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DILEMMAS IN SOLVING ONE TYPE OF EXPONENTIAL EQUATIONS IN MATHEMATICS TEACHING

Abstract: In this paper we considered the problem of solving equations of the form $f(x)^{g(x)} = f(x)^{h(x)}$, ie. exponential equations in which the unknown is both in the base and the exponent. We analysed how solving these so-called power-exponential equations shown in textbooks and collections of math problems for the second grade of vocational schools and grammar schools, as well as in some collections of math problems intended for the preparation of the entrance exam at technical faculties in the Republic of Serbia. We realised that in these textbooks there are two approaches to these equations, which results in obtaining different sets of solutions. Namely, in some collections of math problems the starting point is the fact that the real solutions to an equation are all real numbers for which the given equation becomes an exact equality, including those real numbers for which the base $f(x)$ is a negative number, while in others the possibility of a negative base is excluded due to the area of definition of the function $y = f(x)^{g(x)}$. Obtaining different sets of solutions is a problem for both students and teachers because they do not know which approach is correct and which set of solutions is correct. In the paper, we also indicated a possible solution to this problem.

Keywords: *exponential functions, exponential equations and inequalities, power-exponential equations, the solution to the equation*.

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Introduction

Exponential functions, equations and inequalities are part of the compulsory curriculum of mathematics teaching in grammar schools and vocational high schools. In the light of the current educational agenda (the applicability of mathematical contents in everyday life), learning about exponential functions, unlike many other mathematical contents intended for high school students, is a content whose application is visible all around us and present in many spheres of human life. Exponential functions are used to model situations from everyday life such as demographic trends of the population, interest rates on loans, growth of the Internet traffic, radioactive decay, growth of an epidemic (or to overemphasise the danger: "In Serbia, the number of people infected with the corona virus threatens to increase exponentially", said immunologist Srdja Janković, RTS News, 26 October 2020). Therefore, they are widely represented in natural and social sciences, they appear in physics, engineering, mathematical biology, forensics, economics. According to the mathematician V. Rudin, considering the frequency of the appearance of exponential functions in mathematics, and especially in applied mathematics, these are "the most important functions in mathematics" (Rudin, 1987).

However, we must note that most "populations in nature do not grow exponentially or this growth lasts for a very short time. If populations in nature grew exponentially, even populations that reproduce very slowly would reach enormous numbers and would literally cover the Earth in a realtively short period of time" (Roguljić, Mišura, Baras, 2013).

The introduction of exponential functions in the teaching of mathematics, in addition to emphasising its presence in natural and social sciences, can also be accompanied by various historical stories and interesting facts such as the legend of the origin of chess. The Indian Emperor Shirham, delighted by the beauty of this game, wanted to reward his subject Sissa, who, according to legend, is the creator of chess. Sissa wished that the king would give him 1 grain of wheat for the first square of the chessboard, 2 grains for the second, 4 (= 2^2) for the third, 8 (= 2^3) for the fourth and so on up to the 64th square, for which he asked 2^{63} grains of wheat. The emperor immediately agreed and ordered his servants to pay Sissa. They realised very quickly that the payment was impossible because they had to pay $1 + 2^1 + 2^2 + 2^3 + \cdots + 2^{63}$, ie. 18,446,744,073,709,551,615 grains of wheat. For the sake of comparison, one cubic metre of wheat holds approximately 15,000,000 grains, which means that Sissa should have received 12,000 km^3 of grain. If a warehouse with the base 4×10 metres was built, its height would be 300,000,000 kilometres, which is equal to twice the distance from the Earth to the Sun.

A sequence of numbers like the previous one $(1, 2^1, 2^2, 2^3, \dots, 2^{63})$ where each subsequent number is obtained when the previous one is multiplied by the same factor is called a geometric sequence and it describes exponential growth. A mathematical function that Sissa apparently knew was the exponential function.

We can assume that many students, just like Emperor Shirham, would think that Sissa was a modest man who only wished for a symbolic reward, but after this chess-mathematical story we believe that everyone will understand the power of the exponential function.

Today we know that Archimedes (around 287–212BC) in his work *Hourglass* estimated that 10^{63} grains of sand would be needed to fill the entire universe (Kurepa, 1979):

> "There are those [...] who think that the number of grains of sand is infinite [...]. There are also those who do not think it is infinite, but that there is not a large enough number [...]. But I will try to show you numbers that not only exceed the quantity of sand equal to that of the filled Earth ... but also the quantity equal in size to the universe" (Sagan, 1999).

Although Archimedes is credited with the first knowledge about the exponential numbers as well as the discovery of the rule that $10^m \cdot 10^n = 10^{m+n}$, it is important to highlight two names. In the 16th century, Michael Stiefel introduced the word exponent in mathematics in his work *Arithmetica integra*, and René Descartes is responsible for the present notation of exponential numbers (*La Géométrie* 1637) (Kurepa, 1979).

About exponential functions, equations and inequalities

Students often have dilemmas or wrongly acquired knowledge regarding mathematical content. For example, it is not uncommon for students to be confused in their answers such as whether $\sqrt{4} = \pm 2$ or whether $\sqrt{4} = 2$, or whether $\pi = 3{,}14$ (approximately) or whether perhaps $\pi = 180$ (this dilemma occurs in students after a few hours of processing trigonometric contents), whether an inequality can or cannot be multiplied by an unknown number, etc. However, a bigger problem arises when teachers themselves have doubts or dilemmas about some content they need to present. This seems almost impossible when it comes to teaching mathematics at a high school level because mathematical literature is more accessible today than it used to be (Internet searches included), so it is expected that the answer to any theoretical question concerning mathematical issues can be found there. But, the problem is that there are such questions and problems whose answers and results differ from a collection of math problems to a collection, from a textbook to a textbook.

In this paper we will point out one such problem and consider some of its most important aspects. The problem is solving one type of exponential equations and inequalities.

We will first define exponential functions because they are necessary both for understanding and solving exponential equations and inequalities.

The function $f: R \to R^+$ is defined by $f(x) = a^x$, where $a > 0$ and $a \neq 1$ is called an exponential function (because the unknown x is in the exponent (the power)). The area of definition of exponential functions is all real numbers. If $a > 1$ the exponential function is increasing (Figure 1), and for $0 < a < 1$ the exponential function is decreasing (Figure 2).

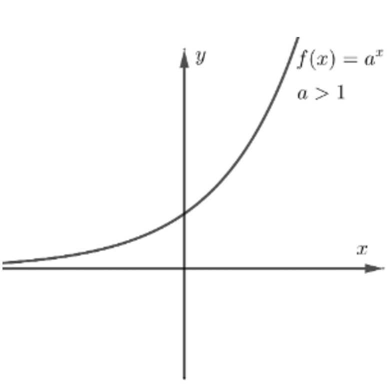

Figure 1.
Increasing exponential function

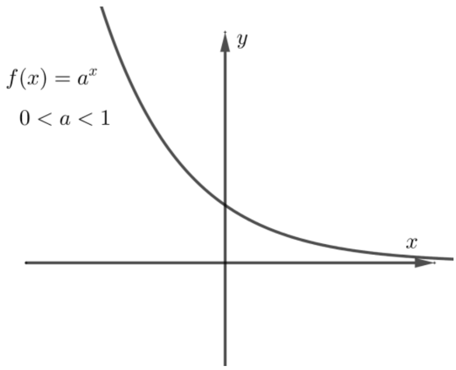

Figure 2.
Decreasing exponential function

The function $f(x) = 1^x$ is also a function, but not an exponential one but a constant function because $1^x = 1$.

Note that for $a \leq 0$ the exponential function is not defined. This is because it is not possible to exponentiate negative numbers with rational numbers with even denominators.

Exponential equations and inequalities are defined using properties of exponential functions. Exponential equations and inequalities are the equations and inequalities where the unknown is also in the exponent (the power).

Examples of exponential equations are $2^x = 32$, $\left(\sqrt{5}\right)^{x^2-6} = \left(\sqrt{5}\right)^x$, $x + \left(\frac{1}{3}\right)^x = 4$. With exponential equations and with equations of any other type, we are always interested in what the *solution* to the observed equation is. Given that the real solution to an equation is every real number for which that equation becomes an exact equality we conclude that the solution to the equation $2^x = 32$ is number 5 because

$2^5 = 32$. Similirly, the solutions to the equation $\left(\sqrt{5}\right)^{x^2-6} = \left(\sqrt{5}\right)^x$ are numbers -2 and 3 because $\left(\sqrt{5}\right)^{(-2)^2-6} = \left(\sqrt{5}\right)^{-2}$ and $\left(\sqrt{5}\right)^{3^2-6} = \left(\sqrt{5}\right)^3$ are exact equalities.

Exponential equations are most often reduced to the form $a^{g(x)} = a^{h(x)}$, and considering the bijectivity of the exponential function the following equivalence applies:

$$a^{g(x)} = a^{h(x)} \Leftrightarrow g(x) = h(x)$$

We applied the above-mentioned equivalence when determining the solutions to the previous two equations. However, when it comes to the third equation we cannot obtain its exact solutions but only the approximate ones (by some numerical or graphical methods). Equations such as $x + \left(\frac{1}{3}\right)^x = 4$ rarely appear in high school collections of math problems, but somewhat more often in entrance exams for admission to technical faculties, where the specific solutions to the equation are not sought but a number of solutions that we usually obtain by the graphical method.

In this case we would construct the functions $f(x) = \left(\frac{1}{3}\right)^x$ and $g(x) = 4 - x$ and determine the number of intersection points from the graph, which would also be the number of solutions to the given equation (in this case the solutions are the first coordinates of the intersection points) or we would determine the number of zeros of the function $f(x) = x + \left(\frac{1}{3}\right)^x - 4$ from the graph. The graph of the function shows that the function $f(x) = x + \left(\frac{1}{3}\right)^x - 4$ has two zeros, ie. that the equation $x + \left(\frac{1}{3}\right)^x = 4$ has two solutions (Figure 3).

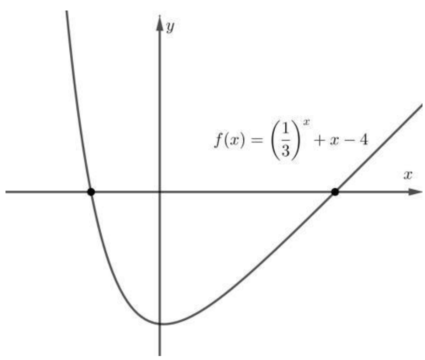

Figure 3. Graph of the function $f(x) = x + \left(\frac{1}{3}\right)^x - 4$

Examples of exponential inequalities are $2^{x^2-2x} < 8$ or $\left(\frac{3}{4}\right)^{6x+10-x^2} > \frac{27}{64}$. To solve an exponential inequality means to find all real numbers for which the given inequality becomes an exact inequality. When solving exponential inequalities we use the monotony of exponential functions, so: for $a > 1$, $a^{f(x)} > a^{g(x)}$ if and only if $f(x) > g(x)$; for $0 < a < 1$, $a^{f(x)} > a^{g(x)}$ if and only if $f(x) < g(x)$. Using the above-mentioned property, we conclude by elementary calculus that the solution to the first of the inequalities previously mentioned are all real numbers belonging to the interval $(-1,3)$, and that the solution to the second inequality is the numbers from the interval $(-\infty,-1) \cup (7,+\infty)$.

The problem and the research results

The problems we will deal with in this paper are some non-standard exponential equations that certain textbook authors (rightly) call *power-exponential equations*.[1] These are equations of the form $f(x)^{g(x)} = f(x)^{h(x)}$. Note that in these equations the unknown is in the base as well as in the exponent. Power equations in which the unknown is in the base, and a real number is in the exponent are generally called power equations ($x^2, x^3, x^{\frac{1}{2}}, x^{\frac{1}{3}}$...). So, the observed equations are in the form of power where the base is unknown (which means they are power equations) and the exponent is unknown (which means they are exponential equations), hence the name *power-exponential equations*. Although the representation of these equations in high school collections of math problems is not large (1–5 exercises), solving them turns out to be a huge puzzler that poses dilemmas for us and leaves us without an answer that we can safely stand behind.

We analysed how the equations of the form $f(x)^{g(x)} = f(x)^{h(x)}$ were solved in the current collections of math problems for the second grade of grammar schools as well as in the collections of math problems which high school students use to prepare for the mathematics entrance exams. The subjects of our research are the following 10 current textbooks and collections of math problems: Analysis with algebra (textbook with exercises for the 2nd grade of Mathematical Grammar School), authors: Z. Kadelburg, V. Mićić, S. Ognjanović (Krug, Belgrade, 2014); Analysis with algebra (Collection of exercises and problems with key 1&2), authors: D. Tošić, M. Albijanić (Zavod za udžbenike, Belgrade 2017); Collection of math problems with key 2, author V. Bogoslavov (Zavod za udžbenike, Belgrade 2014); Mathematics (Collection of problems and tests for the 2nd grade of grammar and vocational schools), authors: S. Ognjanović, Ž. Ivanović (Krug, Belgrade 2021); Collection of problems for the 2nd grade of high schools with key, authors: D. Georgijević,

[1] Collection of math problems for the second grade of high schools with key by D. Georgijević and M. Obradović (Matematiskop, Belgrade 2004).

M. Obradović (Matematiskop, Belgrade 2004); Mathematics 4+ (Problems from the entrance exams at the University of Belgrade 2003–2015 with key), author Dj. Krtinić (Krug, Belgrade 2016); Mathematics 4+ (Problems from the entrance exams at the University of Belgrade 2014–2020 with key), author Dj. Krtinić (Krug, Belgrade 2020); Collection of problems from the entrance exams in Mathematics from 1995 to 2010 with key, authors: Dj. Takač, D. Mašulović (Faculty of Sciences, Novi Sad, 2011); Mathematics for the entrance exams at technical faculties and faculties of sciences, authors: D. Djorić, Dj. Jovanov, R. Lazović (Faculty of Organisational Sciences, Belgrade 2016); Collection of math tests for the entrance exam at the Faculty of Electrical Engineering in Belgrade, authors: N. Cakić, V. Bečejac (N. Cakić, Belgrade 2015); Methodical collection of problems for taking exams in mathematics with key and theory review for admission to technical faculties and faculties of sciences, author M. Jovanović (Akademska misao, M. Jovanović, Belgrade 2021). Note that all the above-mentioned collections also have newer editions, but the problems that are the subject of our study have not changed.

Two different approaches to solving the above-mentioned equations were observed, which in itself would not be significant if different sets of solutions did not result from them.

In his book “Methodical collection of problems for taking exams in mathematics with key and theory review for admission to technical faculties and faculties of sciences”, the author M. Jovanović tackles this problem by starting from the following equivalence:

$$f(x)^{g(x)} = f(x)^{h(x)} \Leftrightarrow (f(x) = 1) \vee (f(x) = 0 \wedge g(x) \neq 0 \wedge h(x) \neq 0) \vee (f(x) \neq 0 \wedge g(x) = h(x))$$

Let us see how this applies on an example from the above-mentioned collection of problems.

The author solves the equation $(x-3)^{x^2-x} = (x-3)^2$ in the following way:

$$(x-3)^{x^2-x} = (x-3)^2 \Leftrightarrow (x-3=1) \vee (x-3=0 \wedge x^2-x \neq 0) \vee (x-3 \neq 0 \wedge x^2-x=2) \Leftrightarrow x=4 \vee x=3 \vee x=2 \vee x=-1$$

So, M. Jovanović concludes that the set of solutions to the given equation is $\{-1, 2, 3, 4\}$. By substituting these numbers into the given equation in all four cases, we will get exact equalities, which, according to the definition of the solution to an equation, leads us to the conclusion that we have solved the equation correctly.

We have a similar consideration, with less precise conditions, in V. Bogoslavov in the collection of problems "Collection of Problems in Mathematics 2 with key", which is traditionally used by students in the Republic of Serbia in math classes. In this collection of problems, the author states the problem $2(x+1)(2x+1)^x-(x-1)^x=(2x+1)^{x+1}$ and comes up with a set of solutions: $\{-2,0\}$.

This approach is also applied by Dj. Golubović in his well-known online lectures (https://youtu.be/wbcmz-oHoCY?si=0BblpyeKf_VIOcM), where he calls them *unusual* exponential equations, and the authors of the study "About a system of equations" think in a very similar way, with the only difference that they deal with the equation of the form $f(x)^{g(x)}=1,(x\in R)$. In the above-mentioned paper it is stated that the equations of this type are solved by differentiating the following cases (Bombardelli, Ilišević, Šipuš, 2005):

$g(x)=0, f(x)$ is any real number;

$f(x)=1$, $g(x)$ is any real number

$f(x)=-1, g(x)$ is an even integer.

Note that (except that the authors omitted $f(x)\neq 0$ in the first case) this solution of the equation is essentially the same as the previous one.

Let us look now at a different way of thinking. The equation $(x-3)^{x^2-x}=(x-3)^2$, which can be found in M. Jovanović's collection of exercises "Mathematics (A collection of exercises and tests for th 2nd grade of grammar and vocational schools)" by S. Ognjanović and Ž. Ivanović. In solving it, these authors, given that the exponential function is defined only for the posivite values of the base, proceed from the following equivalence:

$$f(x)^{g(x)}=f(x)^{h(x)}\Leftrightarrow (f(x)=1)\vee(f(x)>0\wedge g(x)=h(x))$$

Кад се ова еквиваленција примени на наш задатак добијамо:

$$(x-3)^{x^2-x}=(x-3)^2\Leftrightarrow(x-3=1)\vee(x-3>0\wedge x^2-x=2)$$
$$\Leftrightarrow x=4\vee(x-3>0\wedge(x=-1\vee x=2))$$

Since numbers -1 and 2 do not satisfy the condition that they are greater than 3, it follows that the only solution of this equation is number 4.

The justification of this procedure can be verified by constructing the function $y=(x-3)^{x^2-x}-(x-3)^2$ and realising that number 4 is the only solution to this equation. (Figure 4)

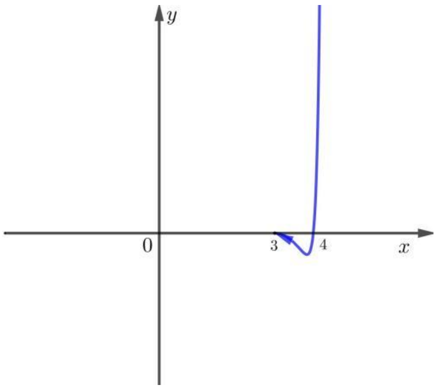

Figure 4. Graph of the function $y = (x-3)^{x^2-x} - (x-3)^2$

In the textbook Analysis with algebra (textbook with a collection of exercises for the 2nd grade of the Mathematical Grammar School), its authors Z. Kadelburg, V. Mićić and S. Ognjanović state: “When solving exponential equations we will always assume that the base of a power is positive (although a power, as we know, is defined in some special cases for negative bases as well)”. The authors, after the aforementioned remark, solve all the equations of the form, $f(x)^{g(x)} = f(x)^{h(x)}$ assuming that the base is $f(x) > 0$. D. Georgijević and M. Obradović solve the problem in the same way (A collection of exercises for the 2nd grade of high schools) and unlike other authors who do not name them, call these equations *power-exponential* equations. In the collections of exercises for the entrance exam in 4+ (Exercises from the entrance exams at the University of Belgrade 2003–2015), (as well as from 2014–2020), Mathematics for the entrance exam at technical faculties and faculties of sciences, power-exponential equations are also tackled with the condition that the base is $f(x) > 0$, but the authors of the last collection of exercises mentioned state the reason for such an approach (they ”do not belong to the area of definition”).

Finally, in the “Collection of exercises from entrance exams in Mathematics from 1995 to 2015 with key” we find the power-exponential equation $x^{x^2-5x+8} = x^2$, which the authors solve by stating that they will observe the case $x \geq 0$ and at the same time they get one less solution compared to the first way of solving the problem (they do not get the solution -1), and one more solution compared to the second way of solving the problem (solution 0). Note that the sentence “We will observe the case when $x \geq 0$” suggests that there may be another case, but it is not stated, nor is there an explanation of why we are observing the specified case. The same condition is stated in the Collection of tests in mathematics for the entrance exam for admission to the Faculty of Electrical Engineering in Belgrade, with the only difference that, in addition to the stated condition, the case that the base is equal to 0 is omitted in the final solution.

Let us note that the power-exponential equation is also found in Analysis with Algebra (Collection of Exercises and Problems 1&2), but there in the text of the problem, the authors emphasise that solutions are sought from the interval $(0, +\infty)$, so we have no doubts about further resolution procedure.

The above-mentioned exercises and the way of solving them best illustrate the problem here and open up a series of questions. How is it possible that different ways of looking at the equation and different procedures lead to different sets of solutions? Is the number for which an equation becomes an exact equality always the solution to that equation? Are all solutions to the equation also zeros of the corresponding function? Which of these actions is incorrect and why?

In the listed textbooks and collections of problems, the authors opt for one of the first two procedures, without questioning them, and we can only assume that students who use different collections of exercises remain confused by this problem.

In his work "Exponential equations" in the mathematics and computing magazine "Tangenta", which is intended for high school students, A. Jegorov makes the following comment about the solution to the equation $(x-3)^x = 3-x$: "For $x < 3$, the base of the power is negative and therefore such x should not be considered, although we can see that by direct inclusion of $x = 2$ into the equation the exact equality is obtained $(2-3)^2 = 3-2$ and that is why number 2 is also the solution of the equation" (Jegorov, 1996). As we can see, in the first part of the sentence, the author recommends not considering the negative base, and then, however, accepts it as a solution (for $x = 2$ we get the negative base -1). This sentence may illustrate the best the contradiction accompanying this type of equation.

We can notice that the power-exponential equations in all the above-mentioned books are also within the system of equations, but that, in most cases, within the problem itself, the condition that the base is positive is emphasised or it can be concluded on the basis of the formulation of the problems.

When it comes to power-exponential inequalities, all of the authors previously mentioned observe two cases (base between 0 and 1 or base greater than 1) – which is in agreement with another approach to solving the above-mentioned equations:

$$f(x)^{g(x)} < f(x)^{h(x)}$$
$$\Leftrightarrow (0 < f(x) < 1 \land g(x) > h(x)) \lor (f(x) > 1 \land g(x) < h(x))$$

Therefore, in all observed textbooks, when it comes to exponential inequalities where both the base and the exponent are unknown, the authors do not consider the possibility of a negative base. The only exception we notice is in an online course where the author solves a power-exponential inequality by a procedure in which the basis can be negative and concludes that in that case the inequality has many solutions that cannot be determined. (https://youtu.be/wbcmz-oHoCY?si=0BblpyeKf_-VIOcM).

Unlike standard exponential equations, power-exponential equations are not everywhere around us and we will not encounter them in everyday life. However, the fact that one cannot clearly see the direct applicability of these equations does not diminish their importance. Let us remember the sophists who attached an educational role to mathematical knowledge which cannot be practically applied, and considered it an excellent means of "formal schooling of reason" (Erić, 2023).

Conclusion

The problem discussed in the paper is not standard for teaching mathematics. At the school level, it is expected that everything is perfectly clear, that various dilemmas and uncertainties such as the one we talked about are meant for dealing with more "serious" mathematics. However, this is not always the case. The question arises whether and to what extent the different understanding of solutions of power-exponential equations confuses students and makes them insecure in mathematics classes or, on the contrary, inspires them. Note that power-exponential equations are content intended for students who already show a remarkable interest and inclination towards mathematics, so such dilemmas can stimulate their curiosity and motivation for further and deeper study of mathematics, rather than scare them. We also believe that teachers should not "hide" this problem, but analyse it together with the students.

The attitude of Davis (Philip J. Davis, 1923–2018) also goes in this direction, which in a broader view absolutely fits into today's understanding of science and education: "Since we are all consumers of mathematics, and since we are both beneficiaries as well as victims, all mathematizations ought to be opened up in the public forums where ideas are debated. These debates ought to begin in the secondary school" (Davis, 1987).

In order to have a unique approach to solving equations and inequalities of the power-exponential form, we believe that when solving the equations we should start from the equivalence:

$$f(x)^{g(x)} = f(x)^{h(x)} \Leftrightarrow (f(x) = 1) \vee (f(x) > 0 \wedge g(x) = h(x))$$

which is justified by the fact that the solutions to the equation $f(x)^{g(x)} = f(x)^{h(x)}$ are actually zeros of the function $y = f(x)^{g(x)} - f(x)^{h(x)}$, as well as by solving the corresponding exponential inequalities. Note that the authors of the above-mentioned textbooks often solve such equations (inequalities) by logarithmisation, which is all the more reason to use the previous equivalence.

In order to remove all doubts, we believe that when setting a task, it is necessary to set a condition in the power-exponential equation as well as the inequality that the base is positive. In this way, we would remove the dilemmas related to the required set of solutions and the final outcome of the task, and through discussion with the students, we could consider the solutions to the equation without the given condition.

Finally, let us think about an interesting comment of Wheeler (David John Wheeler, 1927–2004) about the rules and laws of school mathematics: "But where do these rules come from? Are they like the rules of a game, or like the laws of nature, or like the commandments of God? Could they be different?… [these]… questions have been argued throughout the history of mathematics, and some of them continue to be argued today" (Bibby & Abraham, 1988).

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ДИЛЕМЕ У РЕШАВАЊУ ЈЕДНЕ ВРСТЕ ЕКСПОНЕНЦИЈАЛНИХ ЈЕДНАЧИНА У НАСТАВИ МАТЕМАТИКЕ

РЕЗИМЕ

У овом раду разматрали смо проблем решавања једначина облика $f(x)^{g(x)} = f(x)^{h(x)}$, односно експоненцијалних једначина у којима се непозната налази и у основи и у експоненту. Анализирали смо како је решавање ових тзв. степено-експоненцијалних једначина приказано у уџбеницима и збиркама задатака за други разред средњих школа и гимназија, као и у неким збиркама задатака намењених припреми пријеног испита на техничким факултетима у Републици Србији. Увидели смо да се у поменутим уџбеницима јављају два приступа овим једначинама што за последицу има добијање различитих скупова решења. Наиме, у неким збиркама задатака се полази од чињенице да су реална решења једачине сви реални бројеви за које дата једначина постаје тачна једнакост, укључујући и оне реалне бројеве за које је основа $f(x)$ негативан број, док се у другим искључује могућност негативне основе због области дефинисаности функције $y = f(x)^{g(x)}$. Добијање различитих скупова решења представља проблем како ученицима тако и наставницима јер не знају који приступ је исправан и који скуп решења је тачан. У раду смо навели и могуће разрешење овог проблема.

Кључне речи: *експоненцијалне функције, експоненцијалне једначине и неједначине, степено-експоненцијалне једначине, решење једначине*.