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HOW EFFICIENT IS EXEMPLARY TEACHING IN LEARNING EQUATIONS AND INEQUATIONS IN THE THIRD AND FOURTH GRADE OF PRIMARY SCHOOL?

Abstract: The aim of the study was to investigate the efficiency of learning equations and inequations through the application of exemplary teaching in primary school. In terms of efficiency and achievement levels, we started from the learning outcomes on equations and inequations taught in the third and fourth grade primary school mathematics curriculum and from a taxonomic model of operationalization of the goals and objectives of mathematics teaching in the cognitive domain. According to this model, the performance of the students was examined in the following 5 main taxonomy levels: recognition, reproduction, comprehension, operationalization and creative problem solving. The results of the experimental study, which was carried out on a sample of 100 students, show that the teaching method used helps students to learn equations and inequations more successfully. The use of exemplary teaching had a positive effect on students' performance in reproduction, comprehension, operationalization, and creative

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problem solving, while it had no effect on students whose knowledge was at the recognition level. The results of this study could lead to changes in the presentation of the teaching content, the evaluation of the efficiency of the educational process, a more efficient individualization of the learning process, etc. In addition, the methodological contribution of this work is the creation of exemplary teaching models whose use in a classroom, thanks to analogue reasoning, allows students to find their own approach to mathematics.

Keywords: *exemplary teaching, efficiency, equations, inequations, taxonomy, criterion-referenced tests.*

Introduction

Providing quality education that enables every student to fulfill their potential and acquire appropriate skills in a society that is constantly changing, where knowledge is rapidly becoming obsolete and where it is difficult to predict future changes, is a challenge that almost all education systems worldwide are facing. In response to the current challenges that society poses to education, today's schools "focus on the dominant students' activities, on the development of his/her personality and individuality" (Milijević, 2003: 37).

In mathematics education, examples "play a crucial role in learning about mathematical concepts, techniques, reasoning, and in the development of mathematical competence" (Bills, Mason, Watson, Zaslavsky, 2006: 126). They "open up new horizons and light up new paths, and in this lies their didactic relevance" (Derbolav, 2003: 320). The use of examples in teaching "involves careful choices of specific examples which facilitate the directing of attention appropriately so as to explain and to induce generalizations. Desirable choice of examples depends on many factors, such as the teaching goals and teachers' awareness of their learners' preconceptions and dispositions" (Bills et al., 2006: 128).

As example-based approach, exemplary teaching is a powerful way of parsimoniously and effectively presenting the content that has been reduced to its essence and whose essence is accessible via the example (Van Dyk, 2006: 128). For this method to be effective, appropriate and representative, examples must be selected for the objective to be achieved with the specific teaching content. These examples serve not only as the first stage of learning or as a platform through which progress can be made towards the learning objective, but also because the essence of the matter or phenomenon can already be seen in that specific example. As Van der Stoep and Louw say, "the exemplar must serve to unlock the contents fundamentally, to illuminate essences and principles, to link theory and reality (thinking and acting), and to provide real access to the basic categorical structures of reality" (2005: 343). Furthermore, when using exemplary teaching in the classroom, teachers should be careful to preserve its open, original and creative character without falling for an easy-to-learn schematization (Derbolav, 2003: 332) and not see it as a mechanically followed and applied recipe-like or even chronological arrangement of learning material according to a pre-planned (linear) principle (Van Dyk, 2006: 130).

Research on exemplary teaching has been primarily concerned with student achievement in the lower grades of high school, faculty, and university (e.g., Kalathaki, 2016; Jorgensen, 2013; Wagenschein, 2000) or with gifted students (National Collegiate Honors Council, 2008), with a small number of studies looking at student achievement in the lower grades of primary school (e.g., Ableser, 2011). According to Ableser (2011, 2012), a central theme of exemplary practice in all educational contexts from preschool to higher education (P-20+) (2012: 65) is teaching so that all can learn, and the fundamental outcome of exemplary teaching is that all students are provided with the opportunity to maximize learning success (2011: 23). This is because getting students used to learning new mathematical concepts (such as algorithms, mathematical rules as well as other mathematical facts) on their own and practicing them should begin on the first day of school Malinović-Jovanović & Malinović 2013: 110).

In primary school mathematics lessons, learning resources are arranged in concentric circles, i.e., they represent a system of terms or concepts within a content area as well as within the corresponding topics taught at different grade levels, i.e., at different ages. This means that in a variety of cases, exemplary learning resources can be singled out and used as a model for studying analogous content in that particular content area. In this way, and in line with the aspirations of contemporary education, the teaching process is shifted from the teachers to the students.

This way of teaching mathematics makes the subject more suitable than other school subjects for the use of exemplary teaching and, consequently, for testing its efficiency in the classroom. The main reason for this is the fact that the efficiency of education depends to a large extent on how someone is taught, whether or how someone is motivated to learn and what his/her involvement in this process is, i.e., whether he/she uses all his/her abilities and skills or passively acquires the facts (Malinović-Jovanović & Malinović, 2013: 23).

Background of the study

The main goal of exemplary teaching is to enable students to learn for themselves. Teaching process is organized in such a way that a teacher selects learning resources from the learning topics, which are then presented by the teacher in an exemplary manner and according to the corresponding didactic requirements. These learning resources are representative examples of the content area to which they belong, i.e., the models that students will use as a starting point for independent learning of other topics from that specific content area. If the model is weak, it will affect students' subsequent performance. Therefore, exemplary teachers need to focus on learning outcomes by being intimately familiar with the learning materials and pedagogical knowledge about them. In doing so, they should consider that the exemplary content retains the essential features of the content area and that the principles and features of the exemplary content can be transferred to analogous content. At the same time, "the teacher should take into account the different conditions in which the teaching process takes place in a given class (the size of the class, the heterogeneity of the students and their individual characteristics in relation tolearning, their age, mutual (un)familiarity and possibly pre-existing relationships and climate in the class, the habits of the students and other characteristics of their behaviour, the emotional state of the students, etc.)" (Ristić, Zlatković, Malinović-Jovanović, 2022: 132). In this way, students are enabled, among other things, to work independently on analogue learning materials as well as to research and solve various problems independently outside of school and after graduation (Milijević, 2003: 104).

Given the concentric structure of the mathematical topics and the fact that it is a system of interrelated and connected topics, each topic can be studied and learned within a specific content area, taking into account the previous topics. Because of this topic structure, in most cases the exemplar topics are at the very beginning of a specific content area, which then serve as the basis and foundation for the treatment of the remaining topics in that content area. These are then seen as a model for the following topics, which are then adopted analogously. Each topic chosen in this process "must be carefully judged to determine if the elementals of a matter, problem, view or law can be presented by it according to the real essentials of the matter" (Van der Stoep & Louw, 2005: 343). Apart from the selection of exemplary topics and adequate exemplars, the essence of exemplary teaching lies in the correct choice of didactic and pedagogical methods to be used. Therefore, the teacher must make a good selection of teaching methods, procedures, forms and suitable teaching aids and materials in the preparation phase of the exemplary topics.

According to Van der Stoep & Louw (2005), for a good organization of exemplary teaching, it is necessary that its preparation and application follow these three phases:

(1) Review of the curriculum and identification of exemplary and analogous topics and materials.

(2) Work on the exemplars, which includes preparation, work on the curriculum content, exercises, direct repetition and assessment.

(3) Independent application of the analogies by the students according to the model (2005: 455).

In the first phase, the teacher must thoroughly examine the objectives or outcomes of the specific content area to be covered by the exemplary lesson and research the relevant topics. The next step is to select the topics to be used as exemplars and the topics that can be covered in a similar way, i.e., based on the exemplars. According to Van der Stoep & Louw (2005), in the second phase, the teacher shows the first exemplar and draws attention specifically to the topics and methodological aspects that are relevant to understanding its meaning, explanation and clarification. The additional, linked exemplars are presented as problems for the students in which the teacher and students work together and the teacher assesses the extent to which the explanation in the first exemplar has been successful. In the third phase, students are encouraged to use their own knowledge and insights to apply the insights of the second phase independently in class or at home, extending them to difficult or related exemplars, integrating the details they have already mastered, and so on (Van der Stoep & Louw, 2005: 445). This involves an ongoing process of curriculum design and reflective practice, including setting learning objectives based on standards, curricula, goals, and purposes; assessing students' prior knowledge, interests, and needs; determining and providing methods and approaches to engage students in learning; having a clear criterion for assessment to know whether learning has been effective; evaluating learning to determine whether it has been mastered; or providing additional opportunities for further development (Ableser, 2012: 68). If exemplary teaching is applied with regard to all the didactic requirements mentioned above, it enables students to develop analogical reasoning through independent engagement with analogical content.

"Empirical researchers across disciplines have argued that analogical reasoning may be central to learning abstract concepts (Brown & Kane, 1988; Gentner, Holyoak, Kokinov, 2001), procedures (Goswami, 1992), novel mathematics (Bassok, 2001; Novick & Holyoak, 1991), and for the ability to transfer representations across contexts" (Novick, 1988; Reed, Dempster, Ettinger, 1985, as cited in Richland, Holyoak, Stigler, 2004: 37).

Considering the fact that students' analogical reasoning increases significantly over the first few years of primary school (Siegler & Svetina, 2002) and that in primary school mathematics education, drawing analogical inferences is one of the most important types of reasoning that students use to create mathematical concepts, predictions, etc. (Malinović-Jovanović, Malinović, 2013: 63), the development of analogical reasoning plays an important role in primary school mathematics education. Students learn more about the world they are then able to use that increasing domain knowledge to reason about the relationships between items (Cheshire et al., 2005). Analogy allows students to apply commonalities between mathematical relations to help grasp new problems or concepts through contributing to integral components of mathematical proficiency. Learning via analogy usually involves finding a set of systematic correspondences (a mapping) between a better-known source analog and a more novel target (Somayeh, Parvaneh, Mohammad, 2012: 2918).

On the other hand, "the situation in the practice of teaching mathematics is characterized by the fact that there are large individual differences among students, both in terms of the knowledge they possess and in terms of their mathematical abilities, opportunities and interests to acquire and learn mathematical content" (Bikić, Maričić, & Pikula, 2016: 2786). In this sense, and in accordance with the requirements of contemporary education, the construction of practical models for exemplary teaching must take into account the students to whom the knowledge is addressed. Theoretically, exemplary teaching is suitable as an approach for working with all categories of students because it includes special opportunities for individualization in all its variations, through which the individual student is given the opportunity to follow his or her own disposition and pace to master the contents, and the transition from the concrete to the abstract is very systematic and gradual, which has a direct impact on the quality of performance (Van der Stoep, Louw, 2005: 447). Therefore, we wanted to investigate whether the use of exemplary teaching has a positive impact on the success and achievement of all types of students, i.e., how efficient it is when used in the classroom.

The term "efficiency" here refers to the degree to which educational goals and objectives are achieved (Pedagogical Dictionary, 1967). The criteria for evaluating efficiency can vary: quantity of knowledge, quality of knowledge and skills, positive change in student behavior, etc. In our case, efficiency included the quantity and quality of knowledge as well as the degree of achievement of the goal and objectives of mathematics teaching in primary school. Regarding the influence of exemplary teaching on the efficiency of achieving the objectives, we started from the learning outcomes related to equations and inequations covered in the third and fourth grades as specified in the primary school mathematics curriculum. However, in determining the quality of knowledge acquired by students based on the specified objectives and outcomes, we started from the taxonomic model of operationalization of the goal and objectives of mathematics education (Bogdanović & Malinović-Jovanović, 2009: 620). The taxonomic model corresponds to Bloom's taxonomy (Bloom et al., 1956) in the cognitive domain on the one hand, and to the requirements related to the assessment and evaluation of students' knowledge set out in the mathematics curriculum for the third grade of primary school (*Pravilnik o programu nastave i učenja za treći razred osnovnog obrazovanja i vaspitanja*, 2019, 2020, 2022, 2023) and the mathematics curriculum for the fourth grade of primary school (*Pravilnik o programu nastave i učenja za četvrti razred osnovnog obrazovanja i vaspitanja*, 2019, 2020, 2021, 2023) on the other. The taxonomy consists of five main levels: Recognition (Rec), Reproduction (Rep), Comprehension (C), Operationalization (O) (Application and Generalization) and Creative Problem Solving (CPS) (Analysis, Synthesis and Evaluation according to Bloom). These levels are organized hierarchically according to the qualitative levels of knowledge, and for each category there is a subcategory that defines the educational objectives of the respective category in more detail. Depending on the degree of adoption of facts and generalizations, knowledge can be of different quality.

The value of using Bloom's taxonomy in developing learning outcomes provides a tool for planning, implementing, and assessing teaching (Nayef, Rosila, Yaacob, Ismail, 2013:167). The problem is how to draw a clear line among the categories of knowledge and skills, i.e., how to identify levels of knowledge for which there is no ideal methodology worldwide. Therefore, it is difficult to formulate and validate the test items that would serve as indicators and help measure the knowledge and behavior characteristic of each of the categories. One of many studies that point to the above is the TIMSS study (Trends in International Mathematics and Science Study), where the tasks and determinants of the cognitive domain in the study were changed several times because it is difficult to draw a clear line between the type of knowledge and the mental operations required to understand the subject material (Mullis, Martin, Foy, 2005: 6).

We selected the algebra content covered in primary school mathematics because insufficient preparation for algebra at this age can have long-term consequences for the understanding of this content in the next stages of the teaching process and because "algebra presents the gateway for learning mathematics, but also the barrier, since a large number of students do not get the key to that gate" (Cai, 2004: 107; Lott, 2000; Moses & Cobb, 2001, as cited in Maričić, 2017: 117). The difficulties arise from the abstract nature of the algebraic content itself and the limited cognitive abilities of students in primary school, which entails the need to change the approach to teaching this content and to find appropriate teaching methods for the successful learning of basic algebraic concepts (Anđelković & Maričić, 2023: 252). Research has shown that many students model the solution of equations, i.e., they do not understand the structure underlying the procedure for solving the equations (Brown et al., 1988), and that they solve inequations according to the scheme they have learned, i.e., according to a set of fixed routine procedures which they usually do not summarize and therefore cannot even explain why they have taken certain steps (Anđelković, 2022: 29).

Considering the importance of exemplary teaching in establishing relationships between examples, analogical reasoning and mathematical contexts, as well as the importance of taxonomy in setting educational goals and objectives and improving the teaching process with all students, in this study we establish the appropriate models of exemplary teaching and investigate the efficiency of achievement of objectives in learning equations and inequations in primary school mathematics teaching.

Method

The experimental program was based on the learning objectives and outcomes specified in the mathematics curriculum for the third and fourth grades of primary school, which relate to the content of equations and inequations taught in these grades, and on the taxonomic model of operationalization of the goals and objectives of mathematics education.

The initial research sample consisted of 185 third and fourth grade primary school students. Based on the results of the initial test, two groups of students were formed: the experimental group and the control group. The groups were formed in such a way that each student from the control group had a pair from the experimental group with the same score. The final sample consisted of 100 students: the experimental group (N = 50, age 8.5 to 9.2) and the control group (N = 50, age 9.5 to 10.3), of which 46 were from the third and 53 students from the fourth grade.

The objectives and outcomes of the primary school mathematics curriculum are listed below for further analysis. Upon completion of the course, the student will be able to:

By third grade: Solve the equation with an arithmetic operation on a number line up to 1000 (form: $a \pm x = b$; $x \pm a = b$; $a \cdot x = b$), Solve the problem with an equation, Determine and write down the set of solutions to the inequation with addition and subtraction (form: $a \pm x < b, a \pm x > b, x - a < b, x - a > b$),

Fourth grade: Solve equations and inequations in the set N_0 and check the correctness of the solutions, Solve a problem with an equation or inequation.

The experimental program for the following teaching units was completed in accordance with the specified objectives and the corresponding learning materials: Addition Equations, Subtraction Equations, Multiplication Equations, Inequalities and Inequations, Solving Inequations by Subtraction, Solving Inequations by Addition, in third grade; Equations with one operation, Equations with two operations – multiplication and addition, Equations with two operations – multiplication and subtraction, Equations with two operations - division and addition, Equations with two operations – division and subtraction, Solving inequations by subtraction, Solving inequations by addition, Solving inequations by division and Solving inequations by multiplication in fourth grade.

The development of the models was based on the one hand on the methodological approach of building and forming mathematical concepts in primary school mathematics teaching and on the other hand on the principles of exemplary teaching. The models were not only related to the acquisition, but also to the repetition, practice and systematization of students' knowledge, as this type of teaching is suitable for all types of teaching units. The tasks used in the models are categorized on the basis of knowledge levels and in accordance with the taxonomy. The structure of the model served to design the sequence of student activities, with the main objective being to develop students' cognitive structures so that they can perform a more efficient cognitive activity. In addition, these models planned both the students' and the teachers' activities so that they could be carried out correctly. A total of 15 models were created for the experimental program – 6 for the third grade and 9 for the fourth grade. The learning materials on addition equations, on inequalities and inequations and on solving inequations by subtraction in the third grade, equations with two operations – multiplication and addition, solving inequations by subtraction and solving inequations by multiplication in the fourth grade were selected as exemplars. The remaining learning materials, which were considered analogue, were worked on by the students in the following way: They were given learning materials which they worked on independently and which were intended to have a positive effect on their analogical reasoning. In the control group, the teaching units were processed in a conventional manner.

The implementation of the experimental program and the initial tests were preceded by a pilot study conducted with a sample of 40 students. On this basis, the experimental program was reviewed and the initial forms of the instruments were created.

Criterion-referenced tests were created as an instrument for the purpose of the study. The tests contained different types of tasks. The true-false, multiple-choice, gap-filling, matching tasks were used to test the level of recognition. The gap-filling tasks were used to assess the level of reproduction, but only in cases where students were asked to draw conclusions based on the data in the given tasks by completing the corresponding statements. In all other tests, only questions with constructed responses adapted to the category of knowledge they were intended to measure were used. Two tests were created for each of the grade levels:

Initial Test (IT) – included the tasks on equations as part of the Numbers and Algebra content area, which was covered before the experimental program content. The third grade is about equations for a set of numbers up to 100, while the fourth grade is about equations and inequations for a set of numbers up to 1000.

Final test (FT) – consists of the tasks from the teaching units that were part of the experimental program.

The tasks in the initial and final tests related to different objectives and content. Therefore, the scores obtained in each test differed according to the objectives of each test (Table 1).

Table 1. Score results in the initial and final test per taxonomy level

Number of tasks	Grade	Score results for levels of knowledge of the taxonomic model					
		Rec	Rep	C	O	CPS	Σ
Initial test							
15	III	3	9	12	15	18	57
15	IV	3	9	12	15	18	57
Final test							
20	III	6	11	14	20	29	80
20	IV	5	10	14	16	27	72

One week before the start of the experimental program, the students underwent an initial test (IT), after which the experimental and control groups were formed. The final test (FT) was carried out after the experimental program had been completed. The authors of this study created all tests independently of each other. The validity of the tasks in the tests was determined on the basis of their correspondence to the level of knowledge of the constructed taxonomic model and the objectives specified in the mathematics curriculum. The objectivity of the tests was ensured by placing each student in an approximately similar test situation and by ensuring that the independent examiners followed clear instructions and that the items were scored according to the same key principle. The reliability of the instruments was measured by calculating the Kuder-Richardson formula for each of the levels and for the tests in general. The reliability coefficients for the tests as a whole was $\rho_{IT} = .81$ for IT and $\rho_{FT} = .83$ for FT in third grade and $\rho_{IT} = .81$ for IT and $\rho_{FT} = .88$ for FT in fourth grade. In third grade, the reliability coefficients of the subsample ranged from $\rho = .68$ to $\rho = .94$ for IT and from $\rho = .69$ to $\rho = .96$ for FT. In the fourth grade the reliability coefficients of the subsample ranged from $\rho = .71$ to $\rho = .96$ for IT and from $\rho = .74$ to $\rho = .93$ for FT. Across all tests, the reliability coefficient was highest for the recognition level and then decreased in correlation with the increase in knowledge level, meaning that the reliability coefficient was lowest for the creative problem solving level, which was to be expected as these are hierarchically ordered levels of knowledge.

The data obtained from the study was processed using the SPSS20 statistical software package. The paired t-test was used for the statistical comparison of the experimental and control groups and for the monitoring of the effects of the experimental program.

Results and discussion

To find out how effective exemplary teaching is when it comes to achieving the learning objectives related to equations and inequations in primary school mathematics teaching, we observed the difference in task performance between the experimental and control groups. This difference in test items was analyzed based on each level of the taxonomic model and also based on the overall level of test completion – because the criterion for efficiency was both the quantity and quality of knowledge absorbed.

At the end of the initial tests, both groups achieved approximately similar results in each grade (Table 2): the experimental group (M = 32.30; Sd = 7.89) and the control group (M = 32.04; Sd = 8.15) in the third grade; the experimental group (M = 32.96; Sd = 7.55) and the control group (M = 33.39; Sd = 6.96) in the fourth grade. The calculated t-value of the differences between the mean scores (t(44) = .11; p = .45 in the third grade and t(52) = .20; p = .42 in the fourth grade) indicates that there is no statistically significant difference in the level of knowledge between the students in the experimental and control groups in relation to the initial test. In addition, the results for each individual knowledge category show that there is no statistically significant difference in the level of completion of the initial test items (Recognition, Reproduction, Comprehension, Operationalization and Creative Problem Solving): (t(44) = .38, p = .35; t(44) = .11, p = .45; t(44) = -.16, p = -.43; t(44) = .00, p = .5; t(44) = .11, p = .45 in third grade); (t(52) = .13, p = .44; t(52) = .15, p = .44; t(52) = .14, p = .44; t(52) = -.46, p = -.32; t(52) = -.37, p = -.35 in fourth grade).

Table 2. Descriptive indicators of the success of experimental and control group at the initial testing

Grade	Level of the taxonomic model	Group	N	Mean	Std. Dev	Std. Error	t-value	Sig.	D_f
III	Recognition	E	23	2.43	.66	.15	.38	.35	44
		K	23	2.35	.65	.14			
	Reproduction	E	23	6.07	2.10	.45	.11	.45	
		K	23	6.00	2.21	.47			
	Comprehe-nsion	E	23	7.11	3.73	.8	-.16	-.43	
		K	23	7.26	2.49	.53			
	Operationa-lization	E	23	8.09	5.11	1.09	.00	.50	
		K	23	8.09	3.46	.74			
	Creative problem solving	E	23	8.74	3.93	.84	.11	.45	
		K	23	8.61	3.87	.83			
	Σ (total in the third grade)	E	23	32.30	7.89	1.68	.11	.45	
		K	23	32.04	8.15	1.74			
IV	Recognition	E	27	2.35	.57	.11	.13	.44	52
		K	26	2.33	.53	.11			
	Reproduction	E	27	6.61	2.09	.41	.15	.44	
		K	26	6.52	2.16	.43			
	Comprehe-nsion	E	27	7.22	3.10	.61	.14	.44	
		K	26	7.12	2.07	.41			
	Operationa-lization	E	27	8.48	2.64	.52	-.46	-.32	
		K	26	8.89	3.59	.72			
	Creative problem solving	E	27	8.44	5.06	.99	-.37	-.35	
		K	26	8.89	3.65	.73			
	Σ (total in the fourth grade)	E	27	32.96	7.55	1.48	.20	.42	
		K	26	33.39	6.96	1.39			

After carrying out the experiment, the students were given the final test. The results of this test showed an improvement in the experimental group in both grades. When we look at the average score of the students in the final test in Table 3, we can see that the students in the experimental group in both grades (M = 58.39; Sd = 6.01 in third grade; M = 48.70; Sd = 5.08 in fourth grade) performed better than the students in the control group (M = 44.92; Sd = 4.78 in third grade; M = 39.66; Sd = 4.10 in fourth grade). The t-values obtained (t(44) = 3.80 at p = .000 in the third grade and t = 3.18 at p = .001 in the fourth grade) indicate a statistically significant difference in the processing of the test items in both grades with a high significance level in favor of the experimental group, which is why we can claim that the use of exemplary teaching when teaching equations and inequations improves students' knowledge.

Table 3. Descriptive indicators of the success of experimental and control group at the final testing

Grade	Group	N	Mean	Std. Dev	Std. Error	Min	Max	t-value	Sig.	D_f
Third	E	23	58.39	6.01	1.69	28	80	3.80**	.000	44
	K	23	44.92	4.78	.87	25	71			
Fourth	E	27	48.70	5.08	1.39	21	72	3.18**	.001	52
	K	26	39.66	4.10	.93	20	70			

** The difference is significant at the $p < .001$

To check whether the teaching objectives for the content area of equations and inequations were efficiently achieved, we also observed the success of the experimental and control groups for each category of the taxonomic model (Table 4). The calculated t-values of the differences between the mean scores in the initial tests show that there is no statistically significant difference in terms of the level of accomplishment of the test items for each knowledge category (Table 2), after which we established equivalence between the experimental and control groups. If we look at the results of the final tests (Table 4), we can see that only in the case of the recognition level in both grades is there no statistically significant difference in the processing of the test items (t(44) = 1.92, p = .05 in the third grade and t(52) = .76, p = .22 in the fourth grade), while in all other cases the difference is highly significant in favor of the experimental group (Reproduction, Comprehension, Operationalization and Creative problem solving): (t(44) = 3.74, p = .000; t(44) = 2.77, p = .004; t(44) = 2.96, p = .002; t(44) = 3.99, p = .000; in the third grade); (t(52) = 2.58, p = .006; t(52) = 3.48, p = .001; t(52) = 2.74, p = .004; t(52) = 2.12, p = .01 in the fourth grade).

Table 4. Descriptive indicators for the success of experimental and control groups in the final test according to the taxonomy level

Grade	Level of the taxonomic model	Group	N	Mean	SD	Std. Error	Min	Max	t	Sig.	D_f
III	Recognition	E	23	4.91	.90	.19	3	6	1.92	.05	44
		K	23	4.22	1.44	.31	2	6			
	Reproduction	E	23	8.30	1.77	.38	4	11	3.74**	.000	
		K	23	6.13	2.07	.44	3	11			
	Comprehension	E	23	.65	2.23	.48	6	14	2.77*	.004	
		K	23	8.74	2.34	.50	5	12			
	Operationalization	E	23	14.44	3.60	.77	5	20	2.96*	.002	
		K	23	11.13	3.83	.82	5	20			
	Creative problem solving	E	23	20.09	4.78	1.02	10	29	3.99**	.000	
		K	23	14.70	4.11	.88	10	22			
IV	Recognition	E	27	3.96	.85	.17	3	5	.76	.22	52
		K	26	3.77	.91	.18	2	5			
	Reproduction	E	27	7.89	1.83	.36	4	10	2.58*	.006	
		K	26	6.65	1.57	.31	3	10			
	Comprehension	E	27	10.04	2.89	.57	4	14	3.48**	.001	
		K	26	7.50	2.35	.47	3	14			
	Operationalization	E	27	10.59	2.82	.55	4	16	2.74*	.004	
		K	26	8.62	2.32	.46	6	14			
	Creative problem solving	E	27	16.22	5.46	1.07	6	27	2.12*	.01	
		K	26	13.12	4.93	.99	6	27			

Based on the results obtained, we can conclude that there are no statistically significant differences between the experimental group and the control group when it comes to the level of completion of the test items at the recognition level. Since this level is the lowest in the hierarchical distribution of knowledge, the results obtained are to be expected and justified, because a student can regurgitate a fact when asked to do so without necessarily understanding the meaning of the fact, a level of competence that can be achieved by simple memorization (Lister, 2006: 81), for which the traditional teaching approach is sufficient. Moreover, students with the lowest level of knowledge are not able to study analogical material independently but only with the help of the teacher, because the recognition level requires the most basic knowledge, such as knowledge of facts, concepts, principles, rules, but only up to the level where the student can recall or recognize them with the help of the teacher among the given answers (Nikolić, 2015).

On the other hand, the greatest difference between the experimental and control groups in terms of the level of completion of the test items was found in the third grade for creative problem solving, while in the fourth grade the difference was greatest for students whose knowledge was at the level of comprehension, in both cases in favor of the experimental group.

These results are consistent with some other research findings that have shown that the use of some other innovative teaching models did not have a positive impact on the knowledge of students whose knowledge is at the recognition level. Consistent with this are the findings of Malinović-Jovanović & Janković (2014), who showed that the use of problem-based teaching does not have a positive impact on the knowledge of students belonging to this group when it comes to learning equations, while for all other groups of students the use of this teaching method had a positive effect. The results of the study which looked at the impact and effect of differentiating the content at three levels of complexity and interactive teaching in small groups on the efficiency of achieving the required objectives in the content area of addition and subtraction to hundreds (Malinović-Jovanović, 2015), also showed that differentiated teaching at three levels has no positive effect on the performance of students whose knowledge is at the level of creative problem solving, but has a positive effect on the performance of students whose knowledge is at the level of recognition and the other levels. On the other hand, interactive teaching in a small group does not have a positive effect on students whose knowledge is at the level of recognition and reproduction, but it has a positive effect on all other categories.

In primary school mathematics, algebra is an important part of the mathematics curriculum. Its complexity, understanding, application and the difficulties students face in their studies require specific teaching methods, approaches and strategies when addressing and methodically transforming this content area with students (Maričić, 2017: 119). Surveys have shown that students can successfully adopt algebraic concepts and understand their syntactic rules in the context of primary school mathematics teaching (Blanton & Kaput, 2005; Carpenter & Franke, 2001) and that "success in algebra depends on at least six types of mathematical thinking skills: Generalization, abstraction, analytical thinking, dynamic thinking, modelling and organization" (Lew, 2004: 93). However, the results of some surveys dealing with students' success in solving tasks on equations and inequations indicate that in primary school mathematics teaching, students are mainly taught how to memorize certain procedures for solving tasks, and not how to understand the structure of mathematical tasks, how to analyze the tasks and understand the relationships between the values, which consequently does not enable the development of logical reasoning as one of the basic objectives of mathematics education (Matejević, 1994: 71). Therefore, it is very important to know what teaching methods should be used for teaching in these content areas in order to achieve the best possible results in terms of the quality of knowledge that all categories of students should acquire, i.e., teaching for all to learn.

Conclusion

In order to improve the quality of teaching, this study investigated whether the methodological approach of exemplary teaching has an impact on the quality of students' knowledge, i.e., whether it is efficient in achieving the learning objectives related to the algebraic content area in primary school mathematics teaching. Based on the results of the experimental research, we can conclude that: the teaching method used contributes to better student success in learning equations and inequations; the use of exemplary teaching has a positive impact on students who are at productive levels of knowledge, such as comprehension, operationalization and creative problem solving, while it does not have a positive impact on students whose knowledge is at the level of recognition; the students who make the most progress are those whose knowledge is at the level of creative problem solving in third grade and at the level of comprehension in fourth grade. In addition, by using the exemplary teaching method, students acquired a better quality of knowledge and were more successful in solving tasks than students who followed previously memorized paths in the traditional manner and did not have the opportunity to develop their own approaches (Hines, 2008).

From the results of this and several other studies (e.g., Bikić, Maričić, Pikula, 2016; Malinović-Jovanović & Janković, 2014; Malinović-Jovanović, 2015), it can be concluded that it is very important to choose appropriate teaching methods when preparing and planning mathematics lessons. The methodological contribution of this work is therefore the creation of exemplary teaching models whose use in contemporary teaching enables students to find their own approach to mathematics thanks to analogical thinking, in contrast to the traditional approach in which students memorize certain task-solving processes. When applied in compliance with all necessary didactic requirements, exemplary teaching enables more effective learning, which is reflected in: students' ability to self-study (Milijević, 2003); logical reasoning and the use of analogies, cause-effect relationships and mathematical links between content (Duit, 1991); opportunities for differentiation of students' learning, with each student seeking and finding their own solution to the problem depending on their knowledge and abilities (Vilotijević & Vilotijević, 2008: 121). In addition, it enables the development of analogical reasoning as the ability to recognize and respond to appropriate structural similarities in objects whose surface features are not necessarily similar, which also play a role in children's cognitive repertoire as they learn about the world (Richland et al., 2004: 37).

In addition, the use of taxonomy to set teaching and educational objectives and to improve the teaching process with all groups of students offers the following possibilities: better and more efficient individualization of the learning process, which translates into better learning models for all groups of students; the use of criterion-referenced tests to assess students' level of performance against set curriculum objectives; the establishment of assessment criteria to assess and evaluate the knowledge, skills and habits of mind that students are expected to acquire at a given educational level, etc.

Therefore, the results of this research could lead to changes in the presentation of teaching content, the evaluation of the efficiency of the educational process, a more efficient individualization of the learning process, etc.

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КОЛИКО ЈЕ ЕФИКАСНА ЕГЗЕМПЛАРНА НАСТАВА У ИЗУЧАВАЊУ ЈЕДНАЧИНА И НЕЈЕДНАЧИНА У ТРЕЋЕМ И ЧЕТВРТОМ РАЗРЕДУ ОСНОВНЕ ШКОЛЕ?

РЕЗИМЕ

У раду се испитује ефикасност примене егземпларне наставе у изучавању садржаја о једначинама и неједначинама у оквиру првог циклуса образовања у основној школи. У испитивању ефикасности пошло се од исхода учења о једначинама и неједначинама датим у програму математике за трећи и четврти разред основне школе и таксономског модела операционализације циљева и задатака наставе математике у оквиру когнитивнивног домена конструисаног за потребе истраживања. Постигнућа ученика испитивана су у складу са нивоима знања по квалитету дефинисаним у таксономском моделу и то: препознавање, репродукција, разумевање, операционализација и креативно решавање проблема. Резултати експерименталног истраживања спроведеног на узорку од 100 ученика показују да је примена егземпларне наставе у изучавању ових садржаја позитивно утицала на квалитет знања ученика који поседују знање на нивоу репродукције, схватања, операционализације и креативног решавања проблема, док није имала утицаја на ученике чије је знање на нивоу препознавања. Резултати истраживања могли би да доведу до промена у презентацији наставних садржаја, евалуацији ефикасности образовног процеса, ефикаснијој индивидуализацији процеса учења итд. Осим тога, методички допринос рада огледа се у креирању модела егземпларне наставе чија употреба у савременој настави, захваљујући аналогном закључивању омогућава ученицима да креирају сопствени приступ математици.

Кључне речи: *егземпларна настава*, *ефикасност*, *једначине*, *неједначине*, *таксономија*, *критеријски тестови.*