0.8) \right)$
<i>i</i> = 2	$\left((5.07, 7.89, 10; 1), (5.97, 7.89, 9.31; 0.8) \right)$	$\left((2.80, 4.67, 6.61; 1), (3.44, 4.67, 5.87; 0.8) \right)$	$\left((3.94, 6.24, 8.57; 1), (4.80, 6.24, 7.82; 0.8) \right)$
<i>i</i> = 3	$\left((5.69, 8.70, 10; 1), (6.66, 8.70, 9.60; 0.8) \right)$	$\left((3.65, 5.80, 7.10; 1), (4.41, 5.80, 6.60; 0.8) \right)$	$\left((3.08, 5.47, 7.83; 1), (3.97, 5.47, 7.07; 0.8) \right)$
<i>i</i> = 4	$\left((3.94, 6.61, 8.71; 1), (4.79, 6.61, 8.02; 0.8) \right)$	$\left((1.97, 4.98, 7.94; 1), (2.39, 4.98, 6.88; 0.8) \right)$	$\left((4.27, 6.64, 9.01; 1), (5.18, 6.64, 8.25; 0.8) \right)$
<i>i</i> = 5	$\left((4.77, 7.51, 9.59; 1), (5.64, 7.51, 8.90; 0.8) \right)$	$\left((2.43, 4.17, 6.03; 1), (3.02, 4.17, 5.32; 0.8) \right)$	$\left((4.58, 7.04, 9.20; 1), (5.53, 7.04, 8.53; 0.8) \right)$
<i>i</i> = 6	$\left((3.94, 6.61, 8.71; 1), (4.79, 6.61, 8.02; 0.8) \right)$	$\left((3.28, 5.30, 6.95; 1), (3.98, 5.30, 6.32; 0.8) \right)$	$\left((4.58, 7.04, 9.20; 1), (5.53, 7.04, 8.53; 0.8) \right)$
<i>i</i> = 7	$\left((4.41, 7.03, 9.59; 1), (5.24, 7.03, 8.72; 0.8) \right)$	$\left((2.69, 4.52, 6.61; 1), (3.32, 4.52, 5.81; 0.8) \right)$	$\left((2.40, 4.23, 6.58; 1), (5.53, 4.23, 5.78; 0.8) \right)$

	$p = 1$	$p = 2$	$p = 3$
$i = 8$	$((4.55, 7.21, 9.79; 1), (5.39, 7.21, 8.92; 0.8))$	$((2.97, 4.88, 6.67; 1), (4.01, 4.88, 5.99; 0.8))$	$((4.72, 7.22, 9.40; 1), (5.69, 7.22, 8.76; 0.8))$
$i = 9$	$((5.27, 8.15, 9.59; 1), (6.19, 8.15, 9.41; 0.8))$	$((3.03, 4.97, 6.95; 1), (3.70, 4.97, 6.19; 0.8))$	$((3.76, 5.76, 8.75; 1), (4.60, 5.76, 7.84; 0.8))$
$i = 10$	$((5.69, 8.70, 10; 1), (6.66, 8.70, 9.60; 0.8))$	$((2.91, 4.82, 6.61; 1), (3.57, 4.82, 5.93; 0.8))$	$((5.10, 7.70, 9.40; 1), (6.12, 7.70, 8.90; 0.8))$
$i = 11$	$((4.55, 7.21, 9.79; 1), (5.39, 7.21, 8.92; 0.8))$	$((3.18, 5.17, 6.81; 1), (3.87, 5.17, 6.18; 0.8))$	$((4.91, 7.45, 9.40; 1), (5.90, 7.45, 8.81; 0.8))$
$i = 12$	$((5.27, 8.15, 9.59; 1), (6.19, 8.15, 9.41; 0.8))$	$((2.82, 4.70, 6.47; 1), (3.47, 4.70, 5.80; 0.8))$	$((4.40, 6.81, 9.20; 1), (5.33, 6.81, 8.44; 0.8))$
\tilde{y}_p	$((4.85, 7.64, 9.61; 1), (5.73, 7.64, 9.01; 0.8))$	$((2.82, 4.82, 6.70; 1), (3.49, 4.82, 5.99; 0.8))$	$((4.16, 6.52, 8.81; 1), (5.78, 6.52, 8.09; 0.8))$

Table 4 – The weighted aggregated KPI values and fuzzy arithmetic mean (continuations)

	$p = 4$	$p = 5$	$p = 6$
$i = 1$	$((2.36, 4.40, 6.75; 1), (3.01, 4.40, 5.92; 0.8))$	$((3.25, 5.71, 7.90; 1), (4.03, 5.71, 7.18; 0.8))$	$((1.72, 3.99, 6.85; 1), (2.30, 3.99, 5.93; 0.8))$
$i = 2$	$((2.43, 4.52, 6.89; 1), (3.10, 4.52, 6.05; 0.8))$	$((2.20, 4.47, 6.58; 1), (2.92, 4.47, 5.88; 0.8))$	$((1.85, 4.23, 6.70; 1), (3.20, 4.23, 6.12; 0.8))$
$i = 3$	$((3.14, 5.43, 7.75; 1), (3.34, 5.43, 6.94; 0.8))$	$((2.92, 5.22, 7.74; 1), (3.64, 5.22, 6.88; 0.8))$	$((1.85, 4.23, 6.70; 1), (3.20, 4.23, 6.12; 0.8))$
$i = 4$	$((3.41, 5.78, 8.15; 1), (4.19, 5.78, 7.33; 0.8))$	$((3.01, 5.35, 7.90; 1), (3.75, 5.35, 7.03; 0.8))$	$((1.71, 3.97, 6.66; 1), (2.97, 3.97, 5.80; 0.8))$
$i = 5$	$((3.52, 5.93, 8.32; 1), (4.31, 5.93, 7.49; 0.8))$	$((3.52, 6.10, 7.90; 1), (4.34, 6.10, 7.32; 0.8))$	$((1.59, 3.75, 6.52; 1), (2.79, 3.75, 5.62; 0.8))$
$i = 6$	$((3.52, 5.93, 8.32; 1), (4.31, 5.93, 7.49; 0.8))$	$((2.69, 4.91, 7.36; 1), (3.39, 4.91, 6.52; 0.8))$	$((1.42, 3.44, 6.07; 1), (2.52, 3.44, 5.21; 0.8))$
$i = 7$	$((3.78, 6.28, 8.50; 1), (4.33, 6.28, 7.74; 0.8))$	$((2.92, 5.22, 7.74; 1), (3.64, 5.22, 6.88; 0.8))$	$((1.78, 4.09, 6.99; 1), (3.09, 4.09, 6.06; 0.8))$
$i = 8$	$((3.36, 5.69, 8.50; 1), (4.12, 5.69, 7.50; 0.8))$	$((2.61, 4.79, 7.20; 1), (3.29, 4.79, 6.37; 0.8))$	$((1.54, 3.66, 6.38; 1), (2.71, 3.66, 5.49; 0.8))$
$i = 9$	$((4.24, 6.08, 8.50; 1), (4.44, 6.08, 7.66; 0.8))$	$((2.93, 5.28, 7.20; 1), (3.67, 5.28, 6.57; 0.8))$	$((0.90, 2.72, 5.06; 1), (1.78, 2.72, 4.29; 0.8))$
$i = 10$	$((3.16, 5.42, 8.15; 1), (3.90, 5.42, 7.18; 0.8))$	$((2.91, 5.24, 7.36; 1), (3.64, 5.24, 6.65; 0.8))$	$((1.29, 3.39, 5.95; 1), (2.39, 3.39, 5.13; 0.8))$
$i = 11$	$((3.55, 5.97, 8.15; 1), (2.80, 5.97, 7.40; 0.8))$	$((3.15, 5.57, 7.74; 1), (3.92, 5.57, 7.02; 0.8))$	$((1.48, 3.63, 6.22; 1), (2.64, 3.63, 5.39; 0.8))$
$i = 12$	$((3.15, 5.43, 7.75; 1), (3.89, 5.43, 6.94; 0.8))$	$((3.15, 5.57, 7.74; 1), (3.92, 5.57, 7.02; 0.8))$	$((2.04, 4.59, 6.60; 1), (3.51, 4.59, 6.47; 0.8))$
\tilde{y}_p	$((3.30, 5.61, 7.98; 1), (4.04, 5.61, 7.14; 0.8))$	$((2.94, 5.29, 7.53; 1), (3.68, 5.29, 6.78; 0.8))$	$((1.60, 3.81, 6.39; 1), (2.76, 3.81, 5.64; 0.8))$

Table 5 – The weighted aggregated KPI values and fuzzy arithmetic mean (continuations)

	$p = 7$	$p = 8$	$p = 9$
$i = 1$	$((4.25, 6.50, 7.90; 1), (4.95, 6.50, 7.90; 0.8))$	$((2.77, 3.63, 6.30; 1), (2.48, 3.63, 6.43; 0.8))$	$((1.93, 4.94, 7.23; 1), (3.04, 4.94, 5.64; 0.8))$
$i = 2$	$((3.22, 5.16, 7.90; 1), (3.82, 5.16, 6.94; 0.8))$	$((2.63, 4.64, 7.19; 1), (3.24, 4.64, 6.28; 0.8))$	$((1.22, 2.29, 4.56; 1), (1.57, 2.29, 3.97; 0.8))$
$i = 3$	$((3.19, 5.13, 8.07; 1), (3.79, 5.13, 7.02; 0.8))$	$((3.84, 6.29, 8.30; 1), (4.60, 6.29, 7.62; 0.8))$	$((3.29, 5.18, 8.30; 1), (3.90, 5.18, 7.60; 0.8))$
$i = 4$	$((3.50, 5.52, 8.60; 1), (4.12, 5.52, 7.51; 0.8))$	$((2.77, 4.82, 7.41; 1), (3.39, 4.82, 6.49; 0.8))$	$((3.22, 5.09, 7.96; 1), (3.82, 5.09, 7.34; 0.8))$
$i = 5$	$((4.25, 6.50, 7.90; 1), (4.95, 6.50, 7.90; 0.8))$	$((3.73, 6.14, 8.13; 1), (4.47, 6.14, 7.45; 0.8))$	$((3.31, 5.24, 7.73; 1), (3.15, 5.24, 7.19; 0.8))$
$i = 6$	$((3.50, 5.52, 8.60; 1), (4.12, 5.52, 7.51; 0.8))$	$((3.24, 5.47, 7.79; 1), (3.92, 5.47, 6.98; 0.8))$	$((3.34, 5.26, 7.96; 1), (3.96, 5.26, 7.42; 0.8))$
$i = 7$	$((3.50, 5.52, 8.60; 1), (4.12, 5.52, 7.51; 0.8))$	$((3.29, 5.52, 8.30; 1), (3.97, 5.52, 7.31; 0.8))$	$((2.29, 3.98, 6.73; 1), (2.83, 3.98, 6.07; 0.8))$
$i = 8$	$((3.50, 5.52, 8.60; 1), (4.12, 5.52, 7.51; 0.8))$	$((1.51, 3.39, 5.75; 1), (2.09, 3.39, 4.91; 0.8))$	$((2.47, 4.14, 6.90; 1), (3.00, 4.14, 6.25; 0.8))$
$i = 9$	$((3.96, 6.14, 8.42; 1), (4.64, 6.14, 7.65; 0.8))$	$((3.08, 5.27, 7.57; 1), (3.75, 5.27, 6.77; 0.8))$	$((3.29, 5.18, 8.30; 1), (3.90, 5.18, 7.60; 0.8))$
$i = 10$	$((3.28, 5.27, 7.95; 1), (3.89, 5.27, 6.94; 0.8))$	$((3.12, 5.29, 7.79; 1), (3.78, 5.29, 6.91; 0.8))$	$((2.97, 4.79, 7.57; 1), (3.55, 4.79, 6.96; 0.8))$
$i = 11$	$((3.81, 5.94, 8.42; 1), (4.47, 5.94, 7.57; 0.8))$	$((1.56, 3.27, 5.36; 1), (2.07, 3.27, 4.62; 0.8))$	$((1.88, 3.63, 6.02; 1), (2.44, 3.63, 5.47; 0.8))$
$i = 12$	$((3.96, 6.14, 8.42; 1), (4.64, 6.14, 7.64; 0.8))$	$((3.58, 5.94, 8.13; 1), (4.31, 5.94, 7.38; 0.8))$	$((2.88, 4.67, 7.41; 1), (3.45, 4.67, 6.81; 0.8))$
\tilde{y}_p	$((3.66, 5.74, 8.03; 1), (4.29, 5.74, 7.47; 0.8))$	$((2.93, 4.98, 7.33; 1), (3.51, 4.98, 6.59; 0.8))$	$((2.67, 4.53, 7.22; 1), (3.22, 4.53, 6.53; 0.8))$

According to the proposed procedure (Step 4 of the proposed Algorithm), the obtained results are presented in Table 6.

Table 6 – The standard deviation

	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$	$p = 7$	$p = 8$	$p = 9$
σ_p	0.801	0.498	0.852	0.577	0.391	0.475	0.420	0.953	0.885

The standardized decision matrix (Step 5 of the proposed Algorithm) is presented in Table 7.

Table 7 – The standardized decision matrix

	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$	$p = 7$	$p = 8$	$p = 9$
z_{1p}	0.336	1.922	0.242	2.007	0.387	0.539	1.442	1.020	0.488
z_{2p}	0.336	0.291	0.415	1.809	2.120	0.832	1.157	0.312	2.441
z_{3p}	1.091	1.675	1.333	0.496	0.207	0.832	1.160	1.212	0.854
z_{4p}	2.116	1.185	0.228	0.463	0.320	0.366	0.540	0.144	0.697
z_{5p}	0.127	1.205	0.525	0.532	1.724	0.114	1.442	1.056	0.667
z_{6p}	1.226	0.865	0.525	0.532	0.795	0.697	0.540	0.463	0.842
z_{7p}	0.574	0.444	2.305	0.912	0.207	0.697	0.540	0.613	0.554
z_{8p}	0.411	0.235	0.691	0.315	1.105	0.229	0.540	1.634	0.364
z_{9p}	0.521	0.325	0.723	0.719	0.192	2.295	0.838	0.258	0.854
z_{10p}	1.091	0.080	1.086	0.674	0.182	0.872	1.164	0.325	0.346
z_{11p}	0.411	0.612	0.865	0.636	0.645	0.362	0.481	1.783	1.050
z_{12p}	0.521	0.231	0.380	0.591	0.645	1.408	0.836	0.805	0.208

The remaining elements of the compromise decision matrix are calculated in the same way (Step 6 of the proposed Algorithm) so that:

–	3.196	2.115	2.785	2.194	2.368	2.956	2.781	2.945	2.681	2.431	2.496
3.196	–	3.145	3.567	2.641	2.738	3.489	3.186	3.130	3.240	2.939	3.069
2.115	3.145	–	2.072	2.182	1.624	1.937	2.226	2.392	1.919	1.752	2.130
2.785	3.567	2.072	–	2.780	1.202	2.794	2.648	2.731	1.968	2.567	2.334
2.194	2.641	2.182	2.780	–	1.929	2.785	1.653	3.023	2.489	1.839	2.147
2.368	2.738	1.624	1.202	1.929	–	2.089	1.747	1.978	1.350	1.667	1.435
2.956	3.489	1.937	2.794	2.785	2.089	–	2.266	2.327	1.578	2.032	2.191
2.781	3.186	2.226	2.648	1.653	1.747	2.266	–	2.740	2.035	1.001	1.608
2.945	3.130	2.392	2.731	3.023	1.978	2.327	2.740	–	1.707	2.560	1.361
2.681	3.240	1.919	1.968	2.489	1.350	1.578	2.035	1.707	–	2.088	1.308
2.431	2.939	1.752	2.567	1.839	1.667	2.032	1.001	2.560	2.088	–	1.811
2.496	3.069	2.130	2.334	2.147	1.435	2.191	1.608	1.361	1.308	1.811	–

$$\begin{aligned}
 O_1 &= 2.115 & O_2 &= 2.641 & O_3 &= 1.624 & O_4 &= 1.968 & O_5 &= 1.653 & O_6 &= 1.435 \\
 O_7 &= 1.578 & O_8 &= 1.001 & O_9 &= 1.361 & O_{10} &= 1.308 & O_{11} &= 1.001 & O_{12} &= 1.308
 \end{aligned}$$

Let us check the homogeneity of the alternatives:

$$O_i = 1.583 \pm 2 \cdot 0.473 \in [0.637 - 2.529]$$

It can be concluded that all KPIs, except for ($i = 2$), are homogeneous, meaning they are mutually independent. However, the value of the second KPI does not deviate significantly from the prescribed homogeneity boundaries, so we can further accept that all KPIs are considered mutually independent.

According to the proposed Algorithm (Step 7), the development model is constructed:

$$\begin{array}{llllll} z_{01} = 2.007 & z_{02} = 2.441 & z_{03} = 1.675 & z_{04} = 2.116 & z_{05} = 1.724 & z_{06} = 1.226 \\ z_{07} = 2.305 & z_{08} = 1.634 & z_{09} = 2.295 & z_{010} = 1.164 & z_{011} = 1.783 & z_{012} = 1.408 \end{array}$$

The values C_{i0} are:

$$\begin{array}{llllll} C_{01} = 3.871 & C_{02} = 4.736 & C_{03} = 2.422 & C_{04} = 4.673 & C_{05} = 3.148 & C_{06} = 1.664 \\ C_{07} = 4.943 & C_{08} = 3.338 & C_{09} = 4.975 & C_{010} = 1.962 & C_{011} = 3.310 & C_{012} = 2.571 \end{array}$$

$$C_0 = 3.711 + 2 \cdot 1.282 = 6.275$$

The values of the development attribute, F_i and the ranked KPIs are presented in Table 8.

Table 8 – Rank of KPIs

	F_i	Rank		F_i	Rank
$i = 1$	0.603	5	$i = 7$	0.788	2
$i = 2$	0.755	3	$i = 8$	0.532	6
$i = 3$	0.386	10	$i = 9$	0.793	1
$i = 4$	0.745	4	$i = 10$	0.313	11
$i = 5$	0.502	8	$i = 11$	0.528	7
$i = 6$	0.265	12	$i = 12$	0.410	9

In the general case, maintenance process improvement can be achieved by taking management actions aimed at improving all KPIs. However, in practice, this process cannot be realized due to limited time and financial resources, as well as the availability of workers. Based on the obtained results, DMs determine the set of KPIs that need to be improved. In this way, costs and downtime are significantly reduced, which leads to an increase in the effectiveness of the maintenance process. It can be concluded that it is necessary to improve the KPI in the last position. In this case, it is KPI ($i = 6$) – Required knowledge and skills level of experts. To improve the value of this KPI, it is necessary to have knowledge of equipment, fault detection, component disassembly and assembly. Furthermore, improving this KPI requires increasing worker training for the maintenance of newer equipment. One of the recommended management actions is to organize continuous worker training over time.

Many researchers suggest that management actions should target two or three KPIs simultaneously. In this case, based on good practice experience, it is assumed that three KPIs in the last three positions in the ranking should be improved simultaneously. Therefore, in addition to the aforementioned KPI, the following KPIs need to be improved: (i) KPI ($i =$

10) – Availability of experts. The shortage of experts is present in most companies due to various factors (planned and natural departures, migrations, moves to better positions, retraining for other positions, etc.), inability to replace personnel in case of absence, and the entry of younger and less experienced individuals into the technological process. To improve this KPI, it is necessary to follow one of the basic principles of business known as employee stability; (ii) KPI ($i = 3$) – Material supply of spare parts and consumables. Global issues in the world market can affect the production of spare parts, resulting in delivery delays, supply of smaller quantities than required, and reduced reliability of delivered parts due to frequent failures after repairs. Improving the procurement of reliable spare parts and consumables leads to an improvement in this KPI.

It should be noted that the results obtained in this study closely align with those reported in (Milovanović et al., 2024).

Conclusion

This research proposes a fuzzy MADM model that provides a framework for ranking KPIs under an interval type-2 fuzzy environment and helps define management actions aimed at improving the reliability of the maintenance process. The proposed model has been tested and validated using real-world data from a maintenance depot. The DMs base their assessments on both their experience and the available evidence.

The main contributions of the presented research are: (1) Modeling of existing uncertainties in the relative importance of sub-processes and KPI values, based on IT2FNs; (2) Formulating the assessment of existing uncertainties as a fuzzy group decision-making problem; (3) Determining the weights vector of sub-processes using the fuzzy geometric mean, which offers practical advantages for industrial applications; (4) Ranking the KPIs using the proposed IT2FNs based Taxonomy.

The practical implications of the proposed methodology are directed towards operations managers, who need to prioritize actions to address identified failures. The methodology identifies failures requiring immediate attention, considering the potential costs that may arise if the manufacturing process is interrupted.

The key advantage of the proposed fuzzy MADM model is its ability to precisely identify the set of KPIs that need improvement through targeted management actions. This model can be extended to analyze other production enterprises.

The main limitations of the proposed fuzzy MADM model are: (1) the subjectivity in obtaining input data for the model, and (2) the increased

computational complexity compared to traditional decision-making methods. Future research should focus on developing user-friendly software tools to facilitate the practical application of the proposed fuzzy MADM model in real-world scenarios.

Appendix

In this section, fundamental definitions concerning the fuzzy algebra rules of IT2FNs are introduced (Mendel, 2017).

Definition 1. A type-2 fuzzy set, \tilde{A} in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}$ shown as follows:

$$\tilde{A} = \{(x, u), \mu_{\tilde{A}}(x, u) | \forall x \in X, \forall u \in J_x \subseteq (0, 1), 0 \leq \mu_{\tilde{A}}(x, u) \leq 1\}$$

Definition 2. If X is a set of real numbers, then a type-2 fuzzy set and an interval type-2 fuzzy set in X are referred to as a type-2 fuzzy number and an interval type-2 fuzzy number, respectively.

Definition 3. If the upper and lower membership functions of \tilde{A} are two triangular type-1 fuzzy numbers, then \tilde{A} is called a triangular interval type-2 fuzzy number and is denoted as: $\tilde{A} = (\tilde{A}^U, \tilde{A}^L)$ so that:

$$\tilde{A} = (\tilde{A}^U, \tilde{A}^L) = ((a_1^U, a_2^U, a_3^U; \alpha), (a_1^L, a_2^L, a_3^L; \beta))$$

where: 1) a_1^U and a_3^U represent the lower and upper bounds in the domain, respectively; 2) a_1^L and a_3^L represent the lower and upper bounds of the lower membership function, respectively; 3) a_2^U and a_2^L are the modal values of the upper and lower membership functions, respectively; 4) The values of the membership function are defined as $(\alpha, \beta) \in [0, 1]$.

Definition 4. Let \tilde{A} , and \tilde{B} be two IT2FNs:

$$\tilde{A} = ((a_1^U, a_2^U, a_3^U; \alpha_1), (a_1^L, a_2^L, a_3^L; \beta_1))$$

$$\tilde{B} = ((b_1^U, b_2^U, b_3^U; \alpha_2), (b_1^L, b_2^L, b_3^L; \beta_2))$$

The arithmetic operations are defined as follows:

- Addition:

$$\tilde{A} + \tilde{B} = \left((a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U; \min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2)) \right) \quad (A1)$$

- Subtraction:

$$\tilde{A} - \tilde{B} = \left((a_1^U - b_3^U, a_2^U - b_2^U, a_3^U - b_1^U; \min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2)) \right) \quad (A2)$$

- Multiplication:

$$\tilde{A} \cdot \tilde{B} = \left((a_1^U \cdot b_1^U, a_2^U \cdot b_2^U, a_3^U \cdot b_3^U; \min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2)), (a_1^L \cdot b_1^L, a_2^L \cdot b_2^L, a_3^L \cdot b_3^L; \min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2)) \right) \quad (A3)$$

- Division:

$$\tilde{A} : \tilde{B} = \left((a_1^U : b_3^U, a_2^U : b_2^U, a_3^U : b_1^U; \min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2)), (a_1^L : b_3^L, a_2^L : b_2^L, a_3^L : b_1^L; \min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2)) \right) \quad (A4)$$

Definition 5. Let \tilde{A} be a triangular interval type-2 fuzzy number and let k be a crisp value:

$$k \cdot \tilde{A} = \tilde{A} = \left((k \cdot a_1^U, k \cdot a_2^U, k \cdot a_3^U; \alpha), (k \cdot a_1^L, k \cdot a_2^L, k \cdot a_3^L; \beta) \right) \quad (A5)$$

$$(\tilde{A})^{-1} = \left(\left(\frac{1}{a_3^U}, \frac{1}{a_2^U}, \frac{1}{a_1^U}; \alpha \right), \left(\frac{1}{a_3^L}, \frac{1}{a_2^L}, \frac{1}{a_1^L}; \beta \right) \right) \quad (A6)$$

Definition 6. The Hamming distance between \tilde{A} and \tilde{B} is determined according to the following expression (Nehi & Keikha, 2016; Heidarzade, Mahdavi, & Mahdavi-Amiri, 2016):

$$d_H(\tilde{A}, \tilde{B}) = \frac{d_H^U + d_H^L}{2} \quad (A7)$$

$$d_H^U = \left\{ \frac{1}{4} [|a_1^U - b_1^U| + 2 \cdot |a_2^U - b_2^U| + |a_3^U - b_3^U| + |\alpha_1 - \alpha_2|] \right\} \quad (A8)$$

$$d_H^L = \left\{ \frac{1}{4} [|a_1^L - b_1^L| + 2 \cdot |a_2^L - b_2^L| + |a_3^L - b_3^L| + |\beta_1 - \beta_2|] \right\} \quad (A9)$$

Definition 7. Let there be n interval type-2 triangular fuzzy numbers:

$$\begin{aligned} \tilde{A}_1 &= \left((a_{11}^U, a_{21}^U, a_{31}^U; \alpha_1), (a_{11}^L, a_{21}^L, a_{31}^L; \beta_1) \right), \dots, \tilde{A}_i \\ &= \left((a_{1i}^U, a_{2i}^U, a_{3i}^U; \alpha_i), (a_{1i}^L, a_{2i}^L, a_{3i}^L; \beta_i) \right), \dots \\ \tilde{A}_n &= \left((a_{1n}^U, a_{2n}^U, a_{3n}^U; \alpha_n), (a_{1n}^L, a_{2n}^L, a_{3n}^L; \beta_n) \right) \end{aligned}$$

The normalized values of these IT2FNs are calculated as follows, \tilde{r}_i (Palczewski & Sałabun, 2019):

- a) benefit-type:

$$\tilde{r}_i = \left(\left(\frac{a_{1i}^U}{u^*}, \frac{a_{2i}^U}{u^*}, \frac{a_{3i}^U}{u^*}; \alpha_i \right), \left(\frac{a_{1i}^L}{u^*}, \frac{a_{2i}^L}{u^*}, \frac{a_{3i}^L}{u^*}; \beta_i \right) \right) \quad (A10)$$

- b) cost-type:

$$\tilde{r}_i = \left(\left(\frac{l^*}{a_{3i}^U}, \frac{l^*}{a_{2i}^U}, \frac{l^*}{a_{1i}^U}; \alpha_i \right), \left(\frac{l^*}{a_{3i}^L}, \frac{l^*}{a_{2i}^L}, \frac{l^*}{a_{1i}^L}; \beta_i \right) \right) \quad (\text{A11})$$

where:

$$u^* = \max_{i=1,\dots,n} a_{3i}^U, l^* = \min_{i=1,\dots,n} a_{1i}^U$$

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Višeatributivni pristup za unapređenje procesa održavanja u remontnim zavodima u fazi okruženju tip-2

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OBLAST: matematika, nauke o odlučivanju, industrijsko inženjerstvo
KATEGORIJA (TIP) ČLANKA: originalni naučni rad

Sažetak:

Uvod/cilj: Cilj ovog istraživanja je određivanje prioriteta ključnih indikatora performansi (eng. KPI) na precizan i strukturiran način. Primenom modela odlučivanja koji se zasniva na višeatributivnoj fazi logici, operativni menadžment može identifikovati i prioritarno tretirati aktivnosti koje će u najkraćem mogućem roku poboljšati pouzdanost procesa održavanja, uz istovremeno smanjenje troškova.

Metode: Relativna važnost potprocesa i vrednosti KPI procenjena je korišćenjem unapred definisanih lingvističkih iskaza modelovanih pomoću intervalnih fazi brojeva tipa-2 (eng. IT2FNs). Ove procene su formulisane kroz okvir grupnog odlučivanja u fazi okruženju. Vektori težina određeni su pomoću fazi geometrijske sredine, dok je rangiranje KPI izvršeno primenom metode Taksonomije u kombinaciji sa IT2FNs, što predstavlja glavni naučni doprinos ovog istraživanja.

Rezultati: Realni podaci prikupljeni iz jednog remontnog zavoda korišćeni su za testiranje predloženog modela. U istraživanju je uspešno modelovana neizvesnost u oceni KPI korišćenjem sedam unapred definisanih lingvističkih izraza mapiranih pomoću IT2FNs. Konzistentan vektor težina dobijen je kroz pristup fazi grupnog odlučivanja. Efikasno rangiranje KPI postignuto je kombinovanjem metode Taksonomije i IT2FNs, što je omogućilo identifikaciju najvažnijih oblasti za operativno unapređenje. Validacijom primene potvrđena je sposobnost metode da pruži jasne prioritete za unapređenje pouzdanosti uz smanjenje troškova.

Zaključak: Ključni doprinosi ove studije su: (i) upotreba pravila fazi algebre sa IT2FNs za određivanje grupne korisnosti, i (ii) integracija metode Taksonomije sa IT2FNs u cilju unapređenja procesa odlučivanja.

Ključne reči: ključni indikatori performansi, operativni menadžment, pouzdanost procesa održavanja, intervalni fazi brojevi tipa-2, metoda Taksonomije, grupno odlučivanje.

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