


Estimation of the Moment of Inertia of a Unit in a Thermal Power Plant

Jasna Dragosavac¹, Anita Mijajlović¹, Ilija Klasnić¹, Milan Đorđević²,
Igor Damjanac², Miloš Brdarević²

¹Nikola Tesla Institute of Electrical Engineering, Koste Glavinića 8a, 11000
Belgrade, Serbia

²Electric Power Industry of Serbia, Joint Stock Company, Belgrade, Serbia

jasna.dragosavac@ieent.org

Abstract: Increasing integration of renewable energy sources and the withdrawal of fossil-fuel generators reduce overall power-system inertia, making the grid more vulnerable to disturbances and threatening transient and frequency stability. Low inertia can lead to large frequency deviations, equipment damage, and financial losses, which highlights the need for accurate assessment of inertia contributions from remaining generation units and determination of the minimum inertia required for stable operation. While synchronous generator inertia constants are usually provided by manufacturers, corresponding data for steam turbines are often unavailable. Since post-fault transient stability depends on the true inertia of the combined generator–turbine assembly, this gap complicates stability studies. This paper proposes a method for estimating the inertia of turbogenerators using coast-down measurements from thermal unit B1 of the "Nikola Tesla B" power plant. The method is based on Newton's rotational motion equation and enables more accurate evaluation of inertia in large thermal units.

Key words: moment of inertia, generator, steam turbine, Newton's equation

1. Introduction

Conventional energy sources, such as synchronous generators, possess the kinetic energy of their rotating masses, which forms the basis for system stability and its resilience to unforeseen disturbances. With the integration of Renewable Energy Sources (RES), the existing system becomes decentralized and more sensitive to disturbances, particularly concerning

frequency and transient stability. This happens because RES units connect to the grid via power electronic devices, which possess little to no inertia.

The reduced system inertia leads to an increase in the Rate of Change of Frequency (ROCOF) during a step change in load. When the ROCOF reaches a critical value, it causes undesired system load shedding and shortens the time available for operators to properly react to sudden system imbalances, which are particularly evident due to the variability and unpredictability of RES generation. A high ROCOF value can also cause other negative system consequences, such as the unnecessary tripping of protective relays [1].

From the perspective of grid stability with an increased level of RES integration, generation units must successfully cope with the consequentially larger frequency changes. This situation has led to the introduction of precise requirements in some grids. Some countries have significantly lowered relay response thresholds, or rather, raised the permissible ROCOF values for which relays will not trip, to achieve the 2020 RES integration target, as shown in Figure 1. For example, Denmark raised the acceptable ROCOF value from approximately 0.8 Hz/s to 2.5 Hz/s [2].

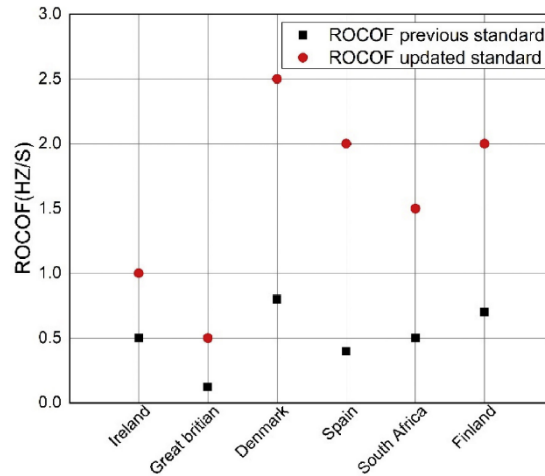


Figure 1. Permissible ROCOF values in various countries around the world [3]

Based on everything enumerated, the significance of knowing the inertia value of system components is evident, since, in addition to the difficulties of storing generated energy, the lack of inertia is the biggest problem faced by the increasingly prevalent Renewable Energy Sources (RES).

This has consequently led to the need for power electronic devices, such as the Virtual Synchronous Generator (VSG), which can emulate inertia. However, it has not been sufficiently investigated whether this emulated inertia has the same effect on the system as the inherent inertia of a synchronous generator, for instance, during a grid fault.

To reach these and similar conclusions, it is necessary to know the inertia value provided by the existing generation units, which is not straightforward. For traditional generation units, the generator inertia data is one of the parameters found in the generator's technical documentation, but it's not possible to obtain the same data for the turbines.

Inertia constants for several thermal power plants and hydropower plants of various nominal powers are shown in Figure 2. It is observed that there is no clear consistency in the inertia constants for a specific type of power plant or power rating. The inertia value of a power plant is therefore case-specific and largely depends on the design of both the generator and the turbine.

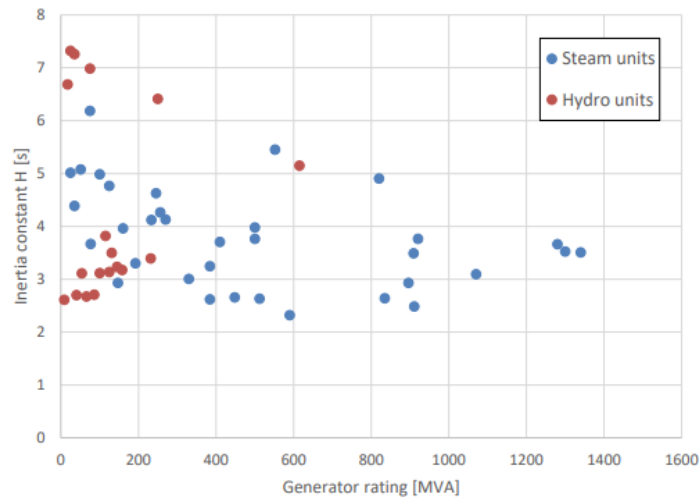


Figure 2. Dependence of the inertia constant on rated power for different types of units [4]

The stability of an electric power system is classified according to the system variable of interest. The significant quantities used to define system stability are power angle, voltage, and frequency. It is shown in [4, 5] that there is no direct connection between the reduction of system inertia and rotor angle or voltage stability. However, inertia plays an important role in system frequency stability. [6] defines frequency stability as the ability of an electric power system to maintain a stable frequency following a severe disturbance between generation and consumption. Frequency instability can lead to sustained frequency oscillations, which may cause the disconnection of generating units or loads if the frequency goes outside a defined range or if the ROCOF (Rate of Change of Frequency) becomes excessively large.

Considering a single synchronous machine, its dynamics can be described by Newton's equation for rotational motion, which is given by expression (1).

$$2H \frac{df}{dt} = P_m - P_e, \quad (1)$$

where P_m and P_e denote the mechanical and electrical power, respectively, while f represents the frequency at the machine terminals. The inertia constant H in (1) is used for the quantitative determination of inertia and is expressed in seconds, as it indicates the time, in seconds, during which the generator can supply its nominal power (P_n) using only the kinetic energy (E_k) stored in the rotating mass of its drive train.

$$H = \frac{\frac{J \omega_0^2}{2}}{S} = \frac{E_k}{S}, \quad (2)$$

where ω_0 is the nominal angular speed (rad/s), and J is the moment of inertia [$\text{kg}\cdot\text{m}^2$].

The inertia of the synchronous machine, designated by its inertia constant H , expresses the resistance to a change in frequency that results from a power imbalance between mechanical and electrical power [7]. Photovoltaic systems and wind farms, with the exception of fixed-speed wind turbines, provide negligible or no inertia due to the power converters that decouple the generator from the system. The system load also contributes to the instantaneous inertia, but this contribution is usually very small. Therefore, as a first estimate in most power system studies, only the inertia of conventional power plants is considered.

The objective of this work is to propose a method for calculating the inertia of a turbogenerator set and thereby contribute to a better assessment of the total system inertia.

The paper is organized as follows: Chapter 2 presents the method for calculating the turbogenerator inertia, while Chapter 3 displays the results of its application. The derived conclusions and directions for future research are presented in Chapter 4.

2. Calculation of Turbogenerator Inertia

Recorded data from five shutdowns of thermal unit B1 at the "Nikola Tesla B" Thermal Power Plant (TENT B) are available, as well as the following generator parameter values:

- Generator efficiency: 98.63%
- Rated rotational speed: 3000 rpm
- Stator winding resistance at 20 °C: 1.079 mΩ
- Rated rotor voltage: 550 V
- Rated rotor current: 5875 A
- Rated stator current: 20 kA

- Turbogenerator inertia: 56 tm²

In the analysis of the rotational dynamics of a turbogenerator, particularly in the shutdown regime, the resistive torque that opposes the rotor's rotation plays a crucial role. The braking torque, denoted as M_k , represents the sum of all resistance torques acting on the machine's rotor and decelerating its motion. Physically, this torque can be expressed as the ratio of the total power losses (P_Σ) and the instantaneous angular speed (ω), according to the expression:

$$M_k = \frac{P_\Sigma}{\omega}. \quad (3)$$

In the context of shutdown and dynamic deceleration, the most significant losses are precisely those that are speed-dependent, as they create a resistance that directly manifests as the resistive torque. However, the losses contributing to the braking torque are not all of the same nature, nor do they share the same speed dependency.

The rotational dynamics of the turbogenerator in shutdown mode can be expressed using Newton's equation for rotational motion in terms of the moment of inertia of the turbogenerator set (J) [kg·m²]:

$$J \frac{d\omega}{dt} = -M_k, \quad (4)$$

where ω [rad/s] is the angular rotational speed. In the shutdown phase, when the driving torque is zero, the resistive torque becomes dominant and causes a negative angular acceleration, i.e., rotor deceleration. It is precisely in this phase that the mechanical interpretation of losses - as the source of resistance that determines the turbogenerator's shutdown dynamics - is highlighted.

To determine the turbogenerator inertia, it is necessary to know the value of the losses that contribute to the deceleration. At synchronous speed (ω_s), these losses can be expressed using (5):

$$P_\gamma(\omega_s) = P_{mt} + P_{lb} + P_{vg}, \quad (5)$$

where P_{mt} are the internal mechanical losses of the turbine, P_{lb} are the bearing losses, and P_{vg} are the generator ventilation losses. [8] states that the turbine losses in a narrow operating range around synchronous speed amount to 1.5% of the nominal turbine power. The values for the power losses from (5) are given in Table 1.

It is important to note that the inertia calculation observes the speed range $[\omega_{min}, \omega_s]$ [rpm], where, based on the demagnetization diagram in Figure 3, it can be stated with certainty that the field has decayed to zero, and thus, magnetic circuit losses are non-existent.

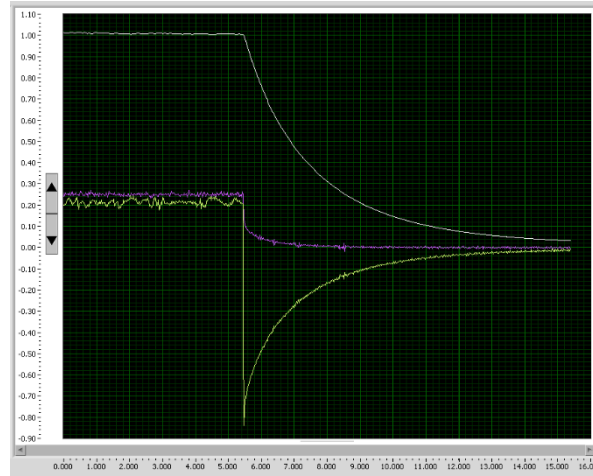


Figure 3. Generator demagnetization diagram within thermal unit B1 at the 'Nikola Tesla' Thermal Power Plant, white – stator voltage, purple – excitation current, green – excitation voltage

Table 1. Power loss values that are dominant during the shutdown of the turbogenerator

Loss power	Value [MW]
P_{mt}	9.750
P_{lb}	1.754
P_{vg}	1.779
$P_{\gamma}(\omega_s)$	13.283

Simultaneously, the value of losses at a given speed is proportional to the cube of that speed. Therefore, for the synchronous speed (ω_s), it can be expressed by the following relation:

$$P_{\gamma}(\omega_s) = k \cdot \omega_s^3, \quad (6)$$

from which the value of the coefficient k is obtained. Newton's equation (4) can now be written using the known coefficient k as:

$$J \frac{d\omega(t)}{dt} = -k \cdot \omega^2(t). \quad (7)$$

Solving the first-order differential equation (7) and considering the initial condition that the speed immediately before shutdown is equal to the synchronous speed (ω_s), it is obtained that:

$$\omega(t) = \frac{1}{\frac{1}{\omega_s} + \frac{k}{J}t}. \quad (8)$$

In accordance with the units of measurement on the speed-time shutdown graphs, relation (8) can be represented by a function of the form (9), using the parameters τ and ω_s , which were obtained as a result of fitting the data from the five available recordings in the software tool OriginLab2024.

$$n(t) = \frac{1}{a + bt}, \quad (9)$$

where n [rpm] is the generator's rotational speed. Since the coefficient b is assumed to be equal to:

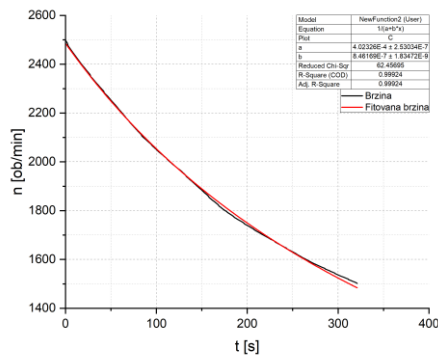
$$b = \frac{\pi}{30} \cdot \frac{k}{J}, \quad (10)$$

it follows that the moment of inertia J is calculated using expression (11).

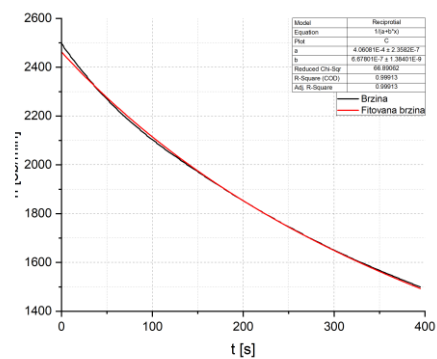
$$J = \frac{\pi \cdot k}{30 \cdot b} \quad (11)$$

3. Results

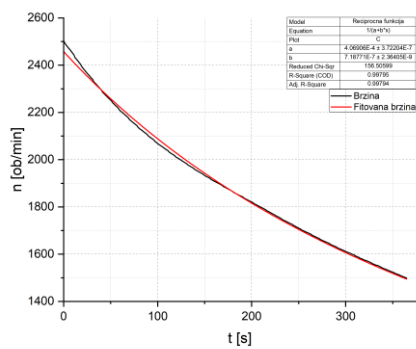
Figure 4 shows the speed-time dependence during shutdown for the selected speed range for five turbogenerator shutdown recordings, along with the corresponding fitting functions obtained using the Origin software program. The values determined for the parameter b are presented in Table 2.



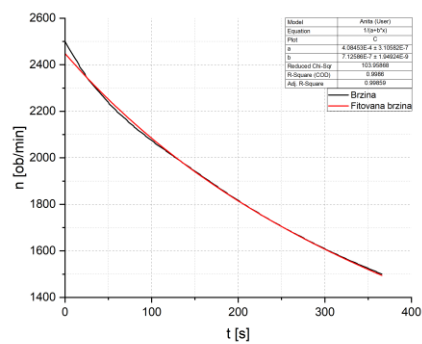
(a)



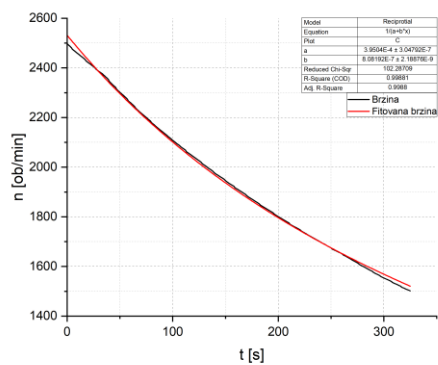
(b)



(c)



(d)



(e)

Figure 4. Results of fitting the function describing the dependence of turbogenerator shutdown speed on time

Table 2. Values of coefficient b obtained by fitting the turbogenerator shutdown speed function

Index number of the shutdown recording	b
1	8.46169 e – 7
2	6.67801 e – 7
3	7.18771 e – 7
4	7.12586 e – 7
5	8.08192 e – 7

Substituting the mean value of the coefficient b into expression (11) yields a turbogenerator inertia value of $J = 59759 \text{ kg}\cdot\text{m}^2$.

4. Conclusion

Maintaining the stability of the electric power system under conditions of reduced inertia requires precise knowledge of the contribution of individual generation units. This paper presents a method for estimating the moment of inertia of a turbogenerator set based on shutdown recordings, which provides insight into the actual inertial contribution of conventional units.

The obtained results can contribute to defining minimum system requirements for inertia and improving the modeling of its dynamic resilience. Further research will focus on improving this method and extending it to a larger number of generating units, as well as developing a separate method for hydro-units, in order to form a comprehensive framework for assessing the inertia of different types of units.

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Kratak sadržaj: U konvencionalnim elektroenergetskim sistemima sinhroni generatori obezbeđuju tranzijentnu i frekvencijsku stabilnost zahvaljujući kinetičkoj energiji obrtnih masa. Stvarna inercija kompletnog sklopa generator–turbina ključna je za ponašanje generatora posle otklanjanja bliskog kratkog spoja, ali se podaci o inerciji, posebno za parne turbine, obično ne nalaze u tehničkoj dokumentaciji. Istovremeno, porast udela obnovljivih izvora i gašenje jedinica na fosilna goriva smanjuju ukupnu inerciju sistema i povećavaju njegovu osetljivost na poremećaje. Nedostatak inercije može dovesti do većih frekvencijskih odstupanja i ozbiljnih finansijskih gubitaka, pa je od presudnog značaja poznavanje doprinosa postojećih agregata i određivanje minimalne kritične inercije potrebne za stabilan rad mreže. U radu je prikazan metod za određivanje inercije turboagregata na osnovu snimaka zaustavljanja termobloka B1 u TE „Nikola Tesla B“. Metod se zasniva na Njutnovoj jednačini obrtnog kretanja i omogućava preciznije sagledavanje ukupne inercije sistema.

Ključne reči: moment inercije, generator, parna turbina, Njutnova jednačina

Određivanje momenta inercije turboagregata

Jasna Dragosavac^{}, Anita Mijajlović^{}, Ilija Klasnić^{}, Milan Đorđević,
Igor Damjanac, Miloš Brdarević

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